An Avoidance of Premature Convergence in IIR Filter Design using PSO

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***Abstract*—In this paper, a design method for the Infinite Impulse Response (IIR) filter using the Particle Swarm Optimization (PSO) is developed. Because the PSO has a strong directivity toward a local minimum, the PSO updating tends to stagnate around such a local minimum and thus indicates a premature convergence. Recently, the Asynchronous Digenetic PSO with Nonlinear dissipative term (N-AD-PSO) has been proposed for a diverse search. It can be expected that the stagnation can be avoided by the N-AD-PSO. In this study, the N-AD-PSO is adapted for the IIR filter design. Several examples are shown to present the effectiveness of the proposed method.**

I. Introduction

IIR filters are used in many applications, such as communication, measurement and control system. However, the design problem of IIR filters is generally a complex approximation problem to a rational function in which both a magnitude response and a phase response are approximated simultaneously. In addition, a filter stability must be paid more attention in the design problem. Thus, the design problem falls into a nonlinear optimization problem, and it makes the design a tough job.

The design methods of IIR filter are classified into two categories. One is an indirect design method, in which the transfer function of analog filter is transformed into the transfer function of digital filter. Although IIR filters can be designed by such a method easily, an adjustment of transformed frequencies is required because the frequency in the continuous domain does not correspond to the frequency in the discrete domain directly. The other is a direct design method, in which IIR filters can be directly designed in the frequency domain. The design problem of direct design is difficult to solve. For example, the method using the linear programming needs an iterative procedure to solve the problem.

To solve the problem, the method using Particle Swarm Optimization (PSO) [1] has been developed [2] [3]. The PSO is one of multipoint search type algorithms. Multiple local minimums that are candidates of optimal solution can be fast enumerated by the method. Because the PSO has a strong directivity toward a local minimum, the PSO updating tends to stagnate around such a local minimum and thus indicates a premature convergence. Recently the Asynchronous Digenetic PSO with Nonlinear dissipative term (N-AD-PSO) [4], which was developed as an extension of the Nonlinear Dissipative Term PSO (NDT-PSO) [5], has been proposed for a diverse search. It can be expected that the stagnation can be avoided by the method.

In this study, the N-AD-PSO is adapted to the IIR filter design problem. Several examples are shown to present the effectiveness of the method.

II. Design problem of IIR filter

The frequency response of IIR filter can be described as

$$H(\omega) = \frac{\displaystyle\sum_{n=0}^{N} a_n e^{-jn\omega}}{1 + \displaystyle\sum_{m=1}^{M} b_m e^{-jm\omega}}, \tag{1}$$

where a_n, b_m are filter coefficients, N is a numerator order and M is a denominator order. In a sense of the Chebyshev approximation criteria, the design problem can be described as following

$$\min_{\boldsymbol{x}} \; F(\boldsymbol{x}), \tag{2}$$

$$F(\boldsymbol{x}) = \max_{\omega \in \Omega} W(\omega)|D(\omega) - H(\omega)|, \tag{3}$$

where $\boldsymbol{x}$=$[a_0, a_1, \cdots, a_N, b_1, b_2, \cdots, b_M]$ is the coefficient vector, ω is a normalized frequency, $D(\omega)$ is a desired frequency response.

III. Particle Swarm Optimization

The PSO is consisted of multiple particles. Each particle has a speed $\boldsymbol{v}$ and a location $\boldsymbol{x}$, the searching is carried out by updating $\boldsymbol{x}$. The updating procedure of i-th particle is described as follows

$$\boldsymbol{x}_i(t+1) = \boldsymbol{x}_i(t) + \boldsymbol{v}_i(t+1), \tag{4}$$

$$\begin{aligned} \boldsymbol{v}_i(t+1) = {} & w\boldsymbol{v}_i(t) + c_1 r_1(\boldsymbol{pbest}_i(t) - \boldsymbol{x}_i(t)) \\ & + c_2 r_2(\boldsymbol{gbest}(t) - \boldsymbol{x}_i(t)), \end{aligned} \tag{5}$$

where $\boldsymbol{pbest}_i$ is the best of location of i-th ($i = 1, \cdots, P$) particle, $\boldsymbol{gbest}$ is the best location among all particle locations up to t-th iteration, t (t=0, 1, $\cdots$, T) is the number of iteration, r_1, r_2 is an uniform random numbers in the interval of [0, 1], w is the inertia weight parameter, c_1 is a weight parameter toward the $\boldsymbol{pbest}_i$ and c_2 is a weight parameter toward the $\boldsymbol{gbest}$.

Such a updating brings the PSO a strong directivity toward the local minimum. However, due to the strong directivity, the PSO tends to stagnate around a local minimum and thus indicates a premature convergence. In addition, diffusion can't be controlled well because a value of each parameter is fixed to a constant value. The error function of 2 order IIR filter is shown as Fig. 1. Because the error function has many local minimums shown in the Fig. 1, the drawbacks of the PSO give a serious damage in the IIR filter design.

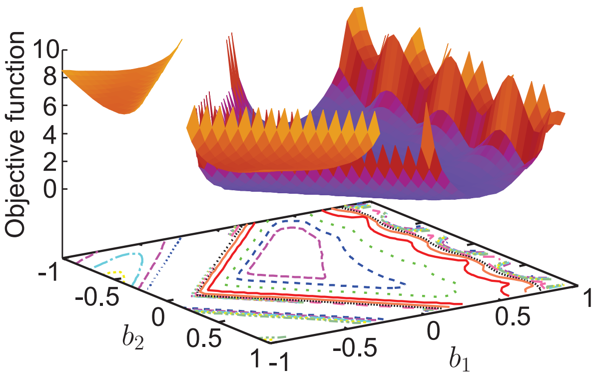

Fig. 1. Error surface of 2 order IIR filter

IV. Asynchronous Digenetic PSO with Nonlinear Dissipative Term

The N-AD-PSO was proposed for a global and a sustainable search. Updating of the location in the N-AD-PSO is shown in Fig. 2. For overcoming two drawbacks in the PSO as stated in section I, “particle age” k_i are given to each particle in the N-AD-PSO. The particle age k_i is the time for judging a debased search ability. The diffusion is controlled with a self-directive and an asynchronous ability. The k_i is updated with an increase of the PSO iteration. Weight parameters $w_i(k_i), c_{1i}(k_i), c_{2i}(k_i)$ are decreased as follows

$$
\begin{aligned}
&w_i(d_{0i}(k_i), \boldsymbol{gbest}(t), \boldsymbol{x}_i(k_i)) \\
&= 1 - d_1 + d_1 d_{0i}(k_i)\exp\left(-\frac{\|\boldsymbol{gbest}(t) - \boldsymbol{x}_i(k_i)\|}{d_2 d_{0i}(k_i)}\right)
\end{aligned} \tag{6}
$$

$$
d_{0i}(k_i) = d_{0\max} - \frac{(d_{0\max} - d_{0\min})k_i}{K_i} \tag{7}
$$

$$
c_{1i}(k_i) = c_{1\max} - \frac{(c_{1\max} - c_{1\min})k_i}{K_i} \tag{8}
$$

$$
c_{2i}(k_i) = c_{2\max} - \frac{(c_{2\max} - c_{2\min})k_i}{K_i}, \tag{9}
$$

where K_i is a maximum number of k_i, d_1 is a weight parameter which decides a minimum value of inertia term and an effect of nonlinear dissipative term, d_2 is a control parameter for the diffusion. When k_i is a small value as the green painted particle in Fig. 2, the particle i searches in a global space. When k_i is a large value as the orange painted particle, the particle i searches in a local space. When a distance between $\boldsymbol{x}_i$ and $\boldsymbol{gbest}$ is close as the blue painted particle, the search ability tends to degenerate. For a continuous search, the inertia weight value is increased. The increment value increases when $\boldsymbol{x}_i(t)$ approaches $\boldsymbol{gbest}(t)$.

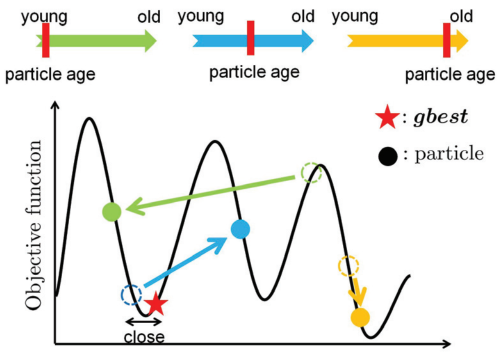

Fig. 2. Updating of the location in the N-AD-PSO

To judge a search condition, $Act_i(t)$ and $Length_i(t)$ is computed as follows

$$
Act_i(t) = \sqrt{\frac{1}{N+M+1}\sum_{j=0}^{N+M} v_{ij}(t)^2} \tag{10}
$$

$$
Length_i(t) = \|\boldsymbol{gbest}(t) - \boldsymbol{pbest}_i(t)\| \tag{11}
$$

where $Act_i(t)$ is an activity of the particle i and $Length_i(t)$ is a distance between $\boldsymbol{pbest}_i(t)$ and $\boldsymbol{gbest}(t)$. $Acount_i(t)$ and $Lcount_i(t)$ is updated as follows

$$
\begin{cases}
Acount_i(t+1) & = Acount_i(t) + 1 \quad (Act_i(t) < \varepsilon) \\
Acount_i(t+1) & = 0 \quad otherwise
\end{cases}, \tag{12}
$$

$$
\begin{cases}
Lcount_i(t+1) & = Lcount_i(t) + 1 \quad (Length_i(t) < \varepsilon') \\
Lcount_i(t+1) & = 0, \quad otherwise
\end{cases}, \tag{13}
$$

where ε, ε' is the threshold value. When either $Acount_i(t)$ or $Lcount_i(t)$ or k_i reaches a maximum number of K_{Act}, K_{Length} and K, respectively, the particle i is reset based on each conditions. This process are called “digenesis”.

The stagnation around a local minimum can be resolved by introducing the concept of the particle age and the digenesis.

A. condition 1

Updating of the location in the condition 1 is shown in Fig. 3. When $Acount_i(t+1) = K_{Act}$ or $k_i = K_i$ as the green painted particle in Fig. 3, the particle i is judged that the activity is degenerated. The location $\boldsymbol{x}_i$ and speed $\boldsymbol{v}_i$ is relocated in random. In addition, $Acount_i(t+1)$, $Lcount_i(t+1)$ and k_i is reset to 0, respectively.

B. condition 2

Updating of the location in the condition 2 is shown in Fig. 4. When $Lcount_i(t+1) = K_{Length}$ as the green painted particle in Fig. 4, the particle i is judged that the search ability is degenerated because the distance between $\boldsymbol{pbest}_i(t)$ and $\boldsymbol{gbest}(t)$ is close each other. The location $\boldsymbol{x}_i$ and the speed $\boldsymbol{v}_i$ is relocated in random. And $\boldsymbol{pbest}_i(t)$ is reset. $Acount_i(t+1)$, $Lcount_i(t+1)$ and k_i is reset to 0, respectively.

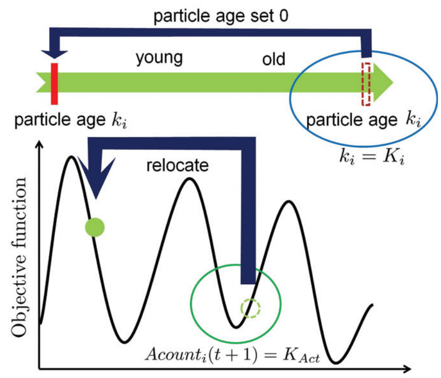

Fig. 3. Updating of the location in the condition 1

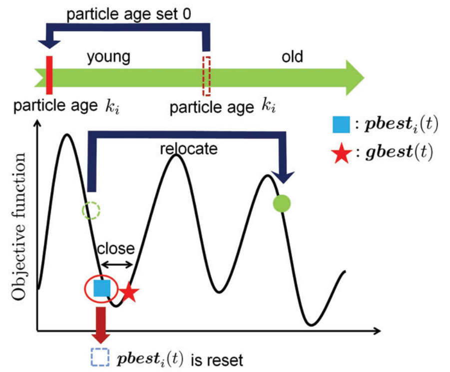

Fig. 4. Updating of the location in the condition 2

V. IIR FILTER DESIGN USING THE N-AD-PSO

In the proposed method, the stability is ensured by adding the penalty function $\varphi(p_{\max})$. The objective function $F'(\boldsymbol{x})$ that the penalty function was given, can be described as follows,

$$\min_{\boldsymbol{x}} F'(\boldsymbol{x}) \tag{14}$$

$$F'(\boldsymbol{x}) = w_f F(\boldsymbol{x}) + c_p \varphi(p_{\max}), \tag{15}$$

where $p_{\max}$ is a maximum pole radius prescribed before, w_f, c_p is a weight parameter of the objective function and the penalty function, respectively.

A. penalty function

When a pole having a maximum radius is within the unit circle on the z-plane, the stability of the IIR filter is ensured. However, an excess magnitude ripple is appeared in a transition band when the poles close to unit circle. In this study, a maximum pole radius is limited by the following penalty function,

$$\varphi(p_{\max}) = \begin{cases} p_{\max}^2 & (p_{\max} > R) \\ 0 & (p_{\max} \leq R) \end{cases}, \tag{16}$$

where R is the prescribed maximum pole radius. The penalty function is shown in Fig. 5. The PSO has a strong directivity toward the local minimum, and thus the particles can search good solutions in the stability region. In addition, the excess magnitude ripple can be reduced.

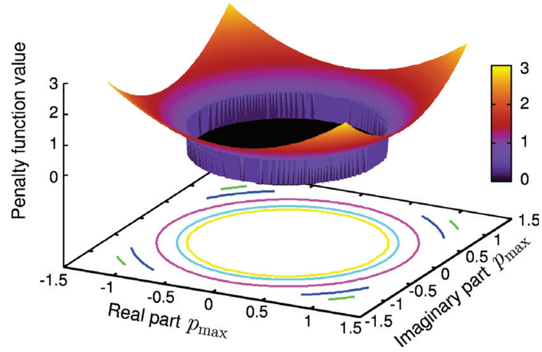

Fig. 5. penalty function $\varphi(p_{\max})(R = 0.9)$

VI. DESIGN EXIAMPLES

In this section, three examples are shown to verify the effectiveness of the proposed method. The desired frequency response $D(\omega_u)$ is given as

$$D(\omega) = \begin{cases} e^{-j\tau\omega} & (0 \leq \omega \leq \omega_p) \\ 0 & (\omega_s \leq \omega \leq 0.5) \end{cases}, \tag{17}$$

where τ is a group delay, .

For all designs, the design parameters were set to $W(\omega) = 1$, $w_f = 1$, $c_p = 1$, $T = 1200$ and the number of frequency dividing $S = 100$. In total, 50 trials were attempted and the best solution among them was chosen. The design parameters are listed in Table I.

TABLE I
DEDIGN CONDITIONS

	N	M	R	τ	ω_p	ω_s
Ex.1	4	4	0.85	2.0	0.198	0.350
Ex.2	8	6	0.90	6.0	0.196	0.272
Ex.3	12	6	0.93	9.0	0.196	0.254

The 3 types of the initial value were used. The type 1 is the uniform random numbers in the interval of [-1, 1], the type 2 is the normal random number (mean is 0 and variance is 0.5) and the type 3 is the solution of linear programming. The proposed method was compared with the N-AD-PSO (type 1), the N-AD-PSO (type 2), the N-AD-PSO (type 3), the PSO (type 1) and the PSO (type3). The N-AD-PSO parameters are listed in Table II. The PSO parameters are listed in Table III.

TABLE III
THE PSO PARAMETERS

	w	c_1	c_2	P
Ex.1	0.6	0.8	1.5	60
Ex.2	0.6	0.8	1.5	90
Ex.3	0.2	1.2	2.8	90

TABLE II
THE N-AD-PSO PARAMETERS

	d_1	d_2	$d_{0\max}$	$d_{0\min}$	$c_{1\max}$	$c_{1\min}$	$c_{2\max}$	$c_{2\min}$	ε	ε'	K
Ex.1	0.65	1.0×10^{-6}	0.8	0.2	2.0	0.8	2.0	1.2	5.0×10^{-4}	3.0×10^{-4}	400
Ex.2	0.65	1.0×10^{-6}	0.8	0.2	2.0	0.8	2.0	1.2	5.0×10^{-4}	3.0×10^{-4}	400
Ex.3	1.00	6.5×10^{-3}	1.0	0.4	2.0	1.0	2.0	1.0	1.2×10^{-4}	1.0×10^{-3}	400

TABLE IV
COMPARISION OF MAXIMUM ERROR

	Ex.1	Ex.2	Ex.3
N-AD-PSO (type 1)	2.81	2.04	1.60
N-AD-PSO (type 2)	2.87	2.06	1.73
N-AD-PSO (type 3)	2.81	2.01	1.53
PSO (type 1)	3.07	3.34	3.89
PSO (type 3)	2.88	2.00	1.53

Maximum error: $\times 10^{-2}$

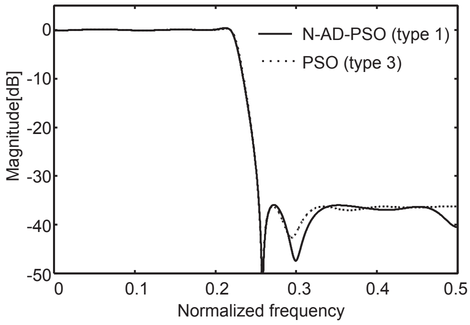

Fig. 6. Magnitude response with the N-AD-PSO (type 1) and the PSO (type 3) in Ex.3

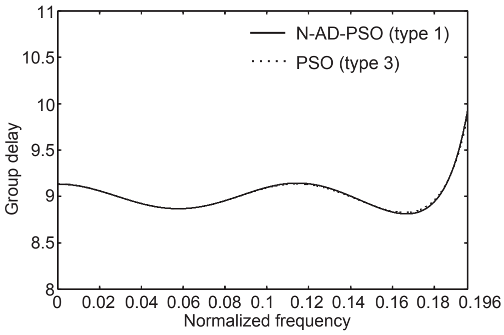

Fig. 7. Group delay response with the N-AD-PSO (type 1) and the PSO (type 3) in Ex.3

A comparison of maximum error are shown in Table IV. The magnitude response, the group delay response and an average error curve in Ex.3 are shown in Fig. 6, 7, 8, respectively.

In Table IV, the best design could be achieved by the PSO (type 3). However, the initial value of the type 3 has to be computed independently. The maximum error of the N-AD-PSO (type 1), the N-AD-PSO (type 2) and The N-AD-PSO (type 3) is almost equal to the maximum error of the PSO

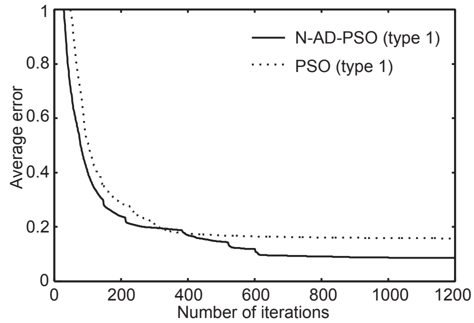

Fig. 8. Transition of average error with the N-AD-PSO (type 1) and the PSO (type 1) in Ex.3

(type 3). It is apparent from the Fig. 8, the local minimum stagnation can be avoided by the N-AD-PSO. Thus, the IIR filter design using the N-AD-PSO doesn't depend on initial value used.

VII. CONCLUSIONS

In this paper, the N-AD-PSO has been adapted to the IIR filter design. The N-AD-PSO can be expected that the stagnation can be avoided. The stability is ensured by adding the penalty function. From several examples, it was shown that the stagnation can be avoided by the proposed method.

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