Hierarchical Multi-stage Interference Alignment for Downlink Heterogeneous Networks

Tomoki AKITAYA and Takahiko SABA Dept. of Computer Science, Chiba Institute of Technology, 2-17-1 Tsudanuma, Narashino, Chiba 275-0016 JAPAN

Abstract—The occurrence of the intercell interference (ICI) is inevitable in downlink heterogeneous networks because user terminals (UTs) often receive the signals transmitted from different base stations (BSs) at the same time. To mitigate ICI, a hierarchical interference alignment (HIA) is proposed. HIA is based on the interference alignment (IA), which aligns the interference signals within a reduced dimensional subspace at each UT by multiplying the signals to be transmitted from each BS by the transmit beamforming matrix. However, HIA can be applied only to a network in which two picocells are placed within a macrocell. In this paper, we propose a hierarchical multi-stage interference alignment (HMIA) to cope with the restriction. In HMIA, by dividing the aligning process into multiple stages, every transmit beamforming matrix can be calculated in closed form. Furthermore, since the alignment is carried out in descending order of signal strength, strong interference signals are aligned preferentially. Simulation results can show that HMIA successfully deal with the network in which more than three picocells are placed within a macrocell although there is slight loss of the per-user capacity.

I. INTRODUCTION

Heterogeneous network is considered as a promising technique for cellular networks to extend the coverage and capacity [1]. In heterogeneous cellular networks, small cells called picocells or femtocells are placed within a macrocell. Base stations (BSs) of such small cells are generally installed inside buildings where coverage is poor or where there is a dense population of users. The use of small cells, however, causes the intercell interference (ICI) at user terminals (UTs) because UTs often receive the signals transmitted from different BSs at the same time. In this paper, we deal with only the small cells whose BSs use the same frequency band as the macrocell BS for downlink signal transmission. Since ICI degrades the demodulation performance of UTs, ICI coordination (ICIC) techniques are required for heterogeneous network [2]. Some ICIC techniques are being studied for standardization [3], [4].

Coordinated multipoint (CoMP) transmission and reception is a candidate technique for Long-Term Evolution Advanced [5]. Downlink CoMP can be classified into two types: joint processing (JP) and coordinated scheduling/beamforming (CS/CB). In the case of JP, data is transmitted to a UT simultaneously from a number of different BSs to improve the received signal quality and strength. However, since the data to be transmitted to the UT needs to be sent to all the corresponding BSs, data traffic among the BSs becomes large. Furthermore, since data for JP such as the channel state information (CSI) must be also shared among the BSs to cancel interference from transmissions that are intended for other UTs, considerable delay may occur in coordination. On the other hand, data to a single UT is transmitted from one BS when CS/CB is employed. As for CS, the transmission timing and frequency to each UT is coordinated among BSs. Furthermore, CB controls the ICI by coordinating the beams among BSs. Since BSs share only data for scheduling and beamforming, delay in coordination is relatively small.

As a kind of CB techniques, hierarchical interference alignment (HIA) is proposed [6]. HIA is based on interference alignment (IA), which aligns all the interference signals in a reduced dimensional subspace at each UT by weighting the signals to be transmitted at BSs [7]. In HIA, the transmit weights for the picocell BSs are calculated first, then those for the macrocell BS are calculated. All the transmit weights can be calculated in closed form by separating the calculations for picocell BSs and for a macrocell BS. Furthermore, it is shown that HIA achieves a higher sum-rate than the other ICIC techniques. Thus, HIA is attractive for heterogeneous network. However, there is a restriction that HIA is applicable only to a network in which two picocells are placed within the macrocell.

In this paper, we propose a hierarchical multi-stage interference alignment (HMIA) as an ICIC technique for heterogeneous network where three or more picocells are placed within the macrocell. Since picocells generally have a narrow coverage and some of them are placed at the coverage holes of the macrocell, signals from BSs do not always strongly interfere with each other. In our proposed HMIA, BSs deal with the interference signals in descending order of signal strength, and calculate the transmit weights through multiple stages. As a result, although some of weak interference signals are not aligned, this multi-stage processing can mitigate the ICI effectively even if the number of picocell BSs within a macrocell is more than two. In addition, the transmit weights can be also calculated in closed form in HMIA. From the simulation results, we can show the proposed HMIA mitigate ICI of the network in which three picocells are placed within the macrocell although there is six percent loss in the per-user capacity.

II. HETEROGENEOUS NETWORK SYSTEM MODEL

We consider a downlink heterogeneous network where K_p picocells are placed within a macrocell. We assume the number of UTs of each picocell is one, and that of a macrocell is two.

Some readers may think that these numbers are small, but BSs can handle more UTs by combining CS with CB. For the sake of convenience, BSs and UTs are assigned numbers, and are denoted by BS-i ($i \in \{1, ..., K_p + 1\}$) and UT-k($k \in \{1, ..., K_p + 1, K_p + 2\}$), respectively. BS- $(K_p + 1)$ is the macrocell BS, and serves UT- $(K_p + 1)$ and UT- $(K_p + 2)$. The others are picocell BSs each of which serves one UT having its corresponding index. In this network, each picocell BS has Mantennas, and the macrocell BS has 2M antennas. Moreover, each UT has M antennas and receives M/2 streams from the corresponding BS. Fig. 1 describes the case of M = 2. The signals to UTs from their corresponding BSs are drawn by solid lines. The others are the interference signals and drawn by dashed lines in Fig. 1. Here, the signal to be transmitted to UT-k is expressed as

$$\boldsymbol{s}_k = P_k \boldsymbol{V}_k \boldsymbol{x}_k, \tag{1}$$

where x_k is the M/2-by-one symbol vector for UT-k. V_k is the transmit beamforming matrix which has a size of M-by-M/2 when $k \leq K_p$ and has a size of 2M-by-M/2 when $k > K_p$. P_k is a variable that limits the power of s_k to a specified transmit power of BSs. Since BSs transmit signals simultaneously, the received signal at UT-k is expressed as

$$\boldsymbol{y}_{k} = \boldsymbol{H}_{k,\text{BS}(k)}\boldsymbol{s}_{k} + \sum_{l=1,l\neq k}^{K_{p}+2} \boldsymbol{H}_{k,\text{BS}(l)}\boldsymbol{s}_{l} + \boldsymbol{n}_{k}, \qquad (2)$$

where $H_{k,i}$ denotes the channel matrix between BS-*i* and UT*k*, and BS(*k*) is the index of BS whose user is UT-*k*. n_k is the noise vector of UT-*k*. The second term on the right-hand side of Eq. (2) expresses the ICI. To eliminate ICI, the received signal is multiplied by the receive beamforming matrix W_k as follows:

$$\hat{\boldsymbol{y}}_{k} = \boldsymbol{W}_{k}^{\dagger} \boldsymbol{H}_{k,\text{BS}(k)} \boldsymbol{s}_{k} + \boldsymbol{W}_{k}^{\dagger} \sum_{l=1,l\neq i}^{K_{p}+2} \boldsymbol{H}_{k,\text{BS}(l)} \boldsymbol{s}_{l} + \boldsymbol{W}_{k}^{\dagger} \boldsymbol{n}_{k}, \quad (3)$$

where $(\cdot)^{\dagger}$ denotes the conjugate transpose of a matrix. We also assume that the global CSI $(\boldsymbol{H}_{k,i}, \forall i, k)$ is available at every BS.

III. HIERARCHICAL MULTI-STAGE INTERFERENCE ALIGNMENT

In this section, we describe our proposed HMIA. HMIA is inspired by HIA, which is an IA-based ICIC technique for downlink heterogeneous network introduced in [6]. In HIA, the calculations of the transmit beamforming matrices are divided into two stages. First is the calculations of the transmit beamforming matrices for the picocell BSs. During this stage, the interference signals due to the picocell BSs are aligned within an M/2 dimensional subspace. The second is the calculations of the transmit beamforming matrices for the macrocell BS. Through these two stages, all the interference signals are aligned at every UT. In HIA, all the transmit beamforming matrices can be calculated in closed form. Moreover, it is shown that HIA can achieve a higher sum-rate than the other ICIC techniques. However, it is proved that BSs cannot



Fig. 1. The system model of the heterogeneous network when M = 2.

align all the interference signals within an M/2 dimensional subspace at each UT when $K_p > 2$ because the transmit beamforming matrices are obtained by solving the generalized eigen-problem, which can be solved only when $K_p = 2$ [6]. Thus, HIA is applicable only to the network where $K_p = 2$.

To cope with the case of $K_p > 2$ in the proposed HMIA, we consider aligning only strong interference signals within a subspace at each UT. Since picocells generally have a narrow coverage and some of them are placed at the coverage holes of the macrocell, signals from BSs do not always strongly interfere with each other in the heterogeneous network. To achieve this, the calculations of the transmit beamforming matrices for the picocell BSs are divided into multiple stages. In HMIA, two picocell BSs which cause the strongest interference to UTs are selected first, and their transmit beamforming matrices are calculated by solving the generalized eigenproblem. Subsequently, one picocell BS is selected, and its transmit beamforming matrix is calculated by the use of the matrices obtained in former stages.

For the formulation of HMIA, we define two subsets of indices as follows.

- B ⊆ {1,...,K_p}: the subset of indices of the picocell BSs whose transmit beamforming matrices are not calculated, and it is initialized as B = {1,...,K_p}.
- U ⊆ {1,..., K_p + 2}: the subset of indices of the UTs whose receive beamforming matrices are calculated, and it is initialized as U = Ø.

Here, the details of the processing of HMIA are explained below.

A. First stage: calculations of the transmit beamforming matrices of two picocell BSs

First, we select two UTs which receive the strongest interference, and also select two picocell BSs which are the sources of the interference in the network. Here, the selected BSs are denoted as $BS-B_{1a}$ and $BS-B_{1b}$ where $B_{1a}, B_{1b} \in \{1, \ldots, K_p\}$, and the selected UTs are denoted as $UT-U_{1a}$ and $UT-U_{1b}$ where $U_{1a}, U_{1b} \in \{1, \ldots, K_p + 2\}$ and $U_{1a}, U_{1b} \notin \{B_{1a}, B_{1b}\}$. At this stage, we calculate the transmit beamforming matrices for $BS-B_{1a}$ and $BS-B_{1b}$ to align the interference signals from $BS-B_{1a}$ and $BS-B_{1b}$ within M/2 dimensional subspaces of $UT-U_{1a}$ and $UT-U_{1b}$. For the sake of convenience, we call this M/2 dimensional subspace of each UT the interference subspace in which the interference signals are aligned. The transmit beamforming matrices for $BS-B_{1a}$ and $BS-B_{1b}$ must satisfy the following conditions.

$$\operatorname{span}\left(\boldsymbol{H}_{U_{1a},B_{1a}}\boldsymbol{V}_{B_{1a}}\right) = \operatorname{span}\left(\boldsymbol{H}_{U_{1a},B_{1b}}\boldsymbol{V}_{B_{1b}}\right), \quad (4)$$

$$\operatorname{span}\left(\boldsymbol{H}_{U_{1b},B_{1a}}\boldsymbol{V}_{B_{1a}}\right) = \operatorname{span}\left(\boldsymbol{H}_{U_{1b},B_{1b}}\boldsymbol{V}_{B_{1b}}\right), \quad (5)$$

where $span(\cdot)$ denotes the subspace spanned by the column vectors of a matrix. From Eqs. (4) and (5), the following equation can be obtained.

$$\operatorname{span}\left(\boldsymbol{V}_{B_{1a}}\right) = \operatorname{span}\left(\boldsymbol{H}_{U_{1a},B_{1a}}^{-1}\boldsymbol{H}_{U_{1a},B_{1b}}\boldsymbol{V}_{B_{1b}}\right)$$
$$= \operatorname{span}\left(\boldsymbol{H}_{U_{1b},B_{1a}}^{-1}\boldsymbol{H}_{U_{1b},B_{1b}}\boldsymbol{V}_{B_{1b}}\right).$$
(6)

Eq. (6) can be seen as the generalized eigen-problem, thus we can calculate $V_{B_{1b}}$ by solving the generalized eigen-problem as

$$V_{B_{1b}} = \operatorname{eig}\left(H_{U_{1b},B_{1b}}^{-1}H_{U_{1b},B_{1a}}H_{U_{1a},B_{1a}}^{-1}H_{U_{1a},B_{1b}}\right), \quad (7)$$

where $eig(\cdot)$ denotes the matrix whose columns are M/2 unit norm eigen vectors of a matrix. Then $V_{B_{1a}}$ can be calculated from Eqs. (6) and (7) as

$$V_{B_{1a}} = \frac{H_{U_{1a},B_{1a}}^{-1} H_{U_{1a},B_{1b}} V_{B_{1b}}}{\left\| H_{U_{1a},B_{1a}}^{-1} H_{U_{1a},B_{1b}} V_{B_{1b}} \right\|}.$$
(8)

We can obtain the transmit beamforming matrices for two picocell BSs in closed form by solving the generalized eigenproblem. After the calculations of $V_{B_{1a}}$ and $V_{B_{1b}}$, B_{1a} and B_{1b} are removed from \mathcal{B}

Before proceeding to the next stage, we calculate the receive beamforming matrices for the UT- U_{1a} , UT- U_{1b} , UT- B_{1a} , and UT- B_{1b} . The receive beamforming matrix for each UT is calculated to eliminate interference signals, as follows:

$$\boldsymbol{W}_{B_{1a}} = \operatorname{null}\left(\left(\boldsymbol{H}_{B_{1a},B_{1b}}\boldsymbol{V}_{B_{1b}}\right)^{\dagger}\right),\tag{9}$$

$$\boldsymbol{W}_{B_{1b}} = \operatorname{null}\left(\left(\boldsymbol{H}_{B_{1b},B_{1a}}\boldsymbol{V}_{B_{1a}}\right)^{\dagger}\right), \qquad (10)$$

$$\boldsymbol{W}_{U_{1a}} = \operatorname{null}\left(\left(\boldsymbol{H}_{U_{1a},B_{1a}}\boldsymbol{V}_{B_{1a}}\right)^{\dagger}\right), \quad (11)$$

$$\boldsymbol{W}_{U_{1b}} = \operatorname{null}\left(\left(\boldsymbol{H}_{U_{1b},B_{1a}}\boldsymbol{V}_{B_{1a}}\right)^{\dagger}\right), \quad (12)$$

where null(·) denotes an orthonormal basis for the null space of a matrix. After the calculations, U_{1a} , U_{1b} , B_{1a} , and B_{1b} are added to \mathcal{U} .

Since the transmit beamforming matrices for two picocell BSs are calculated in closed form, the interference subspaces of these four UTs are defined uniquely. Therefore, the receive beamforming matrices for these UTs can be calculated in closed form.

B. Second to $(K_p - 1)$ th stages: calculations of the transmit beamforming matrices for the remainder picocell BSs

Next, we calculate the transmit beamforming matrices for the rest of the picocell BSs. The calculations of the transmit beamforming matrices for these $(K_p - 2)$ picocell BSs are divided into $(K_p - 2)$ stages. At each stage, one picocell BS is selected from \mathcal{B} , and the transmit beamforming matrix for the selected BS is calculated. The selection is based on \mathcal{U} updated at each stage. That is, we select a picocell BS which causes the strongest interference to one of UTs whose indices are already stored in \mathcal{U} . Here, the picocell BS selected at *n*th stage is denoted as BS- B_n ($B_n \in \mathcal{B}$), and the UT which receives the strongest interference from BS- B_n is denoted as UT- I_n^U ($I_n^U \in \mathcal{U}$).

At the *n*th stage, the transmit beamforming matrix for BS- B_n is calculated as

$$\boldsymbol{V}_{B_n} = \operatorname{null}\left(\boldsymbol{W}_{I_n^U}^{\dagger} \boldsymbol{H}_{I_n^U, B_n}\right), \qquad (13)$$

which aligns the signal from $BS-B_n$ with the interference subspace of $UT-I_n^U$ defined in former stages. Then, B_n is removed from \mathcal{B} .

Furthermore, we also calculate the receive beamforming matrix for another UT. Similar to the calculation of V_{B_n} , we select a UT from the complementary set of \mathcal{U} (denoted as $\overline{\mathcal{U}}$), and calculate its receive beamforming matrix. We select a UT which receives the strongest interference from one of BSs whose indices belong to $\overline{\mathcal{B}}$. The selected UT is denoted as UT- U_n ($U_n \in \overline{\mathcal{U}}$), and the BS which causes the strongest interference to UT- U_n is denoted as BS- I_n^B ($I_n^B \in \overline{\mathcal{B}}$).

The receive beamforming matrix of UT- U_n which cancels out the strongest interference from BS- I_n^B is calculated as

$$\boldsymbol{W}_{U_n} = \operatorname{null}\left(\left(\boldsymbol{H}_{U_n, I_n^B} \boldsymbol{V}_{I_n^B}\right)^{\dagger}\right). \tag{14}$$

Since $V_{I_n^B}$ is calculated in a former stage in closed form, W_{U_n} can be also calculated in closed form. After the calculation, U_n is added to \mathcal{U} . The above operations are successively performed until all the transmit beamforming matrices for the picocell BSs are calculated (i.e., until $\mathcal{B} = \emptyset$).

C. K_p th stage: calculations of the transmit beamforming matrices for the macrocell BS

After the calculations of the transmit beamforming matrices for all the picocell BSs, we finally calculate the transmit beamforming matrices for the macrocell BS. Since the macrocell BS transmits signals to UT- $(K_p + 1)$ and UT- $(K_p + 2)$ simultaneously, inter-user interferences from the macrocell BS must be aligned with an interference subspace of each UT. Moreover, the macrocell BS also has to align its transmitting signals with the interference subspaces of the picocell UTs. Since the macrocell BS has 2M antennas and transmits two M/2 streams, it can align two (= $2M/(2 \cdot M/2)$) interference signals with the interference subspaces of two picocell UTs in addition to aligning a inter-user interference signal at one of two macrocell UTs. Thus, the transmit beamforming matrices for the macrocell BS are obtained in closed form as follows:

$$\boldsymbol{V}_{K_p+1} = \operatorname{null}\left(\begin{bmatrix} \boldsymbol{W}_{m_1}^{\dagger} \boldsymbol{H}_{m_1,K_p+1} \\ \boldsymbol{W}_{m_2}^{\dagger} \boldsymbol{H}_{m_2,K_p+1} \\ \boldsymbol{W}_{K_p+2}^{\dagger} \boldsymbol{H}_{K_p+2,K_p+1} \end{bmatrix} \right), \quad (15)$$

$$\boldsymbol{V}_{K_{p}+2} = \operatorname{null}\left(\begin{bmatrix} \boldsymbol{W}_{m_{1}}^{\dagger} \boldsymbol{H}_{m_{1},K_{p}+1} \\ \boldsymbol{W}_{m_{2}}^{\dagger} \boldsymbol{H}_{m_{2},K_{p}+1} \\ \boldsymbol{W}_{K_{p}+1}^{\dagger} \boldsymbol{H}_{K_{p}+1,K_{p}+1} \end{bmatrix} \right), \quad (16)$$

where $m_1, m_2 \in \{1, \ldots, K_p\}$ denote the indices of two picocell UTs which receive the strongest interference from the macrocell BS.

In HMIA, by dividing the aligning process into multiple stages, every transmit beamforming matrix can be calculated in closed form even when $K_p > 2$. Furthermore, since the alignment is carried out in descending order of signal strength, strong interference signals are aligned preferentially. Thus, ICI is expected to be mitigated effectively.

IV. SIMULATION RESULTS

We assume an isolated macrocell, and thus the interference from the other macrocell is not considered. We also assume that four picocell BSs are placed randomly within the macrocell so that their coverage may not overlap. Each picocell BS is at a distance of 30 to 40 [m] from one picocell, and is at least 130 [m] away from the two remaining picocell BSs. The latter condition ensures that some BSs do not cause strong interference. Furthermore, all the picocell BSs are at least 165 [m] away from the macrocell BS. Each picocell BS is placed at the coverage hole of the macrocell, which is assumed to be a cylindrical building with a radius of 30 [m]. The picocell BS is installed at the center of a cylindrical building, and signals are attenuated by its walls. There are six UTs within the macrocell. Two of them are macrocell UTs, and the others are picocell UTs. The macrocell UTs are uniformly distributed within the macrocell, and are connected to the macrocell BS even if they are within a picocell. That is, we assume the closed subscriber gate access of the picocells [2]. The picocell UTs are also randomly placed within each picocell. The macrocell BS is equipped with four antennas, and the picocell BSs are equipped with two antennas. Moreover, every UT is equipped with two antennas. The simulation parameters are shown in Table I.

In this simulation, we also evaluate the performance of HMIA for the case of $K_p = 3$. In this case, one of four picocells and its associated UT are randomly omitted from the macrocell shown in Fig. 2. Moreover, we compare the performance of HMIA with that of HIA. In the evaluation of HIA, since the number of picocells must be two ($K_p = 2$), two picocells and their UTs are randomly omitted from the macrocell. First, we compare the sum-rate performance of HMIA with that of HIA. Fig. 3 shows the cumulative probability function (CDF) of the sum-rate performance. The



Fig. 2. An example of placement of four picocells within a macrocell.

TABLE I SIMULATION PARAMETERS

the radius of macrocell	289 [m]
the radius of picocell	30 [m]
pathloss from the macrocell BS	$15.3 + 37.6 \log_{10}(d)$ [dB],
	d is distance from the BS [m]
pathloss from the picocell BSs	$30.6 + 36.7 \log_{10}(d) \text{ [dB]}$
transmit power of the macrocell BS	46 [dBm]
transmit power of the picocell BS	30 [dBm]
noise power	-174 [dBm/Hz]
penetration loss	20 – 40 [dB] (randomly changed)
modulation scheme	orthogonal frequency division
	multiplexing
carrier frequency	2 [GHz]
the number of subcarriers	256
subcarrier spacing	15 [kHz]
channel model	4-path Rayleigh fading channel
	with exponential decay
normalized Doppler frequency	0.00001
channel estimation	imperfect

horizontal axis is the sum-rate achieved at the heterogeneous network. The vertical axis is the cumulative probability.

Fig. 3 reveals that fifty percent of transmission with HMIA attains more than 77.1 [bit/s/Hz] in the case of $K_p = 3$, while that with HIA attains more than 65.6 [bit/s/Hz]. Therefore, HMIA achieves a sum-rate about eighteen percent larger than HIA. Moreover, in the case of $K_p = 4$, fifty percent of transmission with HMIA attains more than 87.3 [bit/s/Hz]. Thus, HMIA achieves a sum-rate about thirty three percent larger than HIA in the case of $K_p = 4$. However, these improvements are mainly due to the fact that the network using HMIA serves a larger number of UTs. Thus, for fair comparison, we evaluate the per-user capacity of both systems.

Fig. 4 shows comparison of the average per-user capacity between the network using HMIA and the one using HIA. From Fig. 4, fifty percent of transmission with HMIA attains more than 15.4 [bit/s/Hz] in the case of $K_p = 3$, while



Fig. 3. CDF of sum-rate at the entire network.



Fig. 4. CDF of per-user average capacity.

that with HIA attains more than 16.4 [bit/s/Hz]. Therefore, the per-user capacity of HMIA becomes smaller by about six percent than that of HIA. In the case of $K_p = 4$, fifty percent of transmission with HMIA attains more than 14.6 [bit/s/Hz], thus the per-user capacity of HMIA becomes smaller by about eleven percent that of HIA. These are because all the interference signals cannot be aligned within a reduced dimensional subspace at each UT in HMIA.

Here, we consider the degradation in the sum-rate performance. In the case of $K_p = 3$, there are five UTs in the network using HMIA, while there are four UTs in the network using HIA. That is, HMIA must attain a sum-rate twenty five percent larger than HIA ideally which implies the sum-rate becomes 82 [bit/s/Hz] ideally. Since the actual sum-rate is 77.1 [bit/s/Hz], the sum-rate is smaller by about six percent. This agrees with the result in Fig. 4. Similar agreement can be confirmed in the case of $K_p = 4$. Thus, HMIA can handle more than two picocells at the cost of some per-user capacity performance. However, this is a great advantage because the network using HIA cannot have more than two picocells within the macrocell.

V. CONCLUSION

In this paper, we have proposed a hierarchical multi-stage interference alignment (HMIA) to cope with the network in which more than three picocells are placed within a macrocell. In HMIA, by dividing the calculations of the transmit beamforming matrices into multiple stages, every transmit beamforming matrix can be calculated in closed form. Furthermore, since the alignment is carried out in descending order of signal strength, strong interference signals are aligned even if the number of picocells is more than two. Simulation results can show that HMIA successfully deal with the network in which three or more picocells are placed within a macrocell although there is a slight loss in per-user capacity. From this result, HMIA is effective for downlink heterogeneous networks.

REFERENCES

- A. Khandekar, N. Bhushan, J. Tingfang, and V. Vanghi, "LTE-Advanced: heterogeneous networks," *Proc. European Wireless Conf. 2010*, pp. 978– 982, Lucca, Italy, Apr. 2010.
- [2] D. Lopez-Perez, I. Guvenc, G. de la Roche, M. Kountouris, T. Q. S. Quek, and J. Zhang, "Enhanced intercell interference coordination challenges in heterogeneous networks," *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 22–30, Jun. 2011.
- [3] R.Madan, J. Borran, A. Sampath, N. Bhushan, A. Khandekar, and, T. Ji, "Cell association and interference coordination in heterogeneous LTE-A cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1479–1489, Dec. 2010.
- [4] 3GPP TR 36.819 v11.1.0, "Technical specification group radio access network; coordinated multi-point operation for LTE physical layer aspects (release 11)," Dec. 2011.
- [5] D. Lee, H. Seo, B. Clerckx, E. Hardouin, D. Mazzarese, S. Nagata, and K. Sayana, "Coordinated multipoint transmission and reception in LTE-Advanced: deployment scenarios and operational challenges," *IEEE Commun. Mag.*, vol. 50, no. 2, pp. 148–155, Feb. 2012.
- [6] W. Shin, W. Noh, K. Jang, and H. Choi, "Hierarchical interference alignment for downlink heterogeneous networks," *IEEE Trans. Wireless commun.*, vol. 11, no. 12, pp. 4549–4559, Dec. 2012.
- [7] V. R. Cadambe, and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.