Image Inpainting by Block-Based Linear Regression with Optimal Block Selection

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***Abstract*—Estimation of missing entries in a multivariate data is one of classical problems in the field of statistical science. One of most popular approaches for this problem is linear regression based on the EM algorithm. When we consider to apply this approach to block-based image inpainting problems, we have additional information, that is, a target lost pixel could be included in multiple blocks, which implies that we have multiple candidates of estimates for the pixel. In such cases, we have to choose a good estimate among the multiple candidates. In this paper, we propose a novel image inpainting method incorporating optimal block selection in terms of the expected squared errors among multiple candidates of the estimate for the target pixel. Results of numerical examples are also shown to verify the efficacy of the proposed method.**

I. Introduction

Image inpainting is a technique to estimate the information (intensities in gray scale images, and 3-dimensional vectors corresponding to RGB values in color images) of lost pixels, which is applied to removal of unnecessary objects from given images for instance. There are so many different approaches for this problem such as PDE-based ones [1], exemplar-based ones [2] and statistic-based ones [3]. They adopted different models, different prior information, and different criteria for the optimization. As long as the aim of inpainting is to minimize the squared errors between an unknown true image and an estimated one, adopting statistic-based one with Gaussian prior should be reasonable. In the field of statistical science, estimation of missing entries in a multivariate data is one of classical problems; and one of most popular approaches for the problem is linear regression with the EM algorithm [4], [5], which can be applied to image inpainting problems. Note that it is trivial that a linear-based inpainting method such as the recent work [3] never outperforms the linear regression with an accurate estimate of second order statistics in terms of the squared errors as pointed out in our previous work [6].

When we consider to apply the linear regression with the EM algorithm to block-based image inpainting problems, we have additional information, that is, a target lost pixel could be included in multiple blocks, which implies that we have multiple candidates of estimates for the pixel. In such cases, we have to choose a good estimate among the multiple candidates of estimate. So the performance of the linear-regression-based inpainting could be improved with an appropriate selection of the optimal block among all blocks including the target pixel. In this paper, we propose a novel image inpainting method based on the linear regression with the EM algorithm, incorporating optimal block selection in terms of the expected squared errors among multiple estimates for a target pixel by estimating the variance of the errors in each estimates. Numerical examples for removal of unnecessary texts from natural images are also shown to verify the efficacy of the proposed method.

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II. Estimation of Missing Data by Linear Regression with the EM Algorithm and Its Application to Image Inapainting

In this section, we briefly review the estimation of missing entries in a multivariate data based on the linear regression with the EM algorithm shown in [4], [5]; and also discuss its application to image inpainting problems.

Let $\mathcal{I} = \{1, \ldots, n\}$ be an index set and let $\mathcal{X} = \{\boldsymbol{x}_i \in \mathbf{R}^d \mid i \in \mathcal{I}\}$ be a complete data set of d-dimensional vectors. We assume that the observation $\boldsymbol{y}_k$ $(k \in \mathcal{J} \subset \mathcal{I})$ of $\boldsymbol{x}_k$ has some missing entries modeled as

$$\boldsymbol{y}_k = A_k \boldsymbol{x}_k, \tag{1}$$

where $A_k \in \mathbf{R}^{d \times d}$ denotes diagonal matrix whose diagonal elements are unity or zero, which means that missing entries of $\boldsymbol{x}_k$ are replaced by zero in the observation $\boldsymbol{y}_k$. Note that we assume that A_k is given. The objective is to estimate the missing entries of $\boldsymbol{x}_k$.

Here, we consider a linear regression model to estimate the missing entries. Let $\mathrm{rank}(A_k) = r_k < d$ and let $M_k \in \mathbf{R}^{r_k \times d}$ be the matrix consisting of rows of A_k excluding zero row vectors. Also let $N_k \in \mathbf{R}^{(d-r_k) \times d}$ be the matrix consisting of the rows of I_d, that is the identity matrix of degree d, excluding the rows included in M_k. For instance, when A_k is given as

$$A_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

then M_k and N_k are reduced to

$$M_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \ N_k = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Note that N_k extracts missing entries and M_k extracts the others by multiplying $\boldsymbol{x}_k$. Also note that

$$M_k' M_k + N_k' N_k = I_d \tag{2}$$

and

$$N_k M_k' = O_{d-r_k, r_k} \tag{3}$$

hold, where the superscript $'$ denotes the transposition operator and $O_{m,n}$ denotes the zero matrix in $\mathbf{R}^{m\times n}$. In this paper, we assume that all vectors in $\mathcal{X}$ are realizations of some random vector $\boldsymbol{x}$.

Estimation of $\boldsymbol{x}_k$ is formulated as the problem of finding the matrix B_k satisfying

$$B_k = \arg\min_B J(B), \tag{4}$$

with

$$J(B) = E_{\boldsymbol{x}_k} ||BM_k \boldsymbol{y}_k - N_k \boldsymbol{x}_k||^2, \tag{5}$$

where $||\cdot||$ denotes the Euclidean norm of a vector, and $E_{\boldsymbol{x}_k}$ denotes the expectation operator over the random vector $\boldsymbol{x}_k$. From $M_k \boldsymbol{y}_k = M_k \boldsymbol{x}_k$ and regarding $\boldsymbol{x}_k$ as a random vector same with $\boldsymbol{x}$, (5) is reduced to

$$\begin{aligned} J(B_k) &= E_{\boldsymbol{x}_k} ||BM_k \boldsymbol{x}_k - N_k \boldsymbol{x}_k||^2 \\ &= E_{\boldsymbol{x}} ||BM_k \boldsymbol{x} - N_k \boldsymbol{x}||^2 \qquad (6) \\ &= \mathrm{tr}[(BM_k - N_k) R (BM_k - N_k)'], \qquad (7) \end{aligned}$$

where $\mathrm{tr}[\cdot]$ denotes the trace of a matrix and $R \in \mathbf{R}^{d\times d}$ denotes the correlation matrix of $\boldsymbol{x}$ defined as

$$R = E_{\boldsymbol{x}}[\boldsymbol{x}\boldsymbol{x}']. \tag{8}$$

Since R is unknown, we have to estimate it by $\boldsymbol{x}_k$, $(k \in \mathcal{I}, k \notin \mathcal{J})$ such as

$$\hat{R} = \frac{1}{|\mathcal{I} - \mathcal{J}|} \sum_{k\in\mathcal{I},\ k\notin\mathcal{J}} \boldsymbol{x}_k \boldsymbol{x}_k', \tag{9}$$

where $|\cdot|$ denotes the the number of elements of a given set. The solution of (4) is easily given as

$$B_k = N_k R M_k' (M_k R M_k')^+, \tag{10}$$

where the superscript $^+$ denotes the Moore-Penrose generalized inverse matrix [7]. Note that when R is non-singular, $(M_k R M_k')$ is also non-singular and $(M_k R M_k')^+$ is reduced to $(M_k R M_k')^{-1}$ since M_k is a full row rank matrix. Thus, the estimated missing entries are written as

$$\boldsymbol{z}_k = B_k M_k \boldsymbol{y}_k \tag{11}$$

and the final recovered d-dimensional vector is given as

$$\hat{\boldsymbol{x}}_k = M_k' M_k \boldsymbol{y}_k + N_k' \boldsymbol{z}_k. \tag{12}$$

Note that the first term of (12) places un-missing entries to their original positions and the second term of (12) places estimated missing entries to their corresponding positions.

Once all missing entries in $\mathcal{X}$ are estimated, we can update $\hat{R}$ by

$$\hat{R} = \frac{1}{|\mathcal{I}|} \left(\sum_{k\in\mathcal{I},\ k\notin\mathcal{J}} \boldsymbol{x}_k \boldsymbol{x}_k' + \sum_{k\in\mathcal{J}} \hat{\boldsymbol{x}}_k \hat{\boldsymbol{x}}_k' \right), \tag{13}$$

and we can also update $\hat{\boldsymbol{x}}_k$, $(k \in \mathcal{J})$ based on the updated $\hat{R}$. We iteratively conduct these two steps, that is, estimation of $\hat{\boldsymbol{x}}_k$ and update of $\hat{R}$ until $\hat{R}$ converges.

Next, we briefly review the application of the above estimation scheme to image inpainting problems. Let $H \in \mathbf{R}^{p\times q}$ be a given gray scale image with missing region. We can obtain multiple blocks from H, named $X_k \in \mathbf{R}^{\alpha\times\beta}$, $(k \in \{1, \ldots, n\})$ by shifting the $\alpha \times \beta$ sized window. The number of blocks n depends on the number of sliding pixels of the window. We can also construct a mask matrix $S_k \in \mathbf{R}^{\alpha\times\beta}$ for X_k whose element are zeros for missing pixels and ones for the other pixels.

Here we introduce some mathematical preliminaries.

Definition 1: [8] Let $A = [\boldsymbol{a}_1, \ldots, \boldsymbol{a}_n] \in \mathbf{R}^{m\times n}$, then the vectored version of A, written as $\mathrm{vec}A$ is defined as

$$\mathrm{vec}(A) = [\boldsymbol{a}_1', \ldots, \boldsymbol{a}_n']' \in \mathbf{R}^{mn}.$$

Definition 2: [8] Let $A = (a_{ij}) \in \mathbf{R}^{m\times n}$ and $B \in \mathbf{R}^{p\times q}$. The Kronecker product of A and B, written as $A \otimes B$, is defined as

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix} \in \mathbf{R}^{mp\times nq}.$$

Let $\boldsymbol{x}_k = \mathrm{vec}(X_k)$ and $A_k = \mathrm{diag}(\mathrm{vec}(S_k))$, where $\mathrm{diag}(\cdot)$ denotes the diagonal matrix whose diagonal entries are those of given vector. Then, the image inpainting for H is reduced to the estimation problem with $d = \alpha\beta$ overviewed above. When we are given a color image, the same discussions can be applied by stacking three d-dimensional vectors corresponding to RGB elements for $\boldsymbol{x}_k$ and using $I_3 \otimes A_k$ for A_k.

When H is divided into disjoint (un-overlapping) blocks, we have no room to improve the estimation process in this framework. We abbreviate the inpainting algorithm by linear regression with the EM-algorithm for this case (un-overlapping blocks) to 'baseline'-algorithm in the following contents.

III. Optimal Block Selection for Image Inpainting by Linear Regression

When we consider to apply the linear regression with the EM algorithm shown in the previous section to the image inpainting problems, we usually have additional information, that is, each target lost pixel may be included in multiple blocks by considering overlapping blocks, which implies that we have multiple candidates of estimate for the lost pixel. In this section, we propose a method for selecting the optimal block, adopted for estimation of each target lost pixel, among the set of blocks including the target lost pixel.

Assume that a target pixel is included in ℓ blocks written as $\{\boldsymbol{x}_{k_t} \mid t \in \{1, \ldots, \ell\}, k_t \in \mathcal{J}\}$ and assume that the target pixel is the p_t-th element of $\boldsymbol{x}_{k_t}$. Let $\boldsymbol{e}_{p_t}$ be the d-dimensional unit vector whose p_t-th element is unity and the others are zeros. Thus, the true value v of the target pixel is represented by

$$v = \boldsymbol{e}'_{p_1}\boldsymbol{x}_{k_1} = \cdots = \boldsymbol{e}'_{p_\ell}\boldsymbol{x}_{k_\ell}. \tag{14}$$

By applying the linear regression shown in the previous section, we have ℓ candidates of estimate written as

$$\hat{v}_t = \boldsymbol{e}'_{p_t}\hat{\boldsymbol{x}}_{k_t},\ (t \in \{1, \ldots, \ell\}). \tag{15}$$

Thus, by using the facts that $\boldsymbol{e}'_{p_t}M'_{k_t}$ vanishes (since $\boldsymbol{e}'_{p_t}$ is a certain row of N_{k_t}) and $M_{k_t}\boldsymbol{y}_{k_t} = M_{k_t}\boldsymbol{x}_{k_t}$, and the assumption of the random vector $\boldsymbol{x}_{k_t}$ being identical to the random vector $\boldsymbol{x}$, the expected squared error of $\hat{v}_t$ can be obtained as

$$\begin{aligned}
w_t &= E_{\boldsymbol{x}_{k_t}}||\hat{v}_t - v||^2 \\
&= E_{\boldsymbol{x}_{k_t}}||\boldsymbol{e}'_{p_t}(\hat{\boldsymbol{x}}_{k_t} - \boldsymbol{x}_{k_t})||^2 \\
&= E_{\boldsymbol{x}_{k_t}}||\boldsymbol{e}'_{p_t}(M'_{k_t}M_{k_t}\boldsymbol{y}_{k_t} + N'_{k_t}\boldsymbol{z}_{k_t} - \boldsymbol{x}_{k_t})||^2 \\
&= E_{\boldsymbol{x}_{k_t}}||\boldsymbol{e}'_{p_t}(N'_{k_t}B_{k_t}M_{k_t}\boldsymbol{y}_{k_t} - \boldsymbol{x}_{k_t})||^2 \\
&= E_{\boldsymbol{x}_{k_t}}||\boldsymbol{e}'_{p_t}(N'_{k_t}B_{k_t}M_{k_t} - I_d)\boldsymbol{x}_{k_t}||^2 \\
&= E_{\boldsymbol{x}}||\boldsymbol{e}'_{p_t}(N'_{k_t}B_{k_t}M_{k_t} - I_d)\boldsymbol{x}||^2 \\
&= \boldsymbol{e}'_{p_t}(N'_{k_t}B_{k_t}M_{k_t} - I_d)R \\
&\quad \times(N'_{k_t}B_{k_t}M_{k_t} - I_d)'\boldsymbol{e}_{p_t}.
\end{aligned} \tag{16}$$

Therefore, the index of the optimal block in terms of the minimum expected squared error for the target pixel is given as

$$t_{opt} = \arg\min_{t\in\{1,\ldots,\ell\}} w_t \tag{17}$$

and it is concluded that adopting

$$\hat{v} = \boldsymbol{e}'_{p_{t_{opt}}}\hat{\boldsymbol{x}}_{k_{t_{opt}}} \tag{18}$$

for the target pixel is optimal. This scheme is called 'optimal block selection (OBS)' and the overall algorithm of the proposed method, that is, the baseline algorithm with OBS, is abbreviated to 'baseline + OBS' in the following contents.

A naive implementation of our method requires large computational costs. However, we can implement it efficiently by keeping the minimum w_t for each pixel and updating it (and estimated pixel value) only when a newly appearing w_t is less than the kept we_t.

Note that our optimal block selection scheme could be applied to any other linear-based algorithms by changing B_{k_t} in (16) with the corresponding estimation formula.

IV. Numerical Examples

In this section, we verify the efficacy of the proposed method by numerical examples for removal of unnecessary texts from given images. We compare the performance of the 'baseline'-algorithm, 'baseline + OBS'-algorithm (the proposed method), and the method proposed in [2] which is abbreviated to 'exemplar'-algorithm. Although the 'exemplar'-algorithm is based on a quite different approach, we adopt it as a reference for one of popular and successful inpainting methods. Although the recent method proposed in [3] is based on the similar approach with our method, we do not adopt it as a competitor since we proved in [6] that the linear regression shown in Section II outperforms it theoretically.

As the evaluation measure, we adopt the squared error between a true image and an estimated one. The block size is set to 9×9 pixels that is the default setting of the publicized source code [9] of the 'exemplar'-algorithm [2]. In the 'baseline'-algorithm and the 'baseline + OBS'-algorithm, we have maximum number of blocks by shifting 1 pixel horizontally and vertically to calculate $\hat{R}$ in order to keep the same condition for the estimation accuracy of $\hat{R}$. On the other hand for estimation, we obtain a set of disjoint blocks in 'baseline'-algorithm, while we obtain the maximum number of blocks as the same with those for calculation of $\hat{R}$ in 'baseline + OBS'-algorithm in order to obtain multiple candidates of estimation for lost pixels.

Figures 1 - 3 show the given image including unnecessary texts (left) and the inpainted images by 'exemplar' (mid-left), 'baseline' (mid-right), and 'baseline + OBS' (right) for three sample images. Table I shows the squared error between a true image and inpainted images by 'exemplar', 'baseline' and 'baseline + OBS' for the three sample images.

TABLE I
THE SQUARED ERROR BETWEEN A TRUE IMAGE AND INPAINTED IMAGES.

	Image #1	Image #2	Image #3
'exemplar'	1875.0	4695.3	4874.2
'baseline'	1467.1	2793.2	3239.4
'baseline + OBS'	**1145.8**	**2522.3**	**3129.4**

According to Table I, it is concluded that the 'baseline + OBS'-algorithm (the proposed method) yields the best results in terms of the norm of the difference between the true image and the inpainted image. Especially, it is confirmed that our optimal block selection scheme improves the EM-algorithm-based linear regression from the comparison of the norm of the difference of 'baseline' and 'baseline + OBS'.

Figure 4 shows partial zoomed in image of the sample image #1 and its inpainted versions by the three methods. According to Fig.4, the authors believe subjectively that the 'baseline + OBS'-algorithm (the proposed method) yields better results.

V. Conclusions

In this paper, we proposed a novel image inpainting method based on a EM-algorithm-based linear regression with optimal block selection scheme applicable to the cases where a lost pixel is included in multiple blocks. We also verify the efficacy of the proposed method by some numerical examples and confirmed the advantage of the proposed method in terms of the norm of the error.

Fig. 1. Sample image #1, the given image(left) and inpainted images by 'exemplar'(mid-left), 'baseline'(mid-right), and 'baseline + OBS' (right).

Fig. 2. Sample image #2, the given image(left) and inpainted images by 'exemplar'(mid-left), 'baseline'(mid-right), and 'baseline + OBS' (right).

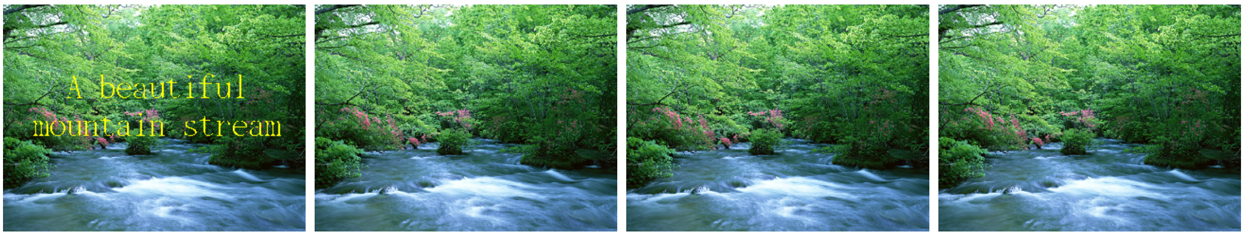

Fig. 3. Sample image #3, the given image(left) and inpainted images by 'exemplar'(mid-left), 'baseline'(mid-right), and 'baseline + OBS' (right).

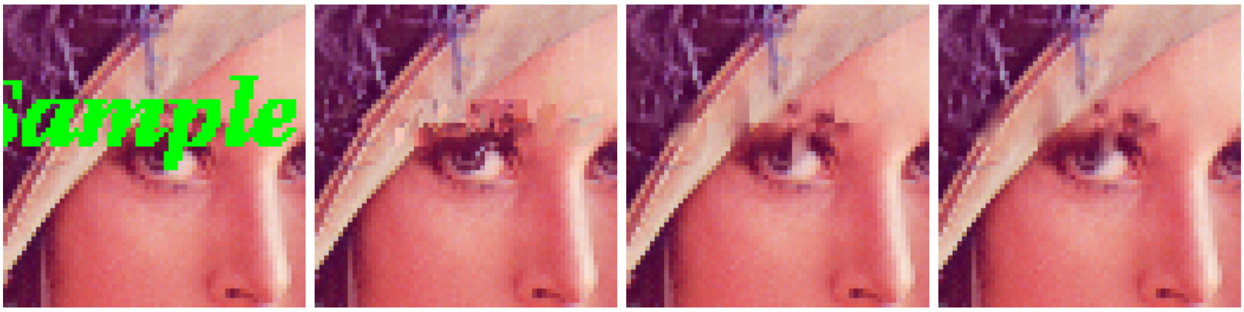

Fig. 4. Zoomed version of the sample image #1, the given image(left) and inpainted images by 'exemplar'(mid-left), 'baseline'(mid-right), and 'baseline + OBS' (right).

REFERENCES

[1] M. Bertalmio, G. Sapiro, V. Caselles, and C. Ballester, "Image Inpainting," *Proceeding of the 27th annual conference on Computer graphics and interactive techniques (SIGGRAPH'00)*, pp. 417–424, 2000.

[2] A. Criminisi, P. Perez, and K. Toyama, "Object removal by exemplar-based inpainting," *Proceedings of 2003 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR'2003)*, vol. 2, no. 1, pp. II–721–II–728, 2003.

[3] T. Amano and Y. Sato, "Image interpolation using BPLP method on the eigenspace," *Systems and Computers in Japan*, vol. 38, no. 1, pp. 87–96, 2007.

[4] S. F. Buck, "A Method of Estimation of Missing Values in Multivariate Data Suitable for use with an Electronic Computer," *Journal of the Royal Statistical Society. Series B (Methodological)*, vol. 22, no. 2, pp. 302–306, 1960.

[5] G. J. McLachlan and T. Krishnan, *The EM Algorithm and Extensions*, John Wiley & Sons, 1997.

[6] A. Tanaka, T. Ogawa, M. Haseyama, and M. Miyakoshi, "Analyses on Interpolation Accuracy of Eigenspace-BPLP," *The IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences (Japanese Edition)*, vol. J94-A, no. 2, pp. 116–126, 2011.

[7] C. R. Rao and S. K. Mitra, *Generalized Inverse of Matrices and Its Applications*, John Wiley & Sons, 1971.

[8] J. R. Magnus and H. Neudecker, *Matrix Differential Calculus with Applications in Statistics and Econometrics*, John Wiley & Sons, 1988.

[9] A. Criminisi, P. Perez, and K. Toyama, "Object removal by exemplar-based inpainting," http://www.cc.gatech.edu/~sooraj/inpainting/.