A Comb Filter with Adaptive Notch Gain for Periodic Noise Reduction

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***Abstract*—A comb filter is used to eliminate a periodic noise signal from an observed signal. For extracting the desired signal, one of the most important factors of the comb filter is the notch gain which controls an elimination quantity of the observed signal at noise frequencies. Conventional comb filters employ a pre-designed notch gain under the assumption that the appropriate notch gain is known. Unfortunately, in many practical situations, the appropriate notch gain is unknown and often changes. In this paper, we propose a new comb filter with the adaptive notch gain to automatically achieve the appropriate notch gain. In the proposed method, we utilize an adaptive line enhancer (ALE) instead of the conventional notch gain multiplier. When the ALE completely estimates the periodic noise signal, the ALE's frequency response directly gives the appropriate notch gain. Simulation results show the effectiveness of the proposed adaptive comb filter.**

I. Introduction

In speech processing, image processing, biomedical signal processing, and many other signal processing fields, one of the most important issues is to extract a desired signal from an observed signal which contains noise. In some practical situations, noise has often periodicity, i.e., the noise consists of a fundamental frequency and its harmonic frequencies. Examples of such noise signals are a car engine noise, a hum noise in an electrocardiograph, a motor noise generated from a digital video camera, and so on.

To achieve the periodic noise reduction with the desired signal retention, it is useful to design the noise reduction filter by using the following two-step procedure. In the first step, we estimate the noise's spectral amplitude and its fundamental frequency. In the second step, we design the frequency gain of the noise reduction filter, which is called a filter gain, by using the estimated noise's spectral amplitude and the observed signal's one. Concretely, we design the filter gain at the harmonic frequencies of the periodic noise signal to eliminate only the noise's spectral amplitude. To retain the desired signal, the filter gain should be set to 1 at the frequencies consisted of only the desired signal.

The several noise reduction filters have been developed to automatically design the filter gain by themselves with the two-step procedure. The typical methods are an adaptive line enhancer (ALE) [1] and cascading adaptive notch filters [2], [3]. In the former, the frequency response of the filter depends on the number of filter order, e.g., the elimination bandwidth in which the filter gain is under $1/\sqrt{2}$ becomes narrow with the large number of filter order. Hence, when the frequency of the desired signal are close to the periodic noise signal, the ALE needs the large number of filter order to retain the desired signal. In this case, the convergence speed becomes slow and computational cost becomes large. In the latter, the narrow elimination bandwidth is achieved regardless of the number of filter order. In this method, the lowest null frequency depends on the number of filter order. Hence, when we eliminate the lower fundamental frequency of the periodic noise signal, we need the lager number of filter order. In this case, the same problems as the former occur.

To achieve the narrow elimination bandwidth without the high filter order, a comb filter is used because of its simple IIR structure [4]–[8]. The comb filter has zeros at equally spaced frequencies which are called notch frequencies. By aligning the notch frequencies to the harmonic frequencies of the periodic noise signal, the comb filter can eliminate noise completely. Additionally, the comb filter can retain the desired signal by appropriately adjusting the notch gain which denotes the filter gain of the comb filter for notch frequency. Although the conventional comb filter shown in [7], [8] can estimate the harmonic frequency and adjust the notch frequency, all the notch gains are fixed to a constant. Hence, the desired signals are deteriorated for each harmonic frequency. To adjust the notch gain individually, we previously derived a comb filter with the flexible notch gain [9]. The comb filter includes a linear phase FIR (LP-FIR) filter whose frequency response directly gives the notch gain. To extract the desired signal, we design the LP-FIR filter by using the appropriate notch gain which is designed according to the second step in the filter design procedure. Hence, we should estimate the appropriate notch gain. Furthermore, to reduce the complication of the filter design, it is required to automatically adjust the notch gain.

In this paper, we propose a comb filter with an adaptive notch gain for the periodic noise reduction. The adaptive notch gain is achieved by replacing the LP-FIR filter with the ALE. The ALE determines only the notch gain, i.e., it does not influence the elimination bandwidth. Hence, to achieve the narrow elimination band, the ALE in the proposed method do not required the high filter orders compared with the conventional ALE. To easily derive the proposed method, our discussion is based on the comb filter with the fixed notch frequency [4]. However, the proposed method can be easily

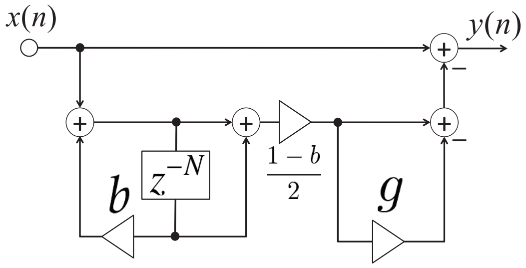


Fig. 1. Structure of basic comb filter.

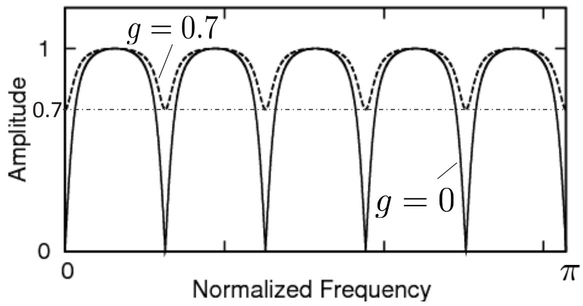


Fig. 2. Magnitude responses of basic comb filter.

extended to the adaptive notch frequency by utilizing the algorithm as shown in [7], [8]. Several computer simulations for the periodic noise reduction show the effectiveness of the proposed adaptive comb filter.

II. Conventional Comb Filter

In this section, we review a basic comb filter with a fixed notch frequency [4] and present a comb filter with a flexible notch gain previously proposed in [9].

A. Basic Comb Filter

The basic comb filter [4] is shown in Fig.1, where $x(n)$ is the input signal at time n, and $y(n)$ is the output signal of the comb filter. The transfer function of this filter is given by

$$C_{\mathrm{conv}}(z) = 1 - \frac{1-b}{2} \cdot \frac{1+z^{-N}}{1-bz^{-N}}(1-g), \tag{1}$$

where N is a natural number, b $(-1 < b < 1)$ is the elimination bandwidth parameter, and g $(0 \leq g \leq 1)$ is the notch gain parameter. The magnitude response of $C_{\mathrm{conv}}(z)$ is completely described by the notch frequency, the elimination bandwidth, and the notch gain. The m-th notch frequency is given by

$$\omega_m = \frac{2\pi m}{N}, \qquad m = 0, 1, \cdots, \lfloor N/2 \rfloor. \tag{2}$$

From the above equation, we see that the first notch frequency ω_1 is uniquely determined by N. The elimination bandwidth becomes narrow when increasing b toward to 1, and becomes wide when decreasing b toward to -1. In addition, the notch gain for ω_m is given by

$$|C_{\mathrm{conv}}(e^{j\omega_m})| = g. \tag{3}$$

We see that all notch gains are fixed to g. Figure 2 shows $|C_{\mathrm{conv}}(e^{j\omega})|$ for the notch gain parameters $g = 0, 0.7$, where we put $N = 10$, $b = 0.3$. Here, we see that the notch gains are fixed to a constant value g for every notch frequency.

B. Comb Filter with Flexible Notch Gain

In many practical situations, both spectral amplitude of the periodic noise signal and one of the desired signal are varied for each notch frequency. To eliminate the noise with the desired signal retention, we should adjust the notch gain for each notch frequency. Although, since all the notch gains of $C_{\mathrm{conv}}(z)$ are the same as shown in (3), the noise remains and the desired signal is deteriorated. To individually design the notch gain, a function $G(e^{j\omega})$ is used as the notch gain parameter instead of g, i.e., $|C_{\mathrm{conv}}(e^{j\omega_m})| = G(e^{j\omega_m})$. Here, $G(e^{j\omega})$ must be a real function satisfying $0 \leq G(e^{j\omega}) \leq 1$. The function $G(z)$ is required to achieve the appropriate notch gain given by

$$\hat{G}(e^{j\omega_m}) = 1 - \frac{|W(e^{j\omega_m})|}{|X(e^{j\omega_m})|}, \tag{4}$$

where $|W(e^{j\omega})|$ denotes the spectral amplitude of the periodic noise signal and $|X(e^{j\omega})|$ denotes one of the observed signal. When we represent $G(z)$ as a filter under the restriction of $0 \leq G(e^{j\omega}) \leq 1$, $G(z)$ becomes a linear phase FIR filter with the zero delay [10], i.e., the transfer function of $G(z)$ is given by

$$G(z) = \sum_{i=-P}^{P} \gamma_i z^{-i}, \tag{5}$$

where the filter order is put as $2P+1$, and γ_i is the filter coefficient satisfying

$$\gamma_i = \gamma_{-i}, \quad -P \leq i \leq P. \tag{6}$$

The filter coefficient γ_i can be calculated by the inverse discrete time Fourier transform of the appropriate notch gain given as

$$\gamma_i = \frac{1}{2\pi}\int_{-\pi}^{\pi} \hat{G}(e^{j\omega}) e^{ji\omega} d\omega \ , \quad -P \leq i \leq P. \tag{7}$$

By replacing g with $G(z)$ in (1), the transfer function of the comb filter with the flexible notch gain is obtained as

$$C_{\mathrm{g}}(z) = \big\{1 - Q(z)\big(1 - G(z)\big)\big\} z^{-P}, \tag{8}$$

$$Q(z) = \frac{1-b}{2}\frac{1+z^{-N}}{1-bz^{-N}}, \tag{9}$$

where P sample delay is introduced to satisfy causality. Figure 3 shows the structure of $C_{\mathrm{g}}(z)$.

A design example of the comb filter with the flexible notch gain is shown in Fig.4. Here, we put $N = 20$, $b = 0.7$, and $P = 20$ (the order of $G(z)$ is 41). In this figure, the dashed line denotes the designed notch gain $\hat{G}(e^{j\omega})$, and the solid line denotes the magnitude response of the comb filter $C_{\mathrm{g}}(e^{j\omega})$. We see from Fig.4 that $C_{\mathrm{g}}(e^{j\omega})$ has nulls along with $\hat{G}(e^{j\omega})$.

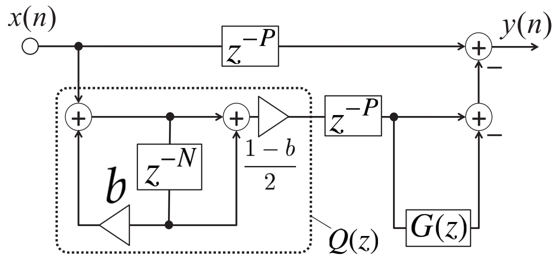


Fig. 3. Structure of comb filter with flexible notch gain.

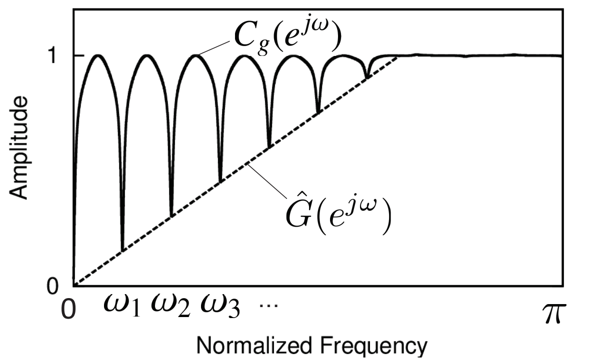


Fig. 4. Example of magnitude response of comb filter with flexible notch gain and designed notch gain.

III. Adaptive Comb Filter with Flexible Notch Gain

The comb filter shown in (8) works efficiently when the appropriate notch gain $\hat{G}(e^{j\omega})$ is known. However, in many cases, $\hat{G}(e^{j\omega})$ in (4) is unknown and changed with time, since the spectral amplitude of the observed signal $|X(e^{j\omega})|$ is often time-variant. Additionally, even so we can obtain $\hat{G}(e^{j\omega})$, it is hardly for the filter designer to design the comb filter for every time according to the time-variant $\hat{G}(e^{j\omega})$. To solve these problems, we propose a comb filter with the adaptive notch gain by using an ALE [1], under the assumption that the desired signal is a wideband random signal. The ALE is an adaptive FIR filter to automatically estimate the periodic noise signal by using the noise correlation. Additionally, the ALE adjusts its filter gain to eliminate the periodic noise signal. When the ALE completely estimates the periodic noise signal, the ALE's frequency response is corresponding to the appropriate notch gain for the harmonic frequencies of the noise. Hence, we can achieve an adaptive notch gain by utilizing the ALE instead of $G(z)$. The structure of the proposed comb filter is shown in Fig.5, where $u(n)$ is the input signal of $G(z)$, and $v(n)$ is the signal which is calculated by subtracting the output signal of $G(z)$ from $u(n)$. Here, the output signal $y(n)$ is given as

$$y(n) = x(n-P) - v(n) \tag{10}$$

$$= x(n-P) - \left\{ u(n) - \sum_{i=-P}^{P} \gamma_i(n) u(n-i) \right\}. \tag{11}$$

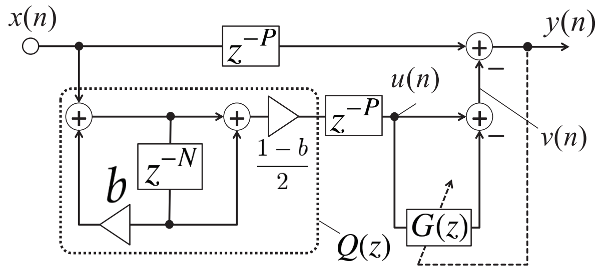
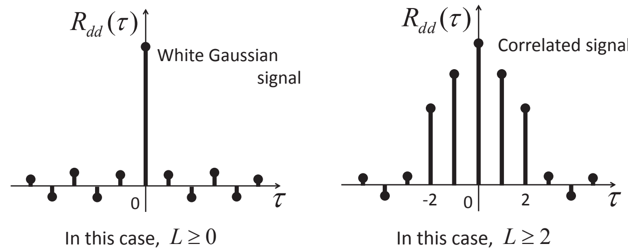


Fig. 5. Structure of adaptive comb filter with flexible notch gain.



(a) White Gaussian signal

(b) Correlated signal

Fig. 6. Autocorrelation of desired signal

Note that the comb filter satisfies causality, since $u(n)$ contains the P sample delay. We can obtain an uncorrelated signal from $y(n)$ by using an LMS algorithm [1]. In this case, the filter coefficient $\gamma_i(n)$ is updated by minimizing the mean-square of $y(n)$. According to the literature [1], we should remove the correlation between $x(n-P)$ and $v(n)$ to estimate the periodic signal appropriately. Since $u(n)$ includes $x(n-P)$, we should set the terms including $u(n)$ in (11) to 0 , i.e., we should set $\gamma_i(n)$ as

$$\gamma_i = \delta_i, \quad -L \leq i \leq L, \tag{12}$$

where L is an uncorrelated parameter satisfying $0 \leq L < P$, and δ_i is Kronecker's delta. The uncorrelated parameter L is introduced to retain the correlated components included in the desired signal. The parameter L is determined by the autocorrelation of the desired signal. Figures 6(a) and 6(b) show the setting examples of L with the white Gaussian desired signal and the correlated desired signal, respectively. Here, τ denotes a time-lag and $R_{dd}(\tau)$ denotes the autocorrelation of the desired signal. In the former, $R_{dd}(\tau)$ has the large value with only $\tau = 0$. In this case, we should set $L \geq 0$. In the latter, $R_{dd}(\tau)$ has the large value with $-2 \leq \tau \leq 2$, and then we should set $L \geq 2$. Based on the gradient method [1], we update the filter coefficients $\gamma_i(n)$ by

$$\gamma_i(n+1) = \begin{cases} \gamma_i(n) + \mu u(n-i) y(n), & L+1 \leq i \leq P \\ \delta_i, & -L \leq i < L+1 \\ \gamma_{-i}(n+1), & -P \leq i < -L \end{cases}, \tag{13}$$

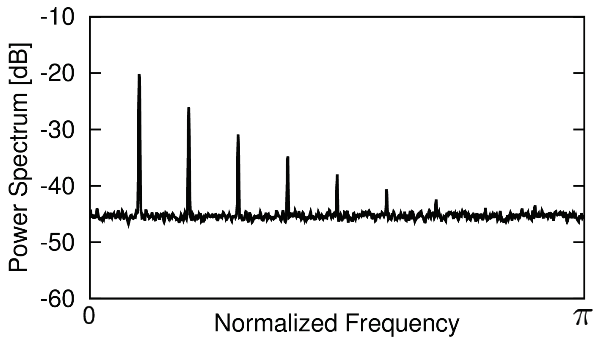


Fig. 7. Power spectrum of input signal.

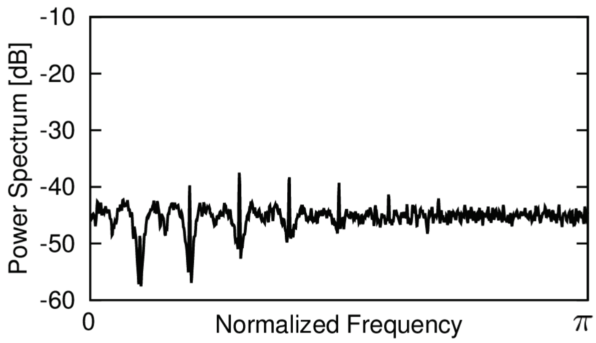


Fig. 8. Power spectral of output signal for ALE.

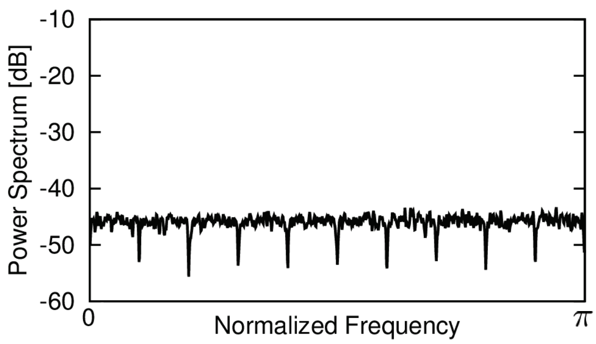


Fig. 9. Power spectral of output signal for conventional comb filter.

where μ is the step size parameter. Here, the middle equation follows (6), and the lower equation follows (12).

The transfer function of the conventional ALE is corresponding to the proposed comb filter by replacing $Q(z)$ in (8) with z^{-L}. The adaptation of the conventional ALE is also corresponding to (13) by setting $\gamma_i(n+1) = 0$ with $-P \leq i < -L$. Comparing the conventional ALE, the proposed method can achieve the narrow bandwidth without high orders as the ALE because of the IIR structure of $Q(z)$.

IV. Simulations

To confirm the effectiveness of the proposed adaptive comb filter, we carried out two types of simulations for the periodic noise reduction from the observed signal. In these simulations, the capability of the proposed method was compared with the conventional comb filter shown in (1) and the conventional ALE explained in section III. For the first simulation, we calculated the power spectrum of the filter outputs. Here, we used a white Gaussian signal as a desired signal, and used a periodic noise signal whose fundamental frequency is $2\pi/20$ with SNR= 0dB shown in Fig.7. Also, we set $N = 20$, $b = 0.7$, $g = 0$, $L = 0$, $\mu = 0.01$, and $P = 20$ (the order of $G(z)$ is 41). Figure 8, 9, and 10 show the simulation results with the ALE, the conventional comb filter, and the proposed comb filter, respectively. From Fig.8, we see that the periodic noise remains and the desired signal is deteriorated in the ALE. Also we see from Fig.9 that the conventional method eliminates both harmonic components of the desired signal and periodic noise. On the other hand, we see from Fig.10 that the proposed method achieves the periodic noise reduction and the desired signal retention, simultaneously. Especially, it is clear over $\pi/2$ frequency. Compared with the ALE under the same orders, the proposed method can also achieve the narrow elimination bandwidth.

For the second simulation, we calculated the improved SNR (ISNR) obtained by subtracting the input SNR from the output SNR, i.e., the ISNR is given by

$$\begin{aligned}\mathrm{ISNR} = &\log_{10}\frac{\sum_{k=0}^{K} d(k)^2}{\sum_{k=0}^{K}\left\{d(k)-y(k)\right\}^2}\\ &-\log_{10}\frac{\sum_{k=0}^{K} d(k)^2}{\sum_{k=0}^{K}\left\{d(k)-x(k)\right\}^2}\quad [\mathrm{dB}], \end{aligned} \tag{14}$$

where K is a signal length. In this simulation, we used two observed signals. For the first observed signal, we used a white Gaussian signal as a desired signal. For the second, as the desired signal, we used a colored signal whose $R_{dd}(\tau)$ has a large number with $-2 \leq \tau \leq 2$. We obtained the observed signals by mixing each desired signal and the periodic noise whose fundamental frequency is $2\pi/20$ with the input SNR from -10dB to 10dB. We set the uncorrelated parameter to $L = 0$ for the first observed signal, and $L = 2$ for the second observed signal. Other setting conditions for the comb filters were the same as the first simulation. Figure 11 shows the simulation results of the first desired signal with the three methods. Here, the horizontal axis denotes the input SNR and the vertical axis denotes the ISNR. We see from Fig.11 that the ISNR of the proposed comb filter is always higher than other conventional methods. Especially it is apparent at the low input SNR. Figure 12 shows the simulation results of the second desired signal with with the three methods. We see from this figure that the ISNR of the proposed method is also higher than other methods for every input SNR. As above

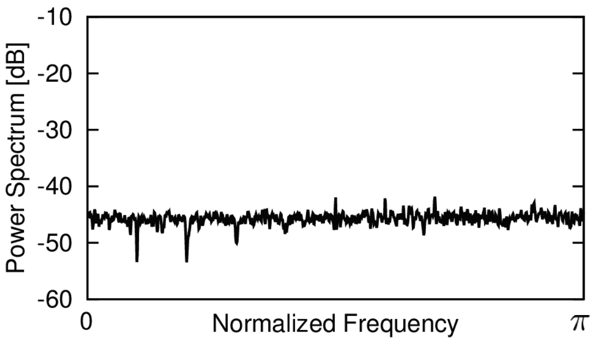


Fig. 10. Power spectral of output signal for proposed adaptive comb filter.

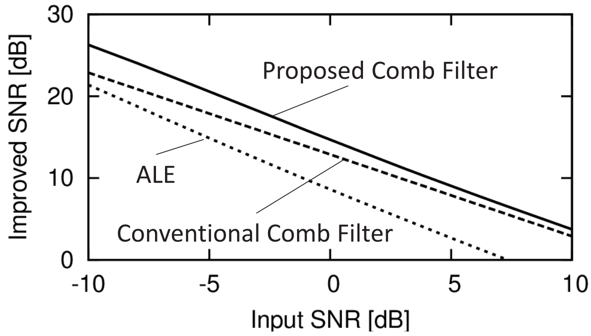


Fig. 11. Improved SNR for desired white Gaussian signal.

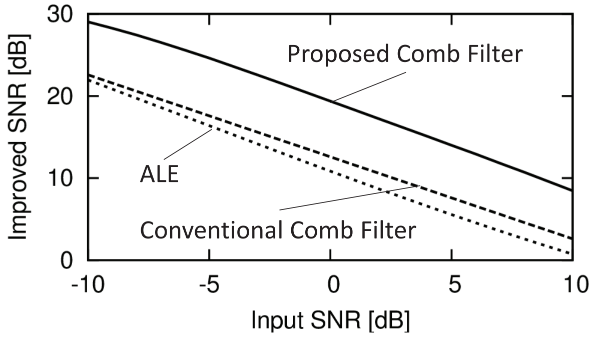


Fig. 12. Improved SNR for desired colored signal.

results, we see that the proposed comb filter can eliminate the periodic noise signal and retain the desired signal more effectively than other two conventional methods.

V. Conclusions

In this paper, we proposed a new comb filter with an adaptive notch gain for periodic noise reduction with desired signal retention. The adaptive notch gain is derived by replacing the FIR filter with an ALE in the conventional comb filter to update the notch gain coefficient. Simulation results showed the effectiveness of the proposed method.

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