Distributed Beamforming with Compressed Feedback in Time-Varying Cooperative Networks

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***Abstract*—In this paper, we investigate distributed beamforming with limited feedback for time varying cooperative network with multiple amplify-and-forward (AF) relays. With perfect channel state information, transmit beamforming has been shown to achieve significant diversity and coding gain in both MIMO or cooperative systems. However, it requires large amount of overhead for receiver to feed back channel information or beamforming coefficients, which makes it impractical. To perform transmit beamforming with limited feedback, the destination can choose the best codeword as beamforming vector from a predetermined codebook. In this work, we adopt the generalized Lloyd algorithm (GLA) to optimize codebook in terms of maximal average SNR. Furthermore, the feedback message can be compressed by exploiting temporal correlation of channel states. Specifically, We model channel states as a first-order finite-state Markov chain, and propose two compression methods according to the property of the transition probabilities among different channel states. Simulations show that distributed beamforming with compressed feedback performs closely to the case with infinite feedback.**

I. Introduction

Cooperative communications have drawn dramatic attention in recent years [1]–[3]. By allowing users cooperate in transmitting each other's messages, even the device with single antenna can attain spatial diversity gain. The cooperative partners serve as relays and mimic multi-input multi-output (MIMO) systems to assist transmission of source user. Thus, transmission technologies proposed in MIMO systems can be applied among relays, e.g., transmit beamforming, antenna selection or space-time coding. Among all transmission technologies, transmit beamforming results in full diversity gain and high coding gain since the signals through different paths can be coherently combined at the receiver. However, it costs large amount of overhead to exchange accurate and instantaneous channel information. With this regards, transmit beamforming with limited feedback have been proposed in multi-input single-output (MISO) systems [6] and MIMO systems[5]. In these works, vector quantization (VQ) has been adopted to allow finite-bit message representing complex beamforming vectors, and these methods includes Grassmannian line packing method[5] and generalized Lloyd algorithm (GLA)[7].

In the content of amplify and forward (AF) cooperative networks, transmit beamforming have been adopted to achieve diversity gain with perfect channel information known at all nodes [4]. However, exchange of channel information becomes costlier since the virtual antennas provided by cooperative partners are not co-located. Thus, transmit beamforming with limited feedback is more practical in cooperative systems. Koyuncu and Jing employed GLA to design codebook for vector quantization[7]. It shows that transmit beamforming with limited feedback can achieve full diversity gain and performance loss due to quantization errors is acceptable with a well-designed codebook. In this work, we consider transmit beamforming in time-varying cooperative channels. At any time instance, the destination selects the best beamforming vectors within a codebook based on current channel state, and sends the index of the chosen codeword to inform relays about the beamforming coefficients. We also adopt GLA to optimize the codebook in terms of maximal average SNR. The operation of vector quantization is to partition the space spanned by all realizations of channel states into 2^B subspace where each subspace contains channel realizations with the best beamforming vector being a specific codeword and B is number of feedback bits. When the channel varies slowly, it is highly possible that the codeword selected at previous instance is also the best candidate for current channel state. If the codewords which will be selected with higher possibilities simply belong to a small portion of the codebook, the destination can indicate the best codewords with fewer feedback bits. In this work, we model the transition among subspaces which correspond to respective codewords as a finite state Markov chain. By exploiting the information of transition probabilities, we propose two methods to further compress the feedback message. In other words, if we can compressed the feedback message according to the rate of channel variation, the cooperative system can employ the codebook with larger size to reduce quantization errors at the same cost on feedback message. Through computer simulations, we show that distributed beamforming with compressed feedback is able to achieve higher coding gain without introducing further overhead.

II. System Model

Consider a cooperative system with one source , L relays and one destination as shown in Fig.1. Assume that there is no direct link between source and destination due to severe path loss effect. The cooperative transmission takes two phases. In Phase I, source transmits a symbol x with unit power $\mathbf{E}[|x|^2] = 1$. The signal received at the relay ℓ is

$$y_\ell = \sqrt{P_s} f_\ell x + w_\ell, \quad \ell = 1, 2, ..., L, \tag{1}$$

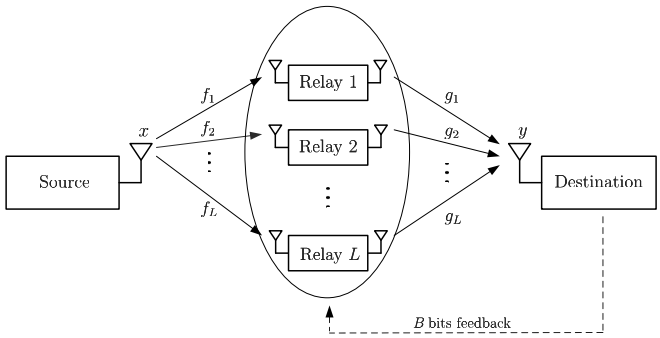

Fig. 1. System model

where P_s is transmission power of the source, f_ℓ is channel coefficient between source and the ℓ-th relay, and w_ℓ is the AWGN at the relay ℓ, which is a circularly symmetric Gaussian distributed with variance N_0. Assume that all channels are Rayleigh faded independently, and $f_\ell \sim \mathcal{CN}(0, \sigma_{f_\ell}^2)$.

During Phase II, each relay first normalizes the received signal y_ℓ and multiplies it with a beamforming coefficient before retransmission. The signal sent by the ℓ-th relay node is $x_\ell = \sqrt{P_r}\alpha_\ell\beta_\ell y_\ell$, where P_r is total transmit power of relays, β_ℓ is beamforming coefficient of the ℓ-th relay, and $\alpha_\ell \triangleq \sqrt{1/(P_s|f_\ell|^2+N_0)}$ is normalization factor. The set of beamforming coefficients $\{\beta_\ell\}$ must satisfy total power constraint at relays, i.e.,

$$\sum_{\ell=1}^{L} \mathbf{E}[|x_\ell|^2] = \sum_{\ell=1}^{L} |\beta_\ell|^2 \leq 1. \tag{2}$$

The signal received at the destination in Phase II is given by

$$y_d = \sum_{\ell=1}^{L} \sqrt{P_s P_r}\alpha_\ell\beta_\ell f_\ell g_\ell x + \sum_{\ell=1}^{L} \sqrt{P_r}\alpha_\ell\beta_\ell g_\ell w_\ell + w_d, \tag{3}$$

where $g_\ell \sim \mathcal{CN}(0, \sigma_{g_\ell}^2)$ is channel coefficient between the ℓ-th relay and the destination, and $w_d \sim \mathcal{CN}(0, N_0)$. We assume that the destination knows global CSI for data detection. The received SNR γ is

$$\begin{aligned}\gamma &= \frac{P_s}{N_0} \cdot \frac{\left|\sum_{\ell=1}^{L} \sqrt{P_r}\alpha_\ell\beta_\ell f_\ell g_\ell\right|^2}{1 + \sum_{\ell=1}^{L} P_r|\alpha_\ell|^2|\beta_\ell|^2|g_\ell|^2} \\ &= \frac{P_s}{N_0} \cdot \frac{|\mathbf{b}^H\mathbf{q}|^2}{1 + \mathbf{b}^H\mathbf{\Phi}\mathbf{\Phi}^H\mathbf{b}},\end{aligned} \tag{4}$$

where $\mathbf{b} \triangleq [\beta_1, \beta_2, \cdots, \beta_L]^T$ is a vector of beamforming coefficients, $\mathbf{\Phi} = \sqrt{P_r} \cdot \mathrm{diag}(\alpha_1 g_1, \alpha_2 g_2, \cdots, \alpha_L g_L)$, and $\mathbf{q} = \sqrt{P_s P_r} \cdot [\alpha_1 f_1 g_1, \alpha_2 f_2 g_2, \cdots, \alpha_L f_L g_L]^T$. The beamforming vector can be optimized to maximize received SNR under power constraint.

III. Distributed Beamforming with Limited feedback

When the instantaneous channel information is perfectly known at relays, the optimal beamforming vectors can be obtained by maximizing the value of received SNR [4]. However, availability of perfect channel information demands large amount of information exchange periodically. Alternatively, after performing channel estimation, the destination sends a feedback message with limited bits to inform the relays about beamforming vector $\mathbf{b}$. More specifically, assume that the feedback message contains B bits, which implies that the feedback message can indicate one of 2^B beamforming vectors $\mathbf{b}$. Let $\mathcal{B} = \{\mathbf{b}_i;\ i = 1, 2, \ldots, 2^B\}$ be a codebook with size 2^B, where each codeword is an L-dimensional complex vector with unit norm which corresponds to a candidate of beamforming vectors. The codebook is known at all nodes and can be constructed offline. Based on current channel state, the destination chooses the codeword with highest SNR and sends the index "i" to inform all relays. For conciseness, let $(\mathbf{f}, \mathbf{g})$ be a realization of channel status, where $\mathbf{f} = [f_1, f_2, \ldots, f_L]$ and $\mathbf{g} = [g_1, g_2, \ldots, g_L]$. The vector quantization (VQ) performed at the destination is to partition the space spanned by all channel realizations into 2^B subspaces $\mathcal{D}_i$, $i = 1, 2, \ldots, 2^B$

$$\mathcal{D}_i = \left\{(\mathbf{f}, \mathbf{g}) \in \mathbb{C}^{2L\times 1} : \frac{|\mathbf{b}_i^H\mathbf{q}|^2}{1+\mathbf{b}_i^H\mathbf{\Phi}\mathbf{\Phi}^H\mathbf{b}_i} \geq \frac{|\mathbf{b}_j^H\mathbf{q}|^2}{1+\mathbf{b}_j^H\mathbf{\Phi}\mathbf{\Phi}^H\mathbf{b}_j}, \forall j\right\}. \tag{5}$$

That is, if the codeword $\mathbf{b}_i$ is the one that maximizes the received SNR under current channel state, then current channel state $(\mathbf{f}, \mathbf{g})$ belongs to the subspace $\mathcal{D}_i$ $(i = 1, 2, \ldots, 2^B)$.

Next step is to construct an efficient codebook. In this work, we adopt GLA to design codebook elements in terms of maximum average SNR [7]. By iteratively partitioning the channel space for a given codebook and optimizing beamforming vector for channel realizations within each subspace, GLA can guarantee convergence of a codebook. The algorithm of constructing the codebook is described as follows:

1) Initialization: Randomly generate an initial codebook $\mathcal{B}^{(0)} = \{\mathbf{b}_i^{(0)};\ i = 1, 2, \ldots, 2^B\}$, and manipulate corresponding received SNR $\{\gamma_i^{(t)}\}$. Set $t = 0$.
2) Iterations:
   a) For a given codebook $\mathcal{B}^{(t)}$, partition the space of all channel states into subspaces $\mathcal{D}_i^{(t)}$ using (5).
   b) For each subspace $\mathcal{D}_i^{(t)}$, find an optimal vector of beamforming coefficients that maximizes the average SNR,
   $$\mathbf{b}_i^{(t+1)} = \arg\max_{\|\mathbf{b}\|^2 \leq 1} \mathrm{E}\left[\frac{|\mathbf{b}^H\mathbf{q}|^2}{1+\mathbf{b}^H\mathbf{\Phi}\mathbf{\Phi}^H\mathbf{b}} \middle| (\mathbf{f}, \mathbf{g}) \in \mathcal{D}_i^{(t)}\right].$$
   Update the optimal beamforming vector as the new codebook element $\mathbf{b}_i^{(t+1)}$.
   c) Compute the received SNR $\gamma_i^{(t+1)}$ corresponding to updated codewords $\mathbf{b}_i^{(t+1)}$. If the maximum increase in received SNR is below ϵ, i.e.,
   $$\max_i |\gamma_i^{(t+1)} - \gamma_i^{(t)}| < \epsilon, \tag{6}$$

where $\epsilon > 0$ is an arbitrary threshold of convergence, terminates the iteration. Otherwise, $t=t+1$ and go to Step 2a.

IV. Compression of Feedback Messages in Time-Varying Channels

In most cases, wireless channel is time varying and temporal correlation of channel coefficient depends on the relative velocities among transmitter, receiver and scatters. If channel varies slowly, channel states of two adjacent time-slots are highly correlated. In this case, we will exploit the temporal correlation of channel states to further compress the feedback message.

In this section, we consider slow fading environment and The time-varying channels can be modeled as first-order Gaussian-Markov processes, and channel coefficients of the link between source and the ℓ-th relay and the link between the ℓ-th relay and destination can be expressed by

$$f_\ell(t) = \rho \cdot f_\ell(t-1) + \sqrt{1-\rho^2} w_{f_\ell}(t), \tag{7}$$

$$g_\ell(t) = \rho \cdot g_\ell(t-1) + \sqrt{1-\rho^2} w_{g_\ell}(t), \tag{8}$$

where $\rho \in [0,1]$ is an autocorrelation coefficient of time-varying channels, $w_{f_\ell}(t)$ and $w_{g_\ell}(t)$ are complex white Gaussian random sequences with power spectral density $\sigma^2_{f_\ell}$ and $\sigma^2_{g_\ell}$, respectively. The random sequences $w_{f_\ell}(t)$ and $w_{g_\ell}(t)$ are statistically independent of $f_\ell(t-1)$ and $g_\ell(t-1)$. The value of autocorrelation coefficient ρ is determined by the rate of channel variation. If the channels vary slowly, the value of ρ is very close to 1. On the contrary, the value of ρ decreases if the channels vary quickly.

Under the assumption that the value of ρ is very close to 1, it is highly possible that the codewords which serves as the beamforming coefficients to maximize the received SNR at two consecutive instants are the same. In other words, the channel states of two consecutive instants are very likely to belong the same subspace corresponding to some codeword. Without loss of generality, the transition among subspaces which channel states belongs to can be modeled as a first-order finite-state Markov chain with state space $\{\mathcal{S}_1, \mathcal{S}_2, \cdots, \mathcal{S}_{2^B}\}$. Let $\mathcal{S}_j$ be the state corresponding to case that $(\mathbf{f}(t), \mathbf{g}(t))$ lies in the subspace $\mathcal{D}_j$, and $\mathbf{P}$ be a $2^B \times 2^B$ transition matrix with the (i,j)-th element being transition probability from state $\mathcal{S}_j$ to state $\mathcal{S}_i$, i.e.,

$$[\mathbf{P}]_{ij} = P\{(\mathbf{f}(t), \mathbf{g}(t)) \in \mathcal{D}_i | (\mathbf{f}(t-1), \mathbf{g}(t-1)) \in \mathcal{D}_j\}. \tag{9}$$

When the autocorrelation ρ is close to 1 (i.e., the case of slow fading channel), most elements of the transition probability matrices are very small. In other words, given the previous state $\mathcal{S}_j$, the codewords which will be selected probably belongs to a minor portion of the codebook. Since relays are aware of previous state, the destination can indicate current state with fewer bits of message if we simply ignore the states with negligible transition probabilities. In the following section, we adopt two compression methods to reduce the number of feedback bits in slowly time-varying channels.

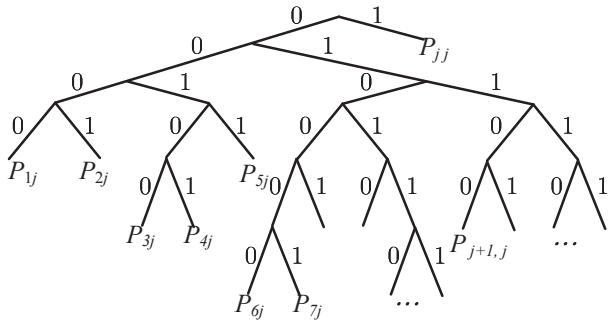

Fig. 2. Huffman tree

A. Fixed compression

It is observed that each column of transition matrix contains many insignificant elements under slow fading environment, which means most codewords will be rarely selected given previous state $\mathcal{S}_j$. Let $\mathcal{R}_j$ be the *neighborhood* of the state $\mathcal{S}_j$ defined by

$$\mathcal{R}_j \triangleq \{\mathcal{S}_i : [\mathbf{P}]_{ij} > p_{th}\}. \tag{10}$$

In other words, given the previous state $\mathcal{S}_j$, the states belongs to $\mathcal{R}_j$ will be selected with probability greater than p_{th}. If we omit the codewords corresponding to the complement of $\mathcal{R}_j$, then it demands only $\log_2(|\mathcal{R}_j|)$ to indicate the codewords which will possibly be selected at next instant. The cost of this method is that both destination and relays have to construct another table of neighborhoods $\{\mathcal{R}_j\}$ according to the transition probabilities. However, if current state does not belong to $\mathcal{R}_j$ unluckily, the destination simply chooses an alternative from the codewords corresponding to states in $\mathcal{R}_j$.

B. Huffman compression

Although the previous method is able to compress the feedback information, there is a tradeoff between compression rate and beamforming performance since the probability of omitted states is nonzero. Motivated from lossless source coding in information theory, we employ Huffman coding [8] to encode the feedback message with variable length. The basic concept of Huffman coding is to indicate the message which is sent with higher probability using fewer number of digits, while encode the message rarely sent with lager number of digits. Given previous state $\mathcal{S}_j$, the feedback message is encoded through a so called Huffman tree, as illustrated in Fig.2. We first set the values of $[\mathbf{P}]_{1,j}$, $[\mathbf{P}]_{2,j}$, $\cdots$, $[\mathbf{P}]_{2^B,j}$ at the bottom of the Huffman tree, and keep connecting two nodes with the least transition probabilities at all times, until all nodes are connected. Then assign 0 or 1 at each branch of connecting points. Through the encoding process, the codeword that will be selected will the highest probability will cost the least bits.

V. Computer Simulations

In this section, we demonstrate bit error rate (BER) performance of the proposed distributed beamforming schemes in Fig.3 and Fig.4. In the computer simulations, all channels are

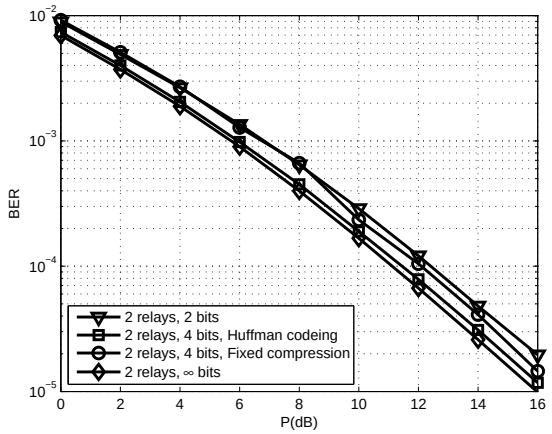

Fig. 3. BER comparison of beamforming schemes in time-varying channel with $L = 2$ and $\rho = 0.99$.

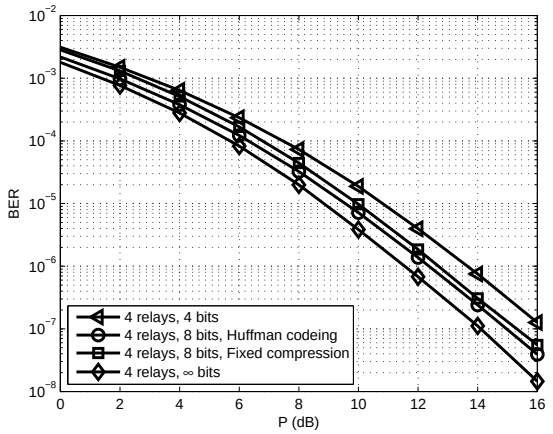

Fig. 4. BER comparison of beamforming schemes in time-varying channel with $L = 4$ and $\rho = 0.99$.

i.i.d. and with unit variance. The autocorrelation coefficient of the channels is $\rho = 0.99$. Transmission power of the source equals to total power of the relays, i.e., $P_s = P_r = P$. The line marked by diamonds indicates the case infinite feedback [4], which is the ideal case with perfect CSI. The line marked by triangles stands for the distributed beamforming with codebook size 2^B. The line marked by squares shows the distirbuted beamforming scheme with codebook size 2^B, and fixe compression is employed here. In this scheme, only the states with $2^{B/2}$ largest transition probabilities are included in the neighboring sets. The line marked by circles demonstrates the distributed beamforming scheme with codebook size 2^B, and Huffman coding is applied to further reduce the size of feedback messages. Fig.3 compares BER of all schemes in cooperative systems with $L = 2$ relays. It shows that all schemes achieve the same diversity order. The ideal case only outperforms the case with $B = 2$ bits feedback by 1.5 dB. The average number of feedback bits in Huffman compression case is 2.4625. It is worth mensioning that the fixed compression scheme requires the same number of feedback bis compared with uncompressed scheme, but results in slightly coding gain. Similar observation can be found in Fig.4, which compares BER for cooperative networks with $L = 4$ relays. In this case, the Huffman compression scheme requires 4.69 bits in average By comparing two figures, it shows that the coding gain with perfect CSI is raised when the number of relays is large. The performance between two compression cases is resulted from the non-neighborhood states are omitted. It shows through comparing both figures that performance degradation of fixed compression becomes insignificant with large size of codebook.

VI. Conclusions

We investigated distributed beamforming schemes with limited feedback in time-varying cooperative systems. To further reduce size of feedback message, we exploit the temporal correlation of channel states. Simulation shows that the proposed schemes provide additional coding gain at the same cost.

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