Base Station Cooperation for Energy Efficiency: A Gauss-Poisson Process Approach

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Abstract—Base station cooperation is an effective means of improving the spectral efficiency of cellular networks. From an energy-efficiency perspective, whether base station cooperation benefits the network performance remains an issue to be answered. In this paper, we adopt tools from stochastic geometry to treat this issue. Specifically, we model the cooperating base stations as clusters in a Gauss-Poisson process, a variant of the usually considered Poisson point processes. We compare the performance in terms of energy efficiency with and without base station cooperation. The results reveal that only when the cooperative base stations account for a large proportion of all the base stations will the cooperation among base stations bring gains to the energy efficiency of the network.

Index Terms—Base station cooperation, energy efficiency, Gauss-Poisson process, mean achievable rate, stochastic geometry

I. INTRODUCTION

With the depletion of non-renewable resources, the energy consumption of cellular network is attracting much attention recently. According to some rough estimation, about 3 percent of the world’s annual electrical energy consumption is caused by the information and communication technology (ICT) infrastructure, which is still growing at 15-20 percent per year, doubling every five years [1].

Cooperation among base stations is a promising technique for improving the spectral efficiency of cellular networks. Indeed, by introducing cooperation, some of interfering transmitters become cooperating transmitters, in which case the power of desired signal increases and that of the interference decreases, thus significantly improving the spectral efficiency. However, as for the energy consumption, the cost from the more sophisticated signal processing and additional backhaul for cooperation among base stations should be considered. Therefore, whether cooperation benefits the energy efficiency of a cellular network remains an open problem. Related work about network coordination can be found in [2]–[5]. The work in [4] summarized challenges of uplink and downlink cooperation, in which backhaul for CoMP is also included. The work in [5] introduced two cooperative schemes, namely zero-forcing transmission and dirty paper coding.

Instead of modeling the spatial distribution of base stations as the hexagonal grid, we turn to use the tools from stochastic geometry, which are often more tractable and convenient to use for deriving closed form results for key performance metrics, such as coverage probability and mean achievable rate in [6] and [7]. Most of the previous work use the Poisson point process (PPP) to model the locations of base stations due to its tractability [8]–[10]. Indeed, as shown in [8], results derived from the PPP model provide a good approximation to the actual performance of the cellular system. In more detail, the PPP model gives a lower bound for the performance of the actual network while the regular hexagonal grid gives an upper bound with comparable accuracy. However, since different points in a PPP are independent, the PPP model can hardly capture the correlation among different base stations when deploying the cellular network. For example, in a practical network with cooperation, some of the base stations are equipped with remote radio heads (RRH), which tend to be in pair with the base station, and the PPP model cannot exactly characterize this coupling nature. Based on this consideration, we propose to use the Gauss-Poisson process to model the cooperation phenomenon. The Gauss-Poisson process, which we will describe in detail in Section II, is a variant of Poisson clustering process, with only one or two points in each cluster.
The analysis of the Gauss-Poisson process model is more complicated than that of the PPP model, and closed form results are usually not available; thus in addition to deriving the exact expressions, we also present lower and upper bounds to simplify the results.

The contributions of our work are as follows. First, we propose a novel network model, in which the spatial distribution of base stations is modeled as a Gauss-Poisson process, to analyze the energy efficiency of cellular networks with cooperation. Second, we derive lower and upper bounds of the mean achievable rate, which are convenient for numerical evaluation. Third, based on the results, we evaluate how the energy efficiency of the cooperative network varies with different system parameters. In the derivations, we make use of a general power consumption model which takes into consideration the additional power consumption caused by the cooperating operation.

The rest of this paper is organized as follows: In Section II, we described the system model in detail. In Section III, we first analyze the area spectral efficiency of our model, and then introduce the power consumption model that we make use of for analysis. Based on which, we derive the energy efficiency of the network. Section IV provides the simulation results. Finally, Section V concludes the paper.

II. SYSTEM MODEL

A. Spatial Distribution

In the downlink cooperative network, we consider two types of access points, the base stations and RRHs attached to them, both of which are assumed to be equipped with omnidirectional antennas. We divided the base stations into two categories, those with one RRH and those without RRH. (Although there are maybe more than one RRH attached to a base station, we take this model for ease of analysis, and the situation for multiple RRHs will be considered in future work.) If a base station has an affiliated RRH, the base station and the corresponding RRH can cooperatively serve a user. We assume that the RRH is at a fixed distance $d$ to the corresponding base station. Based on these considerations, we model the spatial distribution of base stations (including the RRHs) as a Gauss-Poisson process. The Gauss-Poisson process is a variant of Poisson clustering process, in which each cluster has only one or two points with probabilities $1-p$ and $p$ respectively. The centers of the clusters are assumed to be distributed according to a stationary PPP $\Phi_p$ of intensity $\lambda_p$, which is called the parent process. For each cluster center $x \in \Phi_p$, the set of daughter points is denoted by $\Phi^x$, which has one or two points. If a cluster consists of one daughter point, that point is at the parent’s location (the center of that cluster). If it has two daughter points, the two points are separated by a fixed distance $d$ and have the parent’s location as the location of one of the two points. The orientation of the line connecting between the two points is uniformly distributed in $[0, 2\pi]$. We model the base stations as the daughter points at the parents’ locations and model the RRHs as other daughter points. Then the Gauss-Poisson process (or the locations of both base stations and RRHs) can be expressed as follows:

$$\Phi = \bigcup_{x \in \Phi_p} \Phi^x.$$  

Without loss of generality, we take a typical user located at the origin. Our analysis below is conditioned on that there is one typical cluster $\Phi^{x_0}$ of the Gauss-Poisson process whose center is located at $x_0 = (r_0, 0)$, i.e., $x_0 \in \Phi_p$ (see Figure 1). In the cooperative case, the typical user is solely served by the base station if the typical cluster has only one daughter point or is cooperatively served by both of the base station and the RRH if the typical cluster has two daughter points. Since the
parent process is a PPP, by the Slivnyak-Mecke
Theorem [6, page 168], conditioning on the typical
cluster at \((r_0, 0)\) does not change the distribution of
the rest of the Gauss-Poisson process, which will
simplify the analysis of the interference. A possible
alternative assumption would be that the serving
base station is taken from the Gauss-Poisson process
with maximal average received power. However, in
that case the only difference is that there will be
one more integral on \(r_0\), which does not result in
essential difference to the analysis.

B. Path Loss and Power Consumption Model

We assume that the available spectrum with band-
width \(B\) is shared by both the base stations and
RRHs and the transmit power is denoted by \(P_{tx}\). For
sake of convenience, we adopt a standard path loss
propagation model with path loss exponent \(\alpha > 2\).
As for fading, we assume that all the links experi-
ence independent and identically distributed (i.i.d.)
Rayleigh fading with parameter \(\mu = 1\); therefore,
the fading coefficient between the transmitter \(x\) and
the typical user at the origin, denoted by \(h_x\), follows
an exponential distribution, i.e. \(h_x \sim \text{Exp}(1)\).

Introducing cooperation in the network increases
the spectral efficiency; however, the power con-
sumption of the cooperative base station or RRH
also increases because of the additional cost for
backhaul and signal processing. In our work, we
apply the following power consumption model (see
also [11]–[13]):

\[
P_c = N_{PA} \left( \frac{P_{tx}}{\mu_{PA}} + P_{SP} \right) (1+C_c)(1+C_{PSBB}) + P_{BH},
\]

where \(N_{PA}\) is the number of power amplifiers,
\(P_{tx}\) is the transmit power, \(\mu_{PA}\) is the PA efficiency,
\(P_{SP}\) is the power consumption for signal processing with
cooperation, \(P_{BH}\) represents the backhaul power con-
sumption for cooperation, \(C_c\) is the cooling loss,
and \(C_{PSBB}\) is the battery backup and power supply
loss. The power consumption for signal processing
\(P_{SP}\) in the cooperative model is as follows:

\[
P_{SP} = 58(0.87 + 0.1N_c + 0.03N_c^2),
\]

where \(N_c\) is the cooperating set degree (i.e., the
number of cooperative access points). In our model,
if a base station or a RRH is in cooperative mode,
the number of cooperative access points is \(N_c = 2\).

As for the non-cooperative base stations, we only
need to set \(N_c = 1\) and \(P_{BH} = 0\); thus the power
consumption model for the non-cooperative base
stations is as follows:

\[
P_{nc} = N_{PA} \left( \frac{P_{tx}}{\mu_{PA}} + P'_{SP} \right) (1+C_c)(1+C_{PSBB}),
\]

where \(P'_{SP} = 58W\).

C. Non-cooperative Model

In the non-cooperative case, a user is served
by only one access point, either a base station or
a RRH. All points of the Gauss-Poisson process
except for the serving access point \(x_0\) cause inter-
ference to the typical receiver (If the serving cluster
has two points, they will interfere each other), and
the SINR at the typical receiver is

\[
\text{SINR} = \frac{h_{x_0}r_0^{-\alpha}}{\sum_{x \in \Phi \setminus \{x_0\}} h_x \|x\|^{-\alpha} + \frac{\sigma_0^2}{P_{tx}}},
\]

where \(h_{x_0}\) is the fading coefficient between the
typical receiver and the desired transmitter, and \(h_x\)
is the fading coefficients of the interference links.
The noise power is assumed to be constant with
value \(\sigma_0^2\).

D. Cooperative Model

For the cooperative case, if a cluster of the Gauss-
Poisson process has two points (i.e., one base station
and one RRH), the two transmitters jointly transmit
signal to a receiver; therefore, the channel from the
transmitters to the receiver is a MISO channel. We
assume that V-BLAST is used, in which the data
streams from the two cooperative transmitters are
independent. In this case, the power of desired sig-
nal at the typical receiver is the superposition of the
received signal from all points in the serving cluster,
which has either one point or two points. The points
from other clusters of the Gauss-Poisson process are
considered as interferers. The interference form two
different transmitters in a cooperative cluster is also
independent. Therefore, in the cooperative case, the
SINR at the typical receiver is

\[
\text{SINR} = \frac{\sum_{x \in \Phi_{x_0}} h_x \|x\|^{-\alpha}}{\sum_{x \in \Phi \setminus \Phi_{x_0}} h_x \|x\|^{-\alpha} + \frac{\sigma_0^2}{P_{tx}}},
\]

where \(\Phi_{x_0}\) is the typical cooperative cluster located
at \(x_0\) which includes the serving transmitters.
III. ANALYSIS OF NETWORK ENERGY EFFICIENCY

In this section, we analyze the network energy efficiency in both the cooperative and non-cooperative cases. The energy efficiency is defined as the ratio of throughput to the energy consumption of the network. In the following analysis, we first derive the mean achievable rate of the network, then we calculate the energy efficiency of the network. We consider the interference-limited network, in which the noise power is much smaller compared to the interference, so that we ignore the thermal noise by setting $\sigma_0^2 = 0$ for simplicity, and we also assume the path loss exponent to be $\alpha = 4$.

A. Mean achievable rate

The following theorem gives the mean achievable rates of our cooperative and non-cooperative model.

**Theorem 1:** The mean achievable rate of the cooperative model is

$$
\tau_c = \frac{1}{\ln 2} \int_0^\infty \left[ (1-p) \mathcal{L}_{I_r}(r_0^4) + p E_{r_1} \left( \frac{r_0^4 \mathcal{L}_{I_r}(r_0^4)}{r_0^4 - r_1^4} \right) \right] \frac{1}{1 + t} dt.
$$

where $r_1 = \sqrt{\tau_0^2 + d + 2\tau_0 d \cos \theta}$ and $\theta \sim \text{unif}(0, 2\pi)$. The mean achievable rate of the non-cooperative model is

$$
\tau_{nc} = \frac{1}{\ln 2} \int_0^\infty \frac{1}{1 + t} \mathcal{L}_{I_r}(r_0^4)(1-p) + p \frac{\lambda_p}{2 \pi} \int_0^{2\pi} \frac{1}{1 + \tau_0^4 (r_0^2 + d + 2\tau_0 d \cos \beta)^{-2}} d\beta dt.
$$

where $\mathcal{L}_{I_r}(s)$ is the Laplace transform of the interference $I_r$, which is given by

$$
\mathcal{L}_{I_r}(s) = \exp \left( -\frac{\lambda_p}{2} (1-p) \sqrt{s} + \lambda_p p \int_0^\infty \frac{1}{1 + sr^{-4}} dr \right) \int_0^\infty \frac{1}{1 + s(r^2 + d + 2rd \cos \theta)^{-2}} d\theta - 2\pi r dr.
$$

The proof of Theorem 1 is in Appendix A.

The calculation of the mean achievable rate of the cooperative model is complicated. The following corollary gives upper and lower bounds which are much more easier for numerical evaluation.

**Corollary 1:** The upper and lower bounds of the mean achievable rate of our proposed cooperative network is as follows

$$
\tau_{c,h} = \frac{1}{\ln 2} \int_0^\infty \left[ \left(1-p - p r_0^4 \frac{(r_0 - 1)^4}{(r_0 - 1)^4} \right) \mathcal{L}_{I_r}(r_0^4) + p \frac{r_0^4}{r_0^4 - (r_0 - 1)^4} \mathcal{L}_{I_r}(r_0^4) \right] \frac{1}{1 + t} dt,
$$

and

$$
\tau_{c,l} = \frac{1}{\ln 2} \int_0^\infty \left[ \left(1-p - p r_0^4 \frac{(r_0 + 1)^4}{(r_0 + 1)^4} \right) \mathcal{L}_{I_r}(r_0^4) + p \frac{r_0^4}{r_0^4 - (r_0 + 1)^4} \mathcal{L}_{I_r}(r_0^4) \right] \frac{1}{1 + t} dt,
$$

where $\mathcal{L}_{I_r}(s)$ is the Laplace transform of the interference $I_r$, which is given by (8).

Proof: As $r_1$ is the distance between the typical user and the serving base station, we have $|r_0 - d| < r_1 < r_0 + d$. To calculate the expectation respective to $r_1$, we replace $r_1$ by $r_0 + d$ and $|r_0 - d|$ to get the lower and upper bounds respectively.

B. Energy Efficiency

The energy efficiency is defined as the ratio of throughput to the energy consumption of the network. For a typical user in the cooperative network, since we have derived the mean achievable rate $\tau_c$, given the system bandwidth $B$, we can get the throughput of a typical user as $\tau_c B$. In the cooperative network, since all transmitters in a cluster only serve one user jointly, the density of the serving users is equal to the density of the clusters, which is $\lambda_p$. Thus, the total throughput per unit area is $\tau_c \lambda_p B$. As for the energy consumption, since the density of the non-cooperative transmitters is $(1-p)\lambda_p$, and that of the cooperative transmitters is $2p\lambda_p$, the total energy consumption per unit area is $\lambda_p ((1-p)P_{nc} + 2P_c)$. Therefore, the expression of the energy efficiency in the cooperative network is as follows:

$$
\eta_c = \frac{\tau_c \lambda_p B}{\lambda_p ((1-p)P_{nc} + 2P_c)}
$$

To derive the energy efficiency from the upper and lower bounds, we just need to replace $\tau_c$ with $\tau_{c,h}$ or $\tau_{c,l}$. 


TABLE I
PARAMETERS OF POWER CONSUMPTION MODEL

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>system bandwidth</td>
<td>10MHz</td>
</tr>
<tr>
<td>$\lambda_p(1 + p)$</td>
<td>the intensity of all access points</td>
<td>$10^{-5}$ m$^{-2}$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>the distance between the typical user and its serving base station</td>
<td>110m</td>
</tr>
<tr>
<td>$d$</td>
<td>the distance between RRH and its corresponding base station</td>
<td>140m</td>
</tr>
<tr>
<td>$N_{PA}$</td>
<td>power amplifiers per sector</td>
<td>2</td>
</tr>
<tr>
<td>$\mu_{PA}$</td>
<td>power amplifier efficiency</td>
<td>0.38</td>
</tr>
<tr>
<td>$C_c$</td>
<td>cooling loss</td>
<td>0.29</td>
</tr>
<tr>
<td>$C_{PSBB}$</td>
<td>battery backup and power supply loss</td>
<td>0.11</td>
</tr>
<tr>
<td>$P_{BH}$</td>
<td>backhaul power consumption</td>
<td>20W</td>
</tr>
</tbody>
</table>

As for non-cooperative network, each transmitter serves a user independently; thus, the density of the serving users is equal to the density of all access points, i.e., $\lambda_p(1 + p)$. Then, the total throughput per unit area is $\tau_c \lambda_p(1 + p)B$. In non-cooperative network, the total energy consumption per unit area is $\tau_c \lambda_p(1 + p)P_{nc}$. Therefore, the expression of the energy efficiency in the non-cooperative network is as follows:

$$
\eta_{nc} = \frac{\tau_c \lambda_p(1 + p)B}{\lambda_p(1 + p)P_{nc}} = \frac{\tau_c B}{P_{nc}}.
$$

(12)

IV. NUMERICAL RESULTS

In this section, the numerical results are obtained according to the analytical results we have derived. The configurations of system model are as follows (also see Table I). The distance between the desired transmitter and the typical receiver is set to $r_0 = 110$m, and the distance between the RRH and its corresponding base station is $d = 140$m. The spatial density of all transmitters (both the base stations and the RRHs), denoted by $\lambda_p(1 + p)$, is fixed as $10^{-5}$m$^{-2}$. The path loss exponent $\alpha$ is set as 4, and the thermal noise is ignored, i.e., $\sigma_0^2 = 0$.

Figure 2 shows the mean achievable rate $\tau_c$ of a typical user in our proposed cooperative network as a function of $p$ which is the probability that there are two points in a cluster of the Gauss-Poisson process. Also shown in this figure is the upper and lower bounds which are much more convenient for numerical evaluation. In the simulation, the time cost for calculating the bounds is only ten percent of direct calculation of the mean achievable rate in Theorem 1. From Figure 2, we observe that the mean achievable rate increases with the increment of the probability $p$. The curve of the mean achievable rate of non-cooperative networks is not included in this figure as the comparison between the cooperative and non-cooperative networks has been shown in Figure 3.

Figure 3 shows difference between the area spectral efficiencies of the cooperative and non-cooperative networks, i.e., $\tau_c \lambda_p$ and $\tau_{nc} \lambda_p(1 + p)$ respectively. From the figure, we observe that when the probability $p$ is small, the area spectral efficiency of cooperative and non-cooperative model is almost the same, and when $p$ is large, the area spectral efficiency of the cooperative network is much better than the non-cooperative network. Figure 3 verifies that cooperation among base stations indeed improves the area spectral efficiency of the network.

Figure 4 shows the energy efficiencies of the cooperative and non-cooperative networks as a function of the probability $p$. The parameters of the
V. Conclusion

In this paper, we use Gauss-Poisson process to model the spatial distribution of both base stations and RRHs. In our model, we assume that there is only one RRH attached to a base station for ease of analysis. The case that there are multiple RRHs will be included in the future work. We derive the mean achievable rate of a typical user in the cooperative and non-cooperative networks. Based on the results we have derived, we evaluate how the area spectral efficiency and the energy efficiency vary with different system parameters.

From the numerical results, we observed that whether cooperation among base stations improves the energy efficiency depends on the proportion between the number of cooperative base stations and that of all base stations. When there are few cooperative base stations in the network, introducing the cooperation will decrease the energy efficiency; however, when the number of cooperative base stations is large, the cooperation will greatly improve the energy efficiency.

Appendix A

The mean achievable rate of the typical receiver located at \( x_0 = (r_0, 0) \) is

\[
\tau(r_0, \lambda_P, p) = \mathbb{E} \left[ \log_2(1 + \text{SINR}) \right] \\
= \frac{1}{\ln 2} \mathbb{E} \left[ \ln(1 + \text{SINR}) \right] \\
= \frac{1}{\ln 2} \int_{t>0} \mathbb{P} \left[ \ln(1 + \text{SINR}) > t \right] dt \\
= \frac{1}{\ln 2} \int_{t>0} \mathbb{P} \left( \text{SINR} > e^t - 1 \right) dt \\
= \frac{1}{\ln 2} \int_0^\infty \mathbb{P}(t, r_0, \lambda_p, p) \frac{1}{1 + t} dt.
\]

(13)

where \( \mathbb{P}(t, r_0, \lambda_p, p) \) is the coverage probability, \( t \) is the SIR threshold and \( \lambda_p \) is the intensity of the parent process. The probability generating functional (PGFL) of the Gauss-Poisson process is given by

\[
G[v] = \exp \left( \lambda_p \int_{\mathbb{R}^2} ((1 - p)v(x) + pv(x)) f(y) dy - 1 \right) dx.
\]

(14)
where $v(x)$ is a function of $x$, and $f(y)$ is the probability density function of the spatial location of the daughter point at distance $d$ to the parent’s location in a cluster located at the origin with two daughter points. By applying the standardized derivations of the coverage probability in [8], we get the coverage probabilities of the cooperative and non-cooperative networks as follows:

$$p_c(t, r_0, \lambda_p, p) = (1 - p) L_{I_r}(tr_0^4) + p \mathbb{E}_{r_1} \left( \frac{r_0^4 L_{I_r}(tr_0^4)}{r_0^4 - r_1^4} \right).$$

and

$$p_{nc}(t, r_0, \lambda_p, p) = L_{I_r}(tr_0^4) \left( 1 - p + p \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{1 + tr_0^4(r_0^2 + d + 2r_0d \cos \beta)^2} d\beta \right).$$

Thus, we get the results in Theorem 1.

**REFERENCES**


