Splitting Frequency Components of Error Signal in Narrowband Active Noise Control System Design

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Abstract—**A narrowband active noise control (NANC) structure is often applied to reduce undesired noise when multiple tones have close frequencies. A parallel ANC structure is proposed for separating undesired harmonics into a series of adaptive filters. Since the input signal of each adaptive filter has only one harmonic component, a second-order finite impulse response (FIR) filter is sufficient for processing the undesired noise. Therefore, the presented parallel structure for NANC system is greatly simplified. Based on the input signal frequency, a bank of bandpass filters splits the frequency components of the error signal to update the corresponding adaptive filters in the parallel NANC structure, each of which is computationally very simple and unaffected by time delay. A new performance index for a parallel NANC is also proposed. The increased convergence speed obtained by the proposed method is first confirmed by theoretical analysis. Computer simulations are then performed to confirm the enhancement.**

I. INTRODUCTION

Rotating mechanisms periodically generate primary noise containing multiple harmonics in low frequency range. However, passive noise control (PNC) uses sound-absorbing materials, which is only effective in canceling out high frequency noise interference. Therefore, the ANC system is commonly used to generate a secondary noise with equal amplitude and opposite phase of the primary noise to cancel the undesired noise, which is especially good at reducing low frequency noise [1] and is widely used for industrial applications. A NANC application often uses a mechanical tool such as a tachometer to measure synchronization of internally generated input noise in the adaptive filter [2]. Therefore, a NANC system does not require a reference microphone, and acoustic feedback is not problematic. A filtered-X least mean square (FXLMS) algorithm in the second-order adaptive filter of an ANC system has been proven effective for canceling interference with only one harmonic [3]. Thus, most parallel NANC structures have several second-order adaptive filters connected in parallel to cancel the narrowband noise with numerous tones at the fundamental frequency and at some leading harmonic frequencies.

Studies of NANC systems have considered various forms, including direct, direct/parallel and parallel forms. A direct form NANC structure uses one adaptive filter with conventional FXLMS algorithm to handle the undesired narrowband noise, but its performance worsens as the number of harmonics of undesired interference increases [1]. Close frequency separations of harmonics also degrade the direct form NANC. The direct/parallel-form NANC system developed by Yuan *et al.* uses multiple reference signal generators to increase frequency separation at each channel [4]. Therefore, for each channel, frequency separation between two neighboring sinusoids is increased by separating various sinusoids into mutually exclusive sets. Yang *et al.* proposed a parallel structure to cancel the undesired narrowband noise using multiple second-order adaptive filters [5]. The convergence of the structure needs to be further examined. Kuo and Puvvala proved that convergence rate correlates with frequency separation [6] and Xiao *et al.* analyzed the stochastic statistics of a FXLMS based NANC system [7]. However, since these techniques use the same error signal to update all adaptive filters, the performance of the NANC degrades when an error signal approaches zero in some but not all components. In [8], Akhtar and Mitsuhashi presented a hybrid NANC system using direct from structure. The system complexity is high when the number of undesired harmonics increases. The NANC proposed by Glover used a sum of sinusoids as an input signal to an adaptive filter with a length much larger than two [9]. In this controller, convergence is low when the frequency separation between two neighboring harmonics is small. Several signal blocks with input signals along with sinusoidal waves in parallel were presented for NANC in [10]. An additional bandpass filter was used to decompose filter outputs. However, use of the same error signal to update all adaptive filters in the NANC system was still problematic.

In this study, a modified parallel structure NANC system is presented to reduce undesired interference containing close neighboring frequencies. Multiple second-order adaptive filters with modified cost function are derived for the proposed parallel structure ANC system. Multiple delayless bandpass filters are also connected in parallel to split the error signal into corresponding channels according to the frequency components of input signals. Therefore, each second-order adaptive filter is designed to reduce only one harmonic and is updated by the corresponding individual error signal containing the same frequency components as the input signal. The bank of the bandpass filter also has a simple computational load. Therefore, the order of the adaptive filter and the complexity of the parallel NANC system are greatly reduced. Several simulations confirm the effectiveness and enhancement of the proposed parallel NANC system.

II. PARALLEL-FORM NANC SYSTEMS

A general parallel-form NANC system separates the frequency components in the primary noise into several channels. Each channel contains only a single frequency, which can be canceled by a simple two-weight adaptive filter. Figure 1 shows the details of a multiple-frequency NANC in parallel configuration. Suppose the undesired interference contains M sinusoidals at frequencies f_i , $i=1,\cdots,M$ and M second-order adaptive FIR filters $W_m(z)$ with two weights are connected in parallel to attenuate these narrowband noise components. Let $d(n)$ denote the primary noise, $S(z)$ denote the secondary path of the NANC system, respectively, and $\hat{S}(z)$ denote the estimated secondary path. Each independent input signal $x_i(n)$ contains a single cosine or sine wave generated by a synchronization signal generator based on the information provided. That is, the i^{th} signal generator produces

$$x_i(n)=A_i\cos(2\pi f_i n)\ ,\ i=1,\cdots,M\ , \tag{1}$$

which is used as the reference input of the adaptive filter $W_i(z)$ with two weights. Each adaptive filter $W_i(z)$ is connected in parallel, and the error signal is used to update the filtered-X LMS (FXLMS) algorithm. Figure 1 thus shows that canceling signal $u(n)$ is a sum of the output of the M adaptive filters

$$u(n)=\sum_{i=1}^{M}u_i(n)\,, \tag{2}$$

where u_i is the output of each second-order adaptive filter

$$u_i(n)=w_{i,0}(n)x_i(n)+w_{i,1}(n)x_i(n-1)\ , \tag{3}$$

where $w_{i,0}(n)$ and $w_{i,1}(n)$ are the two weights of the i^{th} channel second-order adaptive filter $W_i(z)$ for $i=1,\cdots,M$. Thus, after going through the secondary speaker, the output signal is

$$u'(n)=\sum_{j=0}^{a_1-1}s_j\cdot u(n-j), \tag{4}$$

and the filtered signal used for the FXLMS algorithm is

$$x'_m(n)=\sum_{j=0}^{a_2-1}\hat{s}_j\cdot x_m(n-j), \tag{5}$$

where s_j and $\hat{s}_j$ are the coefficients of the j^{th}-order secondary path $S(z)$ and estimated secondary path $\hat{S}(z)$, respectively, and a_1 and a_2 are the respective filter lengths. The residual noise is $e(n)=d(n)-u'(n)$, and the mean square error (MSE) is

$$\psi(n)=E[e^2(n)]=E[(d(n)-u'(n))^2]\,, \tag{6}$$

where $E[\cdot]$ denotes the expectation operation. Generally, the conventional NANC system uses the FXLMS algorithm to update the two-weight adaptive FIR filters as

$$w_{i,0}(n+1)=w_{i,0}(n)+2\mu e(n)x'_i(n) \quad \text{and}$$
$$w_{i,1}(n+1)=w_{i,1}(n)+2\mu e(n)y'_i(n)\ , \tag{7}$$

where μ is the step size and $y'_i(n)$ is in phase quadrature (90°) with $x'_i(n)$.

The M individual input signals are generated by synchronous signal generators in the parallel-structure NANC system; a linear filter is used in addition to the adaptive FIR filter. Therefore, the filter convolution operation does not change the frequency components of the input signal. The reference signal $x(n)$, the primary noise $d(n)$ and the error signal noise $e(n)$ can then be expressed as multiple frequency components:

$$x(n)=\sum_{i=1}^{M}x_i(n)\,,\ d(n)=\sum_{i=1}^{M}d_i(n)\,,\ \text{ and } e(n)=\sum_{i=1}^{M}e_i(n)\,, \tag{8}$$

where $x_i(n)$, $i=1,\cdots,M$ has only one frequency component and $e_i(n)$ and $d_i(n)$ have the same frequency components associated with $x_i(n)$. Therefore, adaptive filter $W_i(z)$ determines the frequency components in its own input signal. However, the conventional correction term of adaptive filter $W_i(z)$ is the product of its input signal $x'_i(n)$ and error signal $e(n)$ (Eq. (7)). This conflicts with the main purpose of the parallel form NANC system, which is to split frequency components. Further, when only an $e_i(n)$ approaches zero and other components $e(n)$ do not, the other components cause interference in $e_i(n)$, which degrades the performance of the adaptive filter $W_m(z)$ and causes misalignments in the adaptive weights. Instead of Eq. (6), a new cost function is therefore proposed as

$$\psi'(n)=\sum_{i=1}^{M}E[e_i^2(n)]\,. \tag{9}$$

By minimizing the summation of M independent squared error signals, this cost function focuses on the square error summation of each set in different frequency components instead of on the overall squared error signal. Where the processing frequency in the adaptive filter $W_i(z)$ is f_i, the difference between $\psi(n)$ and $\psi'(n)$ can be located at the crossover term

$$\psi(n)-\psi'(n)=2\sum_{\substack{i,j=1\\ i\neq j}}^{M}E[e_i(n)e_j(n)], \qquad (10)$$

which contains the frequency components unrelated to f_i. Therefore, the new cost function is a reasonable performance index for adjusting the adaptive filter. When using the new performance index and gradient estimator, the adaptive FXLMS algorithm for the two-weight FIR filters is changed to

$$w_{i,0}(n+1)=w_{i,0}(n)+2\mu e_i(n)x_i'(n) \quad \text{and}$$
$$w_{i,1}(n+1)=w_{i,1}(n)+2\mu e_i(n)y_i'(n). \qquad (11)$$

To realize the new algorithm, a bank of delayless bandpass filters $B_i(z)$, $i=1,\cdots,M$ (Fig. 1) is used to split the respective frequencies components of error signal $e(n)$. Each bandpass filter has a single passband centered at specific frequencies based on the input signal to that channel. Based on the reference signal frequency, the center frequency is tuned using an adaptive algorithm. The bandpass filter bank operation thus obtains the frequency components of the error signal, which are denoted as $e_i(n)$, $i=1,\cdots,M$.

The structure of bandpass filters $B_m(z)$ is depicted as follows. The center frequency of the i^{th}-channel is f_i, the delayless 2^{nd}-order infinite impulse response (IIR) filter $H_i(z)$ reported in [11]-[12] processes the sinusoidal error signal as

$$H_i(z)=\frac{(1-p_i^2)\left(\dfrac{q_i z}{1+p_i^2}-1\right)}{z^2-q_i z+p_i^2}, \qquad (12)$$

where a parameter p_i ($0<<p_i<1$) close to 1 is typically chosen to achieve a narrow bandwidth and where q_i ($|q_i|<2p_i$) determines the center frequency of the passband. Therefore, the bandpass filter bank is stable because the poles of each filter

$$z_{1,2}^{pole}=\frac{q_i\pm i\sqrt{4p_i^2-q_i^2}}{2} \qquad (13)$$

form a complex-conjugate pair lying on a circle of radius p_i just inside the unit circle. A zero is also observed at $z_1^{zero}=\left(1+p_i^2\right)/q_i$. Properly choosing p_i and q_i reveals the peak passband of $H_i(z)$ at the given frequency

$$f_i=\frac{f_s}{2\pi}\cos^{-1}\left(\frac{q_i}{1+p_i^2}\right), \qquad (14)$$

where f_s is the sampling frequency. Radius p_i of the complex-conjugate pair also determines the filter bandwidth; moreover, $|q_i|<2p_i$, so center frequency f_i of the bandpass filter can be formulated as

$$\frac{f_s}{2\pi}\cos^{-1}\left(\frac{-2p_i}{1+p_i^2}\right)<f_i<\frac{f_s}{2\pi}\cos^{-1}\left(\frac{2p_i}{1+p_i^2}\right). \qquad (15)$$

Since a p_i close to 1 is typically selected to achieve a narrow bandwidth, the bandpass filter design covers most of the frequency range.

The presented bandpass filter bank therefore splits each frequency component of the error signal into several parts and prevents additional delay problems in the NANC system.

III. Simulation Results

Several performance tests were performed to evaluate the proposed parallel structure NANC system and to confirm its fast convergence results. The secondary path $S(z)$ in Fig. 1 is obtained from a practical circular duct to simulate an automotive application of NANC [13]. For each of the reference sinusoidal signals generated by multiple signal generators, one harmonic of the frequency component of the undesired noise is used as a reference input. Consider the case of M harmonics in the undesired noise. Figure 1 shows the M signal generators used to design the parallel structure of the adaptive filters. The M bandpass filters are also designed to split the frequency components of the error signal. In this study, performance comparisons of the proposed parallel form and conventional form of NANC systems were compared in MATLAB simulations.

The first simulation tests an undesired noise with three harmonics (M=3). Three signal generators and second-order bandpass filters in parallel are applied to generate and split the frequency components of the reference and error signals, respectively. For a primary noise containing three sinusoids at 150 Hz, 180 Hz and 200 Hz, the passbands of bandpass filters $H_1(z)\sim H_3(z)$ are centered at 150 Hz, 180 Hz and 200 Hz, respectively, to split the residual noise $e(n)$. The sampling rate is 1 kHz. The parameters for the three bandpass filter systems $H_1(z)\sim H_3(z)$ are set to $p_i=0.98$ ($i=1,\cdots,3$). Since each channel processes one frequency component, each adaptive filter in both the proposed and conventional parallel-form NANC systems is set to a length of 2. Step sizes of 0.4 and 0.05 are used for the conventional and proposed parallel-form NANC systems, respectively.

Figure 2 compares summaries of the error signal obtained by three different NANC algorithms where the dashed, dotted, and solid lines represent the direct form, the conventional parallel form and the proposed parallel form, respectively. Figure 3 shows the convergence speeds of the weights, the dashed lines presented the conventional parallel form and the solid ones were by the proposed algorithm, which also confirm the enhancement achieved by the proposed work. Both computational simulation results obtained by the parallel structure are superior to those obtained by conventional forms.

The second simulation considers primary noise initially containing four harmonics ($M=4$) with varying frequencies during the control. Suppose the initial frequencies of undesired noise are 160 Hz, 180 Hz, 220 Hz and 240 Hz and each increases by 15 Hz after 0.25 seconds. In an actual NANC application, this condition may result from the variation of engine speed. Therefore, four-channel adaptive filters, each of which has two weights, are designed for the proposed parallel form. For the four bandpass filters $H_1(z) \sim H_4(z)$ with $p_i = 0.98$, $q_1 \sim q_4$ are initially 1.3118, 1.1523, 0.7974 and 0.6058, respectively. The sampling frequency is 1.2 KHz.

Figure 4 shows the bandpass filters. One can find all the filters are with magnitude 0 dB and zero phase shift at the specified frequencies. Figure 5 examines convergence speed in terms of parameters $q_1 \sim q_4$. Since the primary noise frequencies change during control, the passband of the bandpass filters shows a corresponding change in the primary noise frequencies. One can find the parameters $q_1 \sim q_4$ quickly converge within hundreds iterations. At last, Fig. 6 compares summaries of residual noises power between the conventional and proposed parallel NANC systems. The top figure in Fig. 6 shows the performance of the conventional parallel NANC and the bottom figure presents results by the proposed method. It is obvious the proposed parallel NANC outperforms the conventional one. Also, the proposed method can track the frequency variation quickly to restrain the undesired noise well when the frequencies of unwanted noise change.

IV. CONCLUSIONS

A parallel-form NANC system with a bandpass filter bank is developed to split the frequency components of error signals without adding delay. A proposed cost function derives the new adaptive algorithm such that each adaptive filter can tune the weights according to its frequency component, which efficiently reduces noise. The noise reductions, convergence speeds of adaptive weights, and mean square errors obtained by the conventional and proposed structure NANC systems are calculated and compared by computer simulations.

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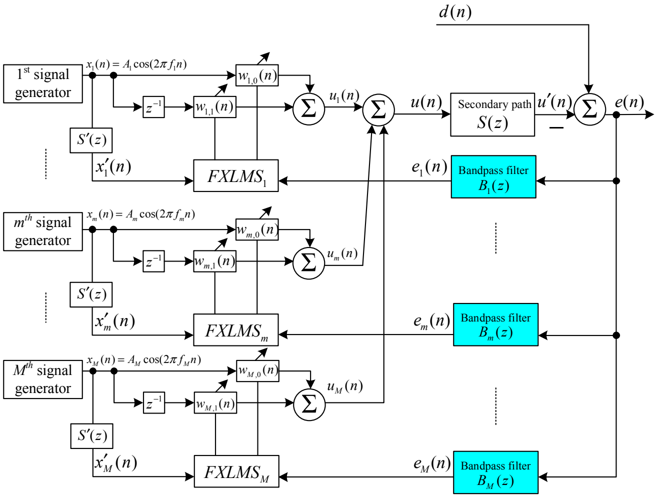

Fig. 1. Proposed parallel form narrowband ANC system.

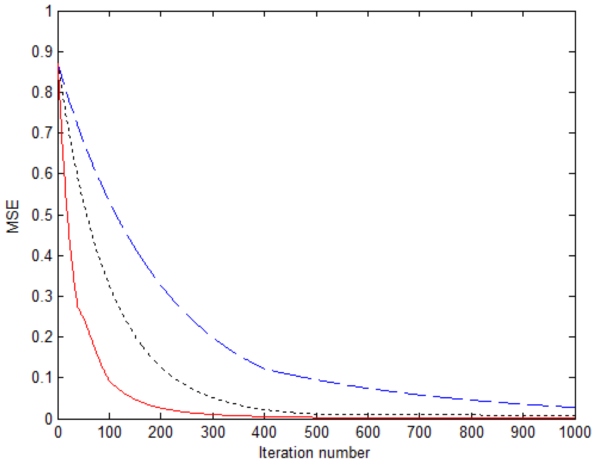

Fig. 2. Learning curve of MSE.

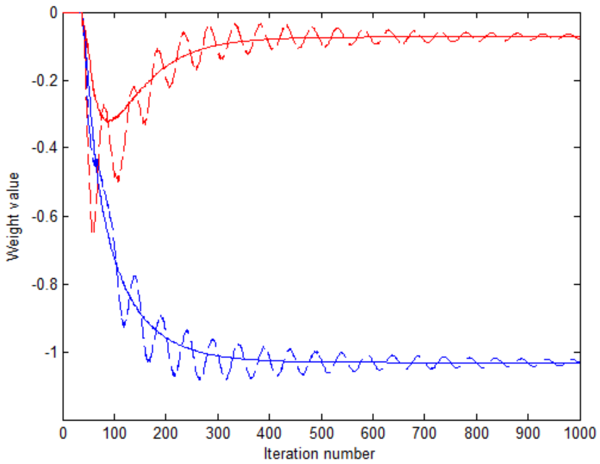

Fig. 3. Convergence analysis of weights.

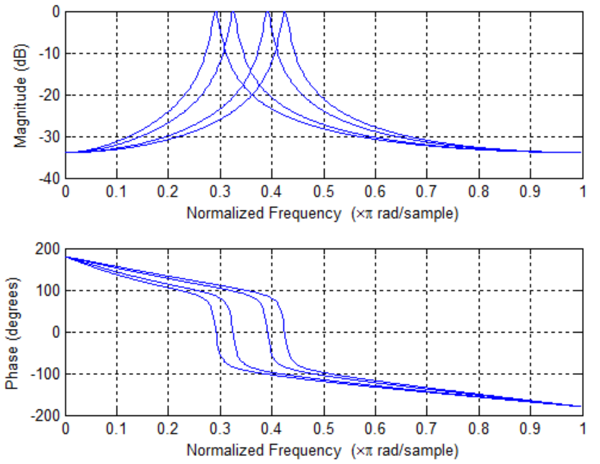

Fig. 4. Bandpass filters analysis, top: magnitude response, bottom: phase response, sampling frequency: 1.2 kHz.

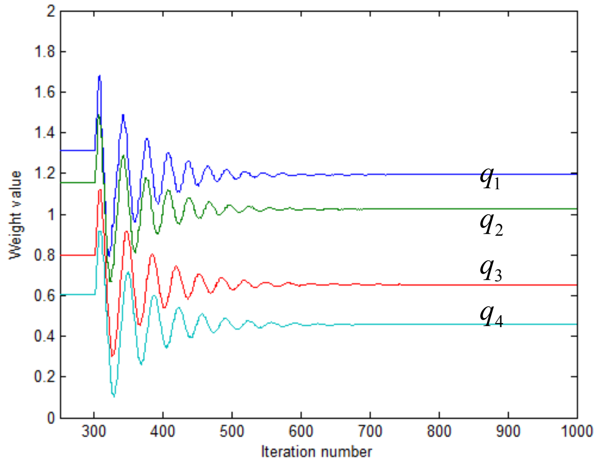

Fig. 5. Convergence of $q_1 \sim q_4$ with passband variation.

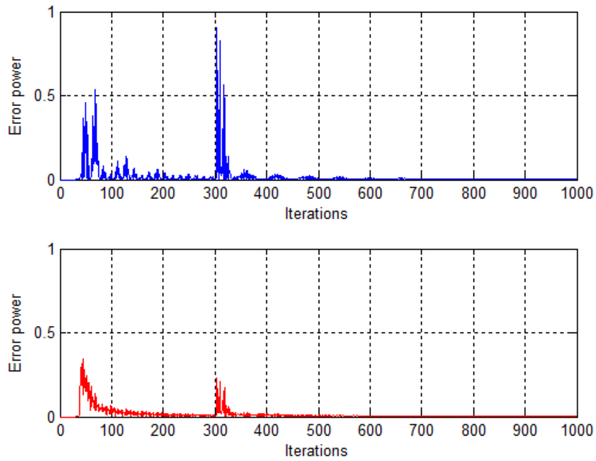

Fig. 6. Noise cancelation performance, top: conventional

method, bottom: proposed method.