

An Effective Allocation of Non-zero Digits for CSD Coefficient FIR Filters Using 0-1PSO

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Abstract—In this paper, a novel method for effective allocation of non-zero digits in design of CSD (Canonic Signed Digit) coefficient FIR (Finite Impulse Response) filters is proposed. The design problem can be formulated as a mixed integer linear programming problem, which is well-known as a NP-hard problem. Recently, a heuristic approach using the PSO (Particle Swarm Optimization) for solving the problem has been proposed, in which the maximum number of non-zero digits is limited in each coefficient. On the other hand, the maximum number of non-zero digits is limited in total in the proposed method and the 0-1PSO is applied. It enables an effective allocation of non-zero digits and provides a good design. Several examples are shown to present an efficiency of the proposed method.

I. INTRODUCTION

FIR filters are often used in a lot of signal processing applications because it is able to realize a perfect linear phase characteristic and to assure a stability. In general, a high order of FIR filters are required for attaining a sharp cut-off characteristic. Then, a circuit complexity tends to be large. In a hardware implementation, each filter coefficient is realized by combining adders and shifters involved in each multiplier. The number of shifters corresponds to the number of non-zero digits involved in each coefficient. Therefore, it is required to reduce the number of non-zero digits of filter coefficients.

The CSD (Canonical Signed Digit) representation is known as one of successful approaches for reducing the number of non-zero digits[1]. The CSD representation has two constraints, in which two non-zero digits are not adjacent and the number of non-zero digits are limited. The design problem of the CSD coefficient FIR filter can be formulated as a MILP (Mixed Integer Linear Programming) problem, which is well-known as a NP-hard problem. It is known that the solving such a problem requires high computational time[2].

Recently, heuristic approaches for solving the problem have been proposed[3][4]. The PSO (Particle Swarm Optimization) is one of the swarm optimization algorithms worked in a continuous search space and easily applicable to the non-convex optimization problem[5]. In [3], a modified objective function with a penalty function is defined to apply the PSO for searching among the CSD set. This function has a lot of local minimums. The penalty function requires to limit the maximum number of non-zero digits in each coefficient and it causes the degradation of frequency response.

In this paper, the maximum number of non-zero digits is limited in total. It is expected that an effective allocation can be achieved dependent on a value of each coefficient.

In the proposed method, the design problem falls into 0-1 combinatorial optimization problem. Then, the 0-1PSO is applied for solving the design problem. The 0-1PSO is binary type of PSO. It is easy to add the constraints of the CSD representation by penalty functions. Several examples are shown to present the efficiency of the proposed method.

II. DESIGN PROBLEM

A structure of N -th order FIR filter is shown in Fig. 1. In Fig.1, h_k , $k \in \{0, 1, \dots, N\}$ is the filter coefficient. The FIR filter has a linear phase characteristic when N is even number and the h_k is even symmetric. The magnitude response $H(\omega)$ of the linear phase FIR filter is described as,

$$H(\omega) = \sum_{n=0}^M a_n \cos n\omega, \quad (1)$$

where $M = N/2$, $a_0 = h_M$, $a_n = 2h_{M-n}$, $n \in \{0, 1, \dots, M\}$.

In a min-max criterion, the design problem can be formulated as following,

$$\min_{a_0, \dots, a_M} \max_{\omega \in \Omega} |D(\omega) - H(\omega)|, \quad (2)$$

where Ω is the approximation frequency band.

In the CSD representation, a_n is expressed as

$$a_n = \sum_{m=1}^p b_{n,m} 2^{-m}, \quad (3)$$

where $b_{n,m} \in \{1, 0, \bar{1} = -1\}$, p is the word length. The adjacent allocation of two non-zero digits is not be allowed in the CSD representation. For example, a binary number $(0.01111)_2$ can be represented as the CSD number $(0.1000\bar{1})_{\text{CSD}}$. A structure of multiplier for the binary number $(0.01111)_2$ is shown in Fig. 2 and the CSD number $(0.1000\bar{1})_{\text{CSD}}$ is shown in Fig. 3.

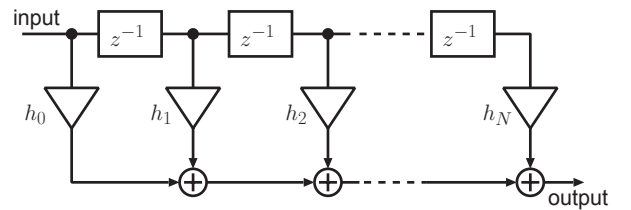


Fig. 1. A structure of N -th order FIR filter

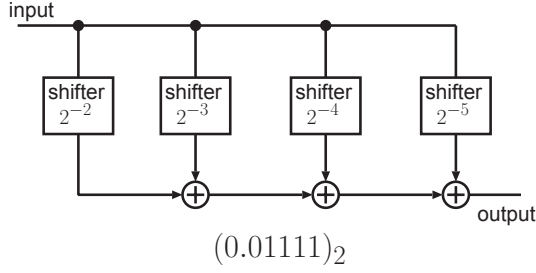


Fig. 2. A structure of multiplier (binary number)

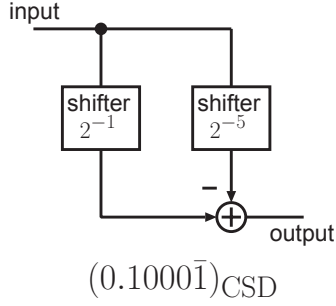


Fig. 3. A structure of multiplier (CSD number)

From Fig. 2, Fig. 3, it is apparent that the CSD representation is able to reduce the number of non-zero digits and thus the reduction of the number of shifters can be achieved.

The limitation of the maximum number of non-zero digits in each coefficient restricts a range of available coefficient values. An effective allocation of non-zero digits based on the value of coefficients makes possible to prevent such a situation. The effective allocation can be accomplished by the limitation of the maximum number of non-zero digits in total.

For example, the number of non-zero digits λ of two CSD numbers are described as follows,

$$1). \quad (0.0100\bar{1}000)_{\text{CSD}}, \quad \lambda = 2, \quad (4)$$

$$2). \quad (0.10\bar{1}0100\bar{1})_{\text{CSD}}, \quad \lambda = 4. \quad (5)$$

In the case that the maximum number of non-zero digits is set to 3 in each coefficient, $(0.10\bar{1}0100\bar{1})_{\text{CSD}}$ is unavailable. In the case that the maximum number of non-zero digits is set to 6 in total, $(0.0100\bar{1}000)_{\text{CSD}}$ and $(0.10\bar{1}0100\bar{1})_{\text{CSD}}$ are both available.

III. PROBLEM FORMULATION

An optimal design problem of CSD coefficient FIR filters that the total number of non-zero digits is constrained is

formulated as the following constrained minimization problem

$$\begin{aligned} \min \quad & \delta \\ \text{sub.to} \quad & |H(\omega) - D(\omega)| \leq \delta, \\ & \sum_{n=0}^M \sum_{m=1}^p (b'_{n,m} + b''_{n,m}) \leq \Lambda, \\ & b'_{n,j} + b''_{n,j} + b'_{n,j+1} + b''_{n,j+1} \leq 1, \quad (6) \\ & b'_{n,m}, b''_{n,m} \in \{0, 1\}, \\ & j \in \{1, 2, \dots, p-1\}, \end{aligned}$$

where

$$a_n = \sum_{m=1}^p (b'_{n,m} - b''_{n,m}) 2^{-m},$$

δ is the maximum absolute error between $H(\omega)$ and $D(\omega)$, Λ is the maximum number of non-zero digits which is available in total. The design problem is the 0-1 combinatorial optimization problem.

IV. PARTICLE SWARM OPTIMIZATION

The PSO (Particle Swarm Optimization) is one of the multipoint search algorithms worked in the continuous search space. The PSO has a high directivity toward a local minimum and is able to enumerate local minimums which are candidates of the optimal solution quickly. The PSO is consisted of multiple particles and they organize one swarm.

A velocity vector $\mathbf{v}_q$, a location vector $\mathbf{x}_q$ and a personal best vector $\mathbf{pbest}_q$ is defined for the particle q . The swarm has a global best solution $\mathbf{gbest}^t$ obtained until t -th iteration. All particles share $\mathbf{gbest}^t$ and search for a minimum solution of the objective function $f(\mathbf{x})$. $\mathbf{v}_q$ and $\mathbf{x}_q$ of particle q is updated as following,

$$\mathbf{v}_q^{t+1} = w_t \mathbf{v}_q^t + c_1 r_1 (\mathbf{pbest}_q^t - \mathbf{x}_q^t) + c_2 r_2 (\mathbf{gbest}^t - \mathbf{x}_q^t), \quad (7)$$

$$\mathbf{x}_q^{t+1} = \mathbf{x}_q^t + \mathbf{v}_q^{t+1}, \quad (8)$$

where r_1, r_2 is a random variable number in $[0, 1]$. The w_t is an inertia weight which is adjusted by the following,

$$w_t = w_{\max} - \frac{T}{w_{\max} - w_{\min}} \times t, \quad (9)$$

where T is the maximum number of iterations, $w_{\max}$, $w_{\min}$ is a constant, respectively.

The $\mathbf{pbest}^t$ at t -th iteration is updated as following,

$$\mathbf{pbest}_q^t = \begin{cases} \mathbf{x}_q^t & , \quad f(\mathbf{x}_q^t) < f(\mathbf{pbest}_q^{t-1}) \\ \mathbf{pbest}_q^{t-1} & , \quad \text{otherwise.} \end{cases} \quad (10)$$

The $\mathbf{gbest}^t$ at t -th iteration is updated as following,

$$\mathbf{gbest}^t = \arg \min_q f(\mathbf{pbest}_q^t). \quad (11)$$

V. 0-1PSO

In this paper, the 0-1PSO is applied for solving the design problem of the CSD coefficient FIR filters. Although the PSO was originally developed for solving the problem having the continuous variables, the PSO has been improved for applying to the 0-1 combinational optimization problem. The location vector $\mathbf{x}_q$ is defined as

$$\mathbf{x}_q = [b'_{0,1}, b''_{0,1}, \dots, b'_{M,p}, b''_{M,p}]^T, \quad (12)$$

where $b'_{n,m}, b''_{n,m} \in \{0, 1\}$. In the 0-1PSO, the continuous variables $u'_{n,m}, u''_{n,m}$ is defined to $b'_{n,m}, b''_{n,m}$ alternatively, and a map function J is applied to make transform from the continuous variable to the binary number. Each digit is expressed by $b_{n,m} = b'_{n,m} - b''_{n,m}$. The $\mathbf{x}_q$ can be redefined by using J as

$$\begin{aligned} \mathbf{x} &= \mathbf{J}(\mathbf{u}) \\ &= [J(u'_{0,1}), J(u''_{0,1}), \dots, J(u'_{M,p}), J(u''_{M,p})]^T, \end{aligned} \quad (13)$$

where $u'_{n,m}, u''_{n,m} \in [-\infty, \infty]$. J is defined as

$$J(u) = \begin{cases} 0 & , \text{ with probability } P(u) \\ 1 & , \text{ with probability } 1 - P(u) \end{cases}. \quad (14)$$

where $P(u)$ is a sigmoid function. The sigmoid function is defined as following,

$$P(u) = \frac{1}{1 + e^{-\alpha u}}, \quad (15)$$

where α is a gain which determines the sharpness of the sigmoid function. The shape of sigmoid function is shown in Fig. 4.

The objective function $F(\mathbf{x})$ of 0-1PSO is defined with two penalties ϕ_1, ϕ_2 in consideration with the CSD representation as following,

$$F(\mathbf{x}) = W\delta + s_1\phi_1(\mathbf{x}) + s_2\phi_2(\mathbf{x}), \quad (16)$$

where W is a weight of δ , s_1 and s_2 is a weight of the penalty function of CSD representation ϕ_1, ϕ_2 , respectively.

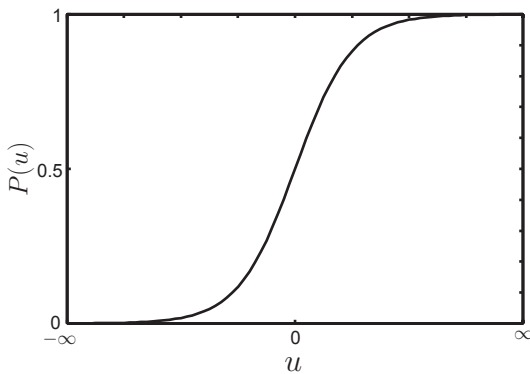


Fig. 4. A shape of sigmoid function

ϕ_1 is the penalty for the number of non-zero digits. ϕ_1 is defined using the number of nonzero digits λ as following,

$$\phi_1 = \begin{cases} 0 & , \lambda \leq \Lambda \\ \lambda - \Lambda & , \text{ otherwise } \end{cases}, \quad (17)$$

where

$$\lambda = \sum_{n=0}^M \sum_{m=1}^p (b'_{n,m} + b''_{n,m}). \quad (18)$$

ϕ_2 is the penalty for forbidding two adjacent non-zero digits. ϕ_2 is defined as

$$\phi_2 = \begin{cases} 0 & , b'_{n,j} + b''_{n,j} + b'_{n,j+1} + b''_{n,j+1} \leq 1 \\ b'_{n,j} + b''_{n,j} + b'_{n,j+1} + b''_{n,j+1} & , \text{ otherwise } \end{cases}. \quad (19)$$

VI. DESIGN EXAMPLES

In this section, several design examples are shown to present the efficiency of the proposed method. The desired response was defined for all examples as following,

$$D(\omega) = \begin{cases} 1, & 0 \leq \omega \leq 2\pi f_p \\ 0, & 2\pi f_s \leq \omega \leq \pi \end{cases}, \quad (20)$$

where f_p is the normalized passband edge frequency, f_s is the normalized stopband edge frequency. The number of dividing frequency $S = 10M$. The maximum number of the non-zero digits Λ is set to $3(M + 1)$ in total. The design conditions in each example are listed in Table I. From preliminary verifications, the optimal 0-1PSO parameters in all examples were decided as shown in Table II. The number of the particles was set to $P = 30$, the maximum number of the iterations was set to $T = 1000$ and the number of trials was set to 100.

As a comparison, the design using StPSO[3] was carried out. In [3], the modified function $F(x)$ has a large number of local minimums and the updating of PSO often tends to stagnate. The number of pioneer particles is set to P_P and the number of neighborhood solutions is set to R . The StPSO parameters[3] used in each example are listed in Table III. The optimal parameters were decided from preliminary verifications. The number of non-zero digits was set to 3 for each coefficient. The maximum number of the iterations was set to $T = 1000$ and the number of trials was set to 100.

TABLE I
DESIGN CONDITIONS

	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5
N	30	56	76	100	130
f_p	0.100	0.225	0.170	0.220	0.210
f_s	0.180	0.265	0.200	0.240	0.228
p	8	8	8	16	16

TABLE II
THE 0-1PSO PARAMETERS

c_1	c_2	$w_{\max}$	$w_{\min}$	α	W	s_1	s_2
2.0	1.0	0.8	0.3	20	1.0	1.0	1.0

TABLE III THE STPSO PARAMETERS					
	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5
$c1$			1.2		
$c2$			2.6		
$w_{\max}$			0.8		
$w_{\min}$			0.3		
W			3.0		
s			1.0		
P	20	30	30	30	30
P_P	2	3	4	5	5
R	15	15	25	30	30

In all examples, the initial values were calculated using a linear programming method in continuous space and the coefficients are simply rounded to the nearest CSD number. The allocation of non-zero digits in each example are shown in Fig. 5–Fig. 14.

The absolute value of coefficient tends to be large for a small n and it needs more non-zero digits. From Fig. 5 and Fig. 6, a similar tendency can be verified. The effectiveness of the proposed method is particularly shown in the design of high order filters. On the other hand, the 0-1PSO is able to allocate more number of non-zero digits while satisfying the constraints. For the large n , the 0-1PSO can further reduce the number of non-zero digits. Thus, the non-zero digits allocation can be achieved effectively by the proposed method.

The $\delta_{\min}$, $\delta_{\max}$ and δ_{mean} is listed in Table IV. The $\delta_{\min}$, $\delta_{\max}$ and δ_{mean} is a minimum δ , a maximum δ , a mean δ in all trials, respectively. From the Table IV, it can be confirmed that a good design can be achieved by the effective non-zero allocation.

The magnitude response of Ex.3 is shown in Fig.15 and its passband magnitude response is shown in Fig.16. The magnitude response of Ex.5 is shown in Fig.17 and its passband magnitude response is shown in Fig.18.



Fig. 5. Allocation of non-zero digits (Ex.1 0-1PSO)

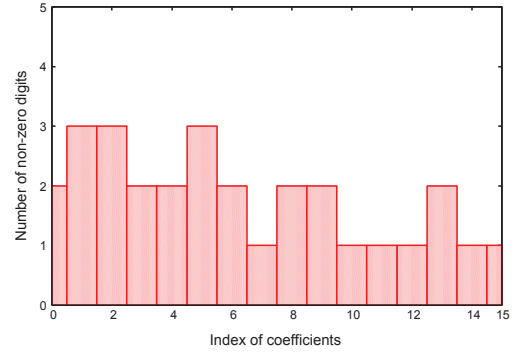


Fig. 6. Allocation of non-zero digits (Ex.1 StPSO)

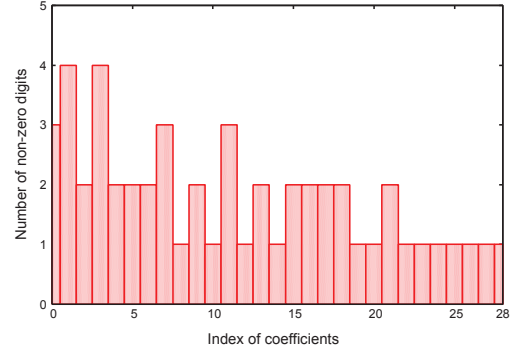


Fig. 7. Allocation of non-zero digits (Ex.2 0-1PSO)

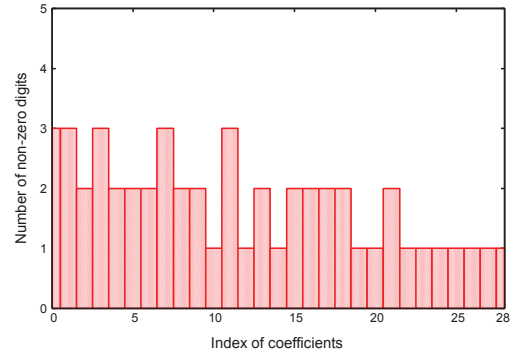


Fig. 8. Allocation of non-zero digits (Ex.2 StPSO)

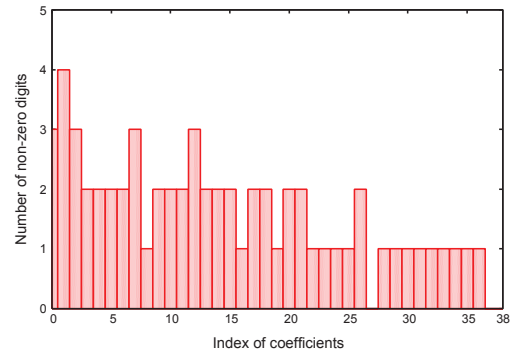


Fig. 9. Allocation of non-zero digits (Ex.3 0-1PSO)

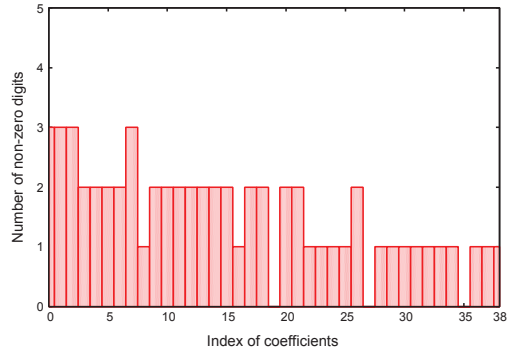


Fig. 10. Allocation of non-zero digits (Ex.3 StPSO)

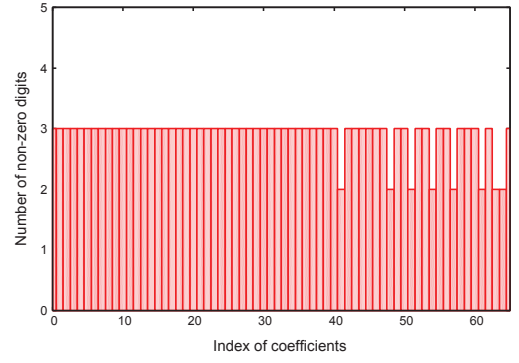


Fig. 14. Allocation of non-zero digits (Ex.5 StPSO)

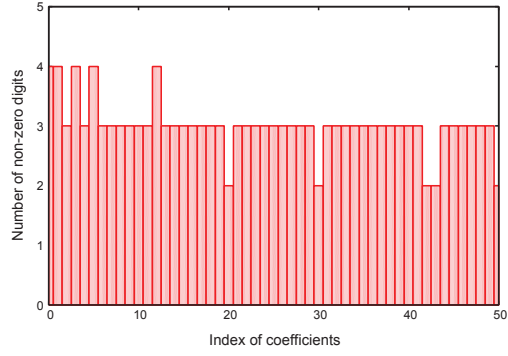


Fig. 11. Allocation of non-zero digits (Ex.4 0-1PSO)

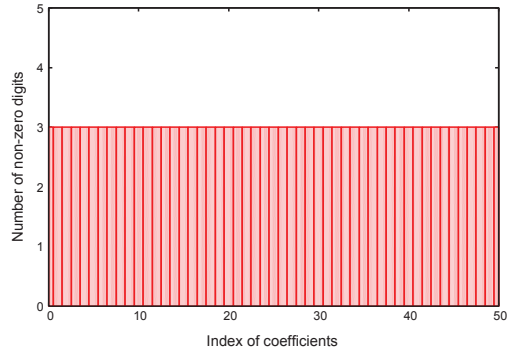


Fig. 12. Allocation of non-zero digits (Ex.4 StPSO)

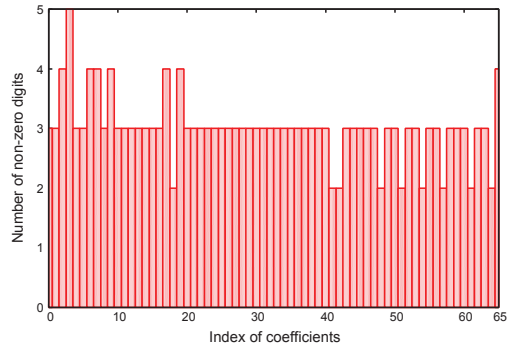


Fig. 13. Allocation of non-zero digits (Ex.5 0-1PSO)

TABLE IV DESIGN RESULTS				
$\times 10^{-2}$	method	$\delta_{\min}$	$\delta_{\max}$	δ_{mean}
Ex.1	0-1PSO	0.886	1.092	0.939
	StPSO	0.886	1.172	1.003
Ex.2	0-1PSO	1.193	1.956	1.532
	StPSO	1.570	2.516	2.137
Ex.3	0-1PSO	1.172	1.880	1.381
	StPSO	1.543	2.270	1.950
Ex.4	0-1PSO	1.131	1.292	1.213
	StPSO	1.167	2.150	1.791
Ex.5	0-1PSO	0.612	0.979	0.772
	StPSO	0.984	1.178	1.057

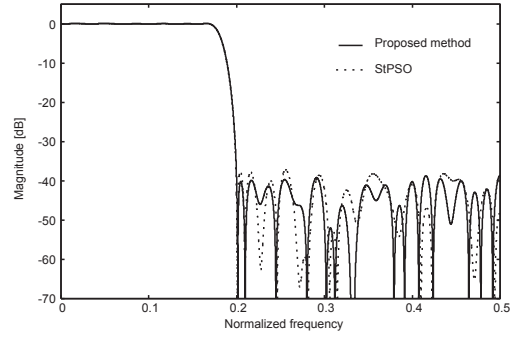


Fig. 15. Magnitude response (Ex.3)

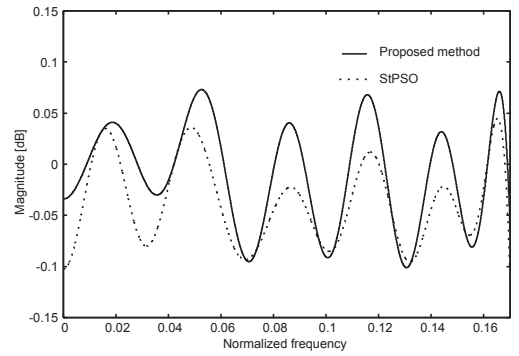


Fig. 16. Passband magnitude response (Ex.3)

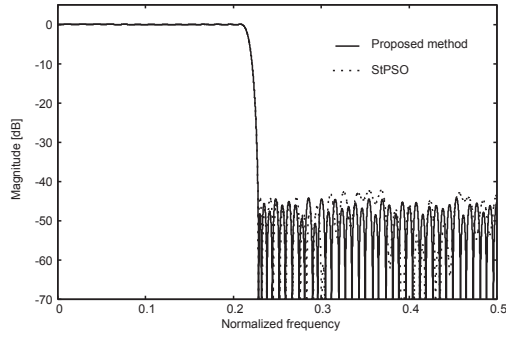


Fig. 17. Magnitude response (Ex.5)

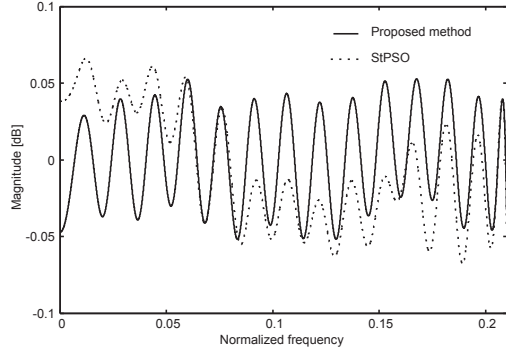


Fig. 18. Passband magnitude response (Ex.5)

VII. COMPARISON WITH GA

In this section, a comparison with the design using the GA is described. The crossover and the mutation of the GA often makes difficult to maintain the CSD structure. Thus, a new rule has been developed[4]. In the comparison, the number of populations was set to 30, and the number of the pairs of crossover was set to 20. The number of generations was set to 1000 and the number of trials was set to 100. The number of non-zero digits was set to $3(M + 1)$ in total.

The $\delta_{\min}$, $\delta_{\max}$ and δ_{mean} obtained are listed in Table V. From the Table V, it can be confirmed that the good design can be achieved by the 0-1PSO comparison with the GA. The 0-1PSO is able to ensure to the CSD structure automatically by only adding the penalty functions. Of course, it can be expected the GA will be able to provide the good design by

adding additional processes.

VIII. CONCLUSION

In this paper, an effective allocation of non-zero digits for CSD coefficient FIR filters using the 0-1PSO is proposed. In the proposed method, the maximum number of non-zero digits is limited for all coefficients and the 0-1PSO is applied. As a result, it was shown that the proposed method achieved an effective allocation of non-zero digits and provided a good design.

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TABLE V
A COMPARISON WITH USING THE GA

$\times 10^{-2}$	method	$\delta_{\min}$	$\delta_{\max}$	δ_{mean}
Ex.1	0-1PSO	0.886	1.092	0.939
	GA	0.886	1.150	0.970
Ex.2	0-1PSO	1.193	1.956	1.532
	GA	1.348	2.250	1.833
Ex.3	0-1PSO	1.172	1.880	1.381
	GA	1.506	2.139	1.823
Ex.4	0-1PSO	1.131	1.292	1.213
	GA	1.240	1.410	1.326
Ex.5	0-1PSO	0.612	0.979	0.772
	GA	0.658	1.427	0.885