Improved Reconstruction for Computer-Generated Hologram from Digital Images

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Abstract—In this paper, the reconstruction process of digital image's Computer Generated Hologram (CGH) is mathematically analyzed from the point of Discrete Fourier Transform (DFT). Based on such analysis, a convenient and effective method is proposed to improve the quality of digital image's CGH reconstruction. By eliminating the Direct Component (DC) of the DFT output of the hologram, the original digital image can be precisely reconstructed from the Burch CGH. Experiments have been done and the results show that the reconstructed realimage is exactly the same as the original digital image used to create the CGH, except that the pixel values are being scaled by some certain ratios. The experiment results also show that the proposed method can effectively improve the reconstruction quality of Huang's CGH and Wai-Hon-Lee's CGH.

Index Terms—Computer-Generated Hologram, DFT, Burch, DC Component

I. INTRODUCTION

The CGH of digital image is increasingly applied in the field of information encryption, such as holographic watermarking [1-4][14-15]. Good image quality of the CGH reconstruction is very important for the holographic watermark extraction. However, there is not much theoretical work on the digital images' CGH reconstruction. In this paper, we give a comprehensive analysis of the digital image's CGH reconstruction from a mathematical angle, especially from the viewpoint of DFT, since we usually use DFT to reconstruct the twin-image from CGHs.

Digital image's CGH reconstruction becomes much more simple when the reconstruction is done by computer. Traditionally, the reconstruction of the hologram requires the reference beam. The fourier transform can be easily implemented by the Fast Fourier Transform (FFT). However, in the digital image's hologram case, it becomes very simple that we do not need the reference beam for the CGH reconstruction, because after a comprehensive theoretical analysis, we find that the reference beam only causes a translation of the reconstructed twin-image in the reconstruction plane.

Direct FFT of the hologram will bring in noise of the DC component [1-4], which decreases the image contrast of the reconstructed twin-image. So we propose a method to locate the position of the DC component and effectively eliminate the DC component of the DFT coefficients before displaying the reconstructed twin-image, instead of the conventional method

to display the log value of the DFT coefficients. After a mathematical analysis of the CGH's reconstruction in the angle of DFT, this paper found that the original digital image can be accurately reconstructed from the Burch CGH by eliminating the DC component.

II. THE RECONSTRUCTION OF COMPUTER GENERATED HOLOGRAM

According to the holographic principle, we define the object wave $O(x, y) = A(x, y) \exp [j\phi(x, y)]$, and the parallel reference beam $R(x, y) = R \exp(j2\pi\alpha x)$ [5-7]. Then the transmittance of the hologram h(x, y) made with the off-axis reference could be represented as follows:

$$h(x,y) = |O(x,y) + R(x,y)|^{2}$$

= $|A(x,y) \exp[j\phi(x,y)] + R\exp(j2\pi\alpha x)|^{2}$
= $A(x,y)^{2} + R^{2}$
 $+ 2RA(x,y) \cos[2\pi\alpha x - \phi(x,y)]$ (1)

This paper mainly discusses the Fourier holograms of the digital image I(p,q), here we define $F\langle\rangle$ as DFT and $F^{-1}\langle\rangle$ as IDFT (Inverse DFT), then

$$\begin{cases} O(x,y) &= F \langle I(p,q) \rangle \\ I(p,q) &= F^{-1} \langle O(x,y) \rangle \end{cases}$$
(2)

The sampling numbers M and N of I(p,q) could be regarded as the bandwidth of $F^{-1} \langle O(x,y) \rangle$. According to the properties of DFT and IDFT, the bandwidth of $F \langle O(x,y) \rangle$ equals to that of $F^{-1} \langle O(x,y) \rangle$, so $B_x = M$, $B_y = N$.

In the following sections, we look into those aspects that would affect the quality of the reconstruction of the CGH from digital images.

A. The Role of the Reference Beam

If $F^{-1}\langle h(x,y)\rangle = I''(p,q)$, then $F^{-1}\langle h(x,y)R\exp(j2\pi\alpha x)\rangle = RI''(p-\alpha M,q)$. That is to say, the reference beam just causes some translation of the CGH reconstruction in the reconstruction plane without the reference beam.

B. CGH Reconstruction by DFT and IDFT

Given a matrix t(x, y), whose size is $M \times N$, according to the properties of the two dimensional DFT, we get

$$\begin{cases} F \langle t(x,y) \rangle &= T(p,q) \\ F^{-1} \langle t(x,y) \rangle &= \frac{1}{MN} T(M+1-p,N+1-q) \end{cases}$$
(3)

Similarly, if the sampling numbers of the DFT are $M \times N$ in the CGH generation and reconstruction processes, then we will get

$$\begin{cases} F^{-1} \langle O(x,y) \rangle &= I(p,q) \\ F \langle O(x,y) \rangle &= F \langle F \langle I(p,q) \rangle \rangle \\ &= MN * I(M+1-p,N+1-q) \end{cases}$$
(4)

From Equation (4), we could see that we can get the virtual version of the original image by two successive transformations of DFT or IDFT. This property of DFT can help us to understand the reconstruction of the Fourier CGH by DFT or IDFT.

We will take the Burch's CGH algorithm for example to illustrate the basic idea of the CGH reconstruction by DFT, the other two cases of Huang's algorithm and Wai Hon Lee's algorithm are similar [8-10].

C. Without the Random Phase Diffusion

The transmittance of the Burch's CGH can be represented as

$$h(x,y) = \frac{1}{2} + \frac{1}{2}A(x,y)\cos[j2\pi\alpha x - \phi(x,y)],$$
 (5)

and the objective wave can be represented as

$$F\langle I(p,q)\rangle = O(x,y) = A(x,y)e^{j\phi(x,y)}.$$
(6)

In practical, A(x, y) should be first normalized to B(x, y),

i.e.,
$$B(x,y) = \frac{A(x,y)}{\max(A(x,y))}$$
, $\max(A(x,y)) = A(1,1) = MN \cdot \overline{I(p,q)}$, where $\overline{I(p,q)}$ represents the average of the gray value of all pixels of $I(p,q)$, thus, the transmittance could be represented as

$$h(x,y) = \frac{1}{2} + \frac{1}{2}B(x,y)\cos\left[2\pi\alpha x - \phi(x,y)\right]$$

= $\frac{1}{2} + \frac{1}{4}B(x,y)e^{j[2\pi\alpha x - \phi(x,y)]}$. (7)
 $+ \frac{1}{4}B(x,y)e^{-j[2\pi\alpha x - \phi(x,y)]}$

The DFT of the transmittance in Equation (7) could be expressed as

$$F \langle h(x,y) \rangle$$

$$= F \left\langle \frac{1}{2} \right\rangle + F \left\langle \frac{1}{4} B(x,y) e^{j[2\pi\alpha x - \phi(x,y)]} \right\rangle$$

$$+ F \left\langle \frac{1}{4} B(x,y) e^{-j[2\pi\alpha x - \phi(x,y)]} \right\rangle$$

$$= \frac{1}{2} M \cdot N \cdot \delta(1,1) + \frac{1}{4} \cdot F \left\langle B(x,y) e^{-j\phi(x,y)} e^{j2\pi\alpha x} \right\rangle$$

$$+ \frac{1}{4} \cdot F \left\langle B(x,y) e^{j\phi(x,y)} e^{-j2\pi\alpha x} \right\rangle$$
(8)

And we know

$$F\left\langle B(x,y)e^{j\phi(x,y)}\right\rangle = F\left\langle \frac{A(x,y)}{MN \cdot \overline{I(p,q)}}e^{j\phi(x,y)}\right\rangle = \frac{1}{MN \cdot \overline{I(p,q)}}F\left\langle O(x,y)\right\rangle = \frac{1}{\overline{I(p,q)}}I(M+1-p,N+1-q)$$
(9)

According to the properties of two dimensional DFT as shown below

$$\begin{cases} F \langle I(p,q) \rangle = A(x,y)e^{j\phi(x,y)} \\ F^{-1} \langle I(p,q) \rangle = \frac{1}{MN}A(x,y)e^{-j\phi(x,y)} , \quad (10) \end{cases}$$

then,

$$F\left\langle B(x,y)e^{-j\phi(x,y)}\right\rangle = \frac{1}{MN \cdot \overline{I(p,q)}}F\left\langle A(x,y)e^{-j\phi(x,y)}\right\rangle$$
$$= \frac{1}{MN \cdot \overline{I(p,q)}}F\left\langle MN \cdot F^{-1}\left\langle I(p,q)\right\rangle \right\rangle = \frac{1}{\overline{I(p,q)}}I(p,q)$$
(11)

thus

$$F \langle h(x,y) \rangle$$

$$= F \left\langle \frac{1}{2} \right\rangle + F \left\langle \frac{1}{4} B(x,y) e^{j[2\pi\alpha x - \phi(x,y)]} \right\rangle$$

$$+ F \left\langle \frac{1}{4} B(x,y) e^{-j[2\pi\alpha x - \phi(x,y)]} \right\rangle$$

$$= \frac{1}{2} MN \cdot \delta(1,1) + \frac{1}{4} \cdot F \left\langle B(x,y) e^{-j\phi(x,y)} e^{j2\pi\alpha x} \right\rangle$$

$$+ \frac{1}{4} \cdot F \left\langle B(x,y) e^{j\phi(x,y)} e^{-j2\pi\alpha x} \right\rangle$$

$$= \frac{1}{2} MN \cdot \delta(1,1) + \frac{1}{4} \cdot \frac{1}{\overline{I(p,q)}} I(p - \alpha M, q)$$

$$+ \frac{1}{4} \cdot \frac{1}{\overline{I(p,q)}} I(M + 1 - p - \alpha M, N + 1 - q)$$
(12)

D. With the Random Phase Diffusion

In practical, in order to distribute the information uniformly in the hologram plane, the digital image should first be multiplied by a random phase component, resembling the role of the ground glass in the optical holography. We define rand(p,q) as the random phase component, and then we obtain $I'(p,q) = I(p,q)e^{j\cdot 2\pi \cdot rand(p,q)}$. From Equation (12), we get the reconstruction of the CGH generated from I'(p,q)as

$$F \langle h(x,y) \rangle = \frac{1}{2} MN \cdot \delta(1,1) + \frac{1}{4} \frac{1}{\overline{I'(p,q)}} \cdot [I'(p-\alpha M,q) + I'(M+1-p-\alpha M,N+1-q)]$$
(13)

Generally, if we assume the size of the original digital image is m and n, and the sampling numbers of DFT or IDFT is . $M = \beta_1 m$, $N = \beta_2 n$, then

$$F \langle h(x,y) \rangle = \frac{1}{4 \cdot \overline{I'(p,q)}} \left[2\beta_1 \beta_2 mn \cdot \overline{I'(p,q)} \cdot \delta(1,1) + I'(p - \alpha\beta_1 m,q) + I'(\beta_1 m + 1 - p - \alpha\beta_1 m, \beta_2 n + 1 - q) \right]$$
(14)



Fig. 1. The Burch algorithm: (a) and (e) are the original images, (a) is a binary image and (e) is a gray scale image; (b) and (f) are the holograms generated from (a) and (e) respectively; (c) and (g) are the reconstructed twin-images by the traditional method, while (d) and (h) are the reconstructed twin-imagess by the proposed reconstruction method.

For the reconstruction of CGH, we only need to detect the amplitude of the output from DFT, which could be expressed as below

$$|F\langle h(x,y)\rangle| = \frac{1}{4\overline{I'(p,q)}} \left| \left[2\beta_1\beta_2mn \cdot \overline{I'(p,q)} \cdot \delta(1,1) + \right. \right]$$
$$I'(p - \alpha\beta_1m,q) + I'(\beta_1m + 1 - p - \alpha\beta_1m,\beta_2n + 1 - q) \right|$$
(15)

Although the three terms in the above equation are complex representations, as long as the sampling numbers are big enough, those three terms could be effectively separated in different locations in the reconstruction plane, so the modulus of the sum of the three terms equals to the sum of the modulus of those three terms. Then the equation above could be put as follows

$$|F \langle h(x,y) \rangle| = \frac{1}{4\overline{I'(p,q)}} [\left| 2\beta_1\beta_2 mn \cdot \overline{I'(p,q)} \cdot \delta(1,1) \right| + |I'(p - \alpha\beta_1 m, q)| + |I'(\beta_1 m + 1 - p - \alpha\beta_1 m, \beta_2 n + 1 - q)|] \\= \frac{1}{4\overline{I'(p,q)}} [2\beta_1\beta_2 mn \cdot \overline{I(p,q)} \cdot \delta(1,1) + I(p - \alpha\beta_1 m, q) + I(\beta_1 m + 1 - p - \alpha\beta_1 m, \beta_2 n + 1 - q)]$$
(16)

E. Eliminating the DC Component

In Equation (16), we can see that the last two terms are the twin-images of the original image, while the first term is the impulse component $2\beta_1\beta_2mn \cdot \overline{I'(p,q)} \cdot \delta(1,1)$. In fact, this impulse component has no use to the original digital image reconstruction, and it will severely reduce the contrast of the reconstructed twin-image. So in order to improve the CGH's reconstruction image quality, we eliminate the first term in Equation (16) before displaying the twin-image in the reconstruction plane. Then the CGH reconstruction by DFT without the impulse component could be simplified as



Fig. 2. The Huang algorithm: (a) and (e) are the original images; (b) and (f) are the CGHs generated from (a) and (e) respectively; (c) and (g) are the reconstructed twin-images by the traditional methods, while (d) and (h) are the reconstructed twin-images by the proposed reconstruction method.

TABLE ITHE PIXEL VALUE RATIO BETWEEN THE RECONSTRUCTED REAL IMAGEAND THE ORIGINAL DIGITAL IMAGE, WITH BURCH ALGORITHM INDIFFERENT SAMPLING CASES. $\alpha = 0.25$.

β_1	β_2	Binary Image	Grayscale Image
2	1	0.0788	0 17089
2	2	0.1534	0.36364
3	2	0.2255	0.48582
3	3	0.3136	0.77702
4	4	0.5375	1.345

following:

$$|F\langle h(x,y)\rangle| = \frac{1}{4} \frac{1}{I'(p,q)} \cdot [I(p-\alpha\beta_1m,q) + I(\beta_1m+1-p-\alpha\beta_1m,\beta_2n+1-q)]$$
(17)

We could see the real-image $I(p - \alpha\beta_1 m, q)$ and the virtual image $I(\beta_1 m + 1 - p - \alpha\beta_1 m, \beta_2 n + 1 - q)$ are accurately reconstructed, with some translation shift in the reconstruction plane.

III. EXPERIMENTS

We use two typical image types (binary image and grayscale image) to test our proposed reconstruction method to see how it improves the reconstructed twin-images.

Figure 1 illustrates the Burch CGH reconstruction comparison between by the traditional method and by the proposed method. In the traditional reconstruction method, the range of the DFT output value is too large, which is caused by

TABLE II THE PSNR BETWEEN THE RECONSTRUCTED REAL-IMAGE AND THE ORIGINAL IMAGE IN THE OTHER TWO CGH ALGORITHMS. B STANDS FOR BINARY, G FOR GRAYSCALE, T FOR TRADITIONAL METHOD, AND P FOR PROPOSED METHOD. $\alpha = 0.25$.

Algorithm	β_1	β_2	B (T)	B (P)	G (T)	G (P)
Huang	4	2	0.1752	21.71	-19.76	27.33
Huang	4	4	-9.859	11.20	-22.84	37.99
W. H. Lee	4	4	-7.072	57.43	-19.87	16.24

the existence of the impulse component. So before displaying the reconstructed twin-image, we need to take the log value of the DFT output. From Figure 1, we see that the contrast of the reconstructed twin-image from the CGH (Figure 1 (b)) made from the binary image (Figure 1 (a)) is acceptable, but the image quality is too bad for the gray scale image case (Figure 1 (e) and (g)). After eliminating the impulse component, we could notice the obvious enhancement of the twin-image contrast, especially in the gray scale image case (Figure 1 (h)).

In fact, for the Burch CGH, after comparing the pixel values between the reconstructed real image and the original image, we find they are exactly the same except there is a scaling =, as denoted in Equation (17). And we listed ratio, $4\overline{I'(p,q)}$

the ratios of different cases in Table 1.

The reconstruction cases of the Huang's algorithm and the Wai-Hon Lee's algorithm are shown in Figure 2 and Figure 3, from which we see that the image quality of the twin-images reconstructed by the proposed method is much better than the traditional method. The Peak Signal-to-Noise Ratio (PSNR) between the reconstructed real-image and the original image is given in Table 2. By comparing the 4th and 5th columns (binary case), and 6th and 7th columns (grayscale case), we can clearly see that our proposed method greatly improved the image quality of the reconstructed twin-image.

IV. CONCLUSION

This paper proposed a method to improve the digital image's CGH reconstruction by eliminating the DC component in the reconstruction plance. The reconstruction of digital image's Fourier CGH is analyzed from the viewpoint of DFT, and we proved that the original digital image can be precisely reconstructed from the Burch CGH, by eliminating the DC of the FFT output of the hologram. Experimental results show that the proposed method can also effectively improve the quality of the reconstructed twin-image of the Huang's CGH and Wai-Hon-Lee CGH.

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Fig. 3. The Wai-Hon-Lee algorithm: (a) and (e) are the original images; (b) and (f) are the CGH generated from (a) and (e) respectively; (c) and (g) are the reconstructed twin-images by the traditional method, while (d) and (h) are the reconstructed twin-images by the proposed method.

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