Performance Analysis and Power Allocation Strategy in Overlay Cognitive Networks

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Abstract—In this paper, we propose a power allocation strategy for overlay cognitive networks, where each node is equipped with single antenna. Owing to the overlay strategy, the secondary user (SU) can help transmitting the primary user’s (PU) data and meanwhile can convey its own data with the superposition coding (SC). We first analyze the bit-error-rate (BER) of the PU and the SU. Then, the power allocation strategy is devised by minimizing total power, providing that the BER of the PU and that of SU are guaranteed. Since the BER formulations are not convex, the optimization is difficult to conduct. We then propose two new tractable BER approximations for the PU and SU, which can sophisticatedly transfer the design into a convex problem. The solution can thus be obtained. Simulations verify that our power allocation design is validated for different channel environments.

Index Terms – Cognitive radio, overlay design, superposition coding, power allocation, convex optimization.

I. INTRODUCTION

Recently, the overlay spectrum strategy has been developed to effectively increase the overall spectral efficiency in the strong interference region of the cognitive radio (CR) networks [1]. The main feature of the overlay technique is that the secondary user (SU) can have the message of the primary user (PU). Thus, the SU can help the primary user (PU) transmitting the PU’s data, and meanwhile transmits its own data.

For the work related to the overlay design, most designs consider coexistence of a single-input single-output (SISO) and a multiple-input single-output (MISO) secondary system. Since the secondary transmitter (ST) is equipped multiple antennas, the cooperative relaying is used to relay the PU’s signal and to transmit the SU’s signal [2]-[5]. In [3], the primary system and the secondary system can operate simultaneously with the space-time coding developed in the MIMO systems. In [4], the zero-forcing (ZF) beamforming is devised at the ST for conveying both PU’s and SU’s data. In [5], the overly structure provided in [4] is extended to a more realistic system where the random vector quantization (RVQ) are considered into the design of the cooperative relaying. In [8], the performance of the PU is first enhanced to meet the predefined quality-of-Service (QoS) with the help of the SU and meanwhile, the SU is allowed to transmit its own signals.

Most existing works focus on the beamforming or space-time coding design at the ST [3]-[5]. For the ST with single antenna, theoretical capacity is often analyzed in the overlay design [2], [8]. The design considers the bit-error-rate (BER) as the QoS is also critical but challengeable for the PU’s and SU’s transceiver pairs equipped with single antenna. As far as we know, it has not been studied yet. In this paper, we will propose a transceiver design based on the QoS of BER for the equipped single antenna overlay systems.

The signal transmission in the considered overlay design is conducted in two phases. In the first phase, the PU’s data is first transmitted to the PR and the ST. In the second phase, the ST uses the decode-and-forward (DF) to detect the PU’s data. Owing to single antenna used in each node, we adopt the superposition coding (SC) for the ST transmitting both PU’s and SU’s data. The SC combines the PU’s and the SU’s data with a power factor and then transmits it to the PR and to the secondary receiver (SR). The PR then combines the two received signals with maximum ratio combining (MRC), and then decodes its data. The SR decodes its data directly.

To design the SC, we first analyze the bit-error-rate (BER) at the PR and at that at the SR. The resultant BERs are expressed as complicated functions of the PU’s and SU’s transmitter power and the SC’s power factor. We then aim at properly adjusting the power factor of the SC to minimize the total transmit power and meanwhile to guarantee the PU’s and SU’s BER. Since the problem is not convex due to the complicated constraints, the solution is essentially difficult to derive. To overcome the difficulty, we then propose two tractable approximations for the BER’s. The problem can finally be transferred to a convex one with the approximations and the optimum power allocation can be finally obtained [7]. Numerical results verify the validity of our design in different channel link conditions.

II. SYSTEM MODEL

A. System Model

We consider an overlay spectrum sharing system, where a primary and a secondary transceiver coexist and each node is equipped with an antenna. The received signals at the PR and the ST, denoted as \( y_{PR,1} \) and \( y_{ST} \), can be respectively expressed as

\[
\begin{align*}
y_{PR,1} &= h_{PT,PR} \sqrt{P_T} s_{PU} + n_{PR,1} \\
y_{ST} &= h_{PT,ST} \sqrt{P_T} s_{PU} + n_{ST}
\end{align*}
\]

(1)

(2)

where \( s_{PU} \) is the PU’s signal; \( h_{PT,PR} \in \mathbb{C}^{1 \times 1} \) and \( h_{PT,ST} \in \mathbb{C}^{1 \times 1} \) are the channel gain of the PT-to-PR and the
PT-to-ST links, respectively, with \( \mathcal{CN}(0, \sigma^2_{P,PR}) \) and \( \mathcal{CN}(0, \sigma^2_{ST,PR}) \). \( n_{PR,1} \sim \mathcal{CN}(0, \sigma^2_{n,PR}) \) and \( n_{ST} \sim \mathcal{CN}(0, \sigma^2_{n,ST}) \) denote the noise received at the PR and the ST. \( P_T \) is the transmit power of PT. The ST then decodes the signal \( s_{PU} \). If signal \( s_{PU} \) is decoded correctly, the ST combines \( s_{PU} \) and \( s_{SU} \) with SC scheme and transmits it to PR and SR simultaneously. Alternatively, if \( s_{SU} \) is decoded incorrectly, then it merely transmits \( s_{SU} \) in the second phase. Herein, ST operates the decode-and-forward relaying protocol.

In the second phase, the ST conducts the SC as the \( s_{PU} \) is decoded correctly; otherwise, ST only transmits its signal to SR and PR keeps silence. The received signal at the PR and the SR, denoted \( y_{PR,2} \) and \( y_{SR} \), are expressed as:

\[
y_{PR,2} = \begin{cases} 
\frac{h_{ST,PR}}{\sqrt{P_S}}(s_{PU} + \sqrt{1 - \delta^2}s_{SU}) + n_{PR,2}, & \text{if ST decodes signal correctly} \\
0, & \text{otherwise}
\end{cases}
\]

and

\[
y_{SR} = \begin{cases} 
\frac{h_{ST,SR}}{\sqrt{P_S}}(s_{PU} + \sqrt{1 - \delta^2}s_{SU}) + n_{SR}, & \text{if ST decodes signal correctly} \\
\frac{h_{ST,SR}}{\sqrt{P_S}}s_{SU} + n_{SR}, & \text{otherwise}
\end{cases}
\]

where \( h_{ST,PR} \sim \mathcal{CN}(0, \sigma^2_{ST,PR}) \) and \( h_{ST,SR} \sim \mathcal{CN}(0, \sigma^2_{ST,SR}) \) are the flat fading channels of the ST-to-PR and the ST-to-SR links, respectively. \( n_{PR,2} \sim \mathcal{CN}(0, \sigma^2_{n,PR}) \) and \( n_{SR} \sim \mathcal{CN}(0, \sigma^2_{n,SR}) \) denote the noise received at the PR and the SR. \( P_s \) is the transmit power of ST and \( \delta \) (\( \delta^2 < 0.5 \)) indicates the power allocation factor by the SC scheme.

### B. BER Performance at PR

The signal detected at the PR is then considered in two cases: the ST correctly decodes \( s_{PU} \) and it incorrectly decodes \( s_{SU} \). In the first case, the SC signal transmits to the PR. Therefore, we have to correctly decode \( s_{SU} \) first and then subtracts \( s_{SU} \) from \( y_{PR,2} \). The MRC can then be adopted to combine pure received signals of \( s_{SU} \) in the two phases. Otherwise, if the \( s_{SU} \) is decoded incorrectly, only \( s_{PR,1} \) is used for detection. Thus, the instantaneous received SNR of \( s_{PU} \) in the second phase, denoted \( \gamma_{PR,2} \), can be expressed as:

\[
\gamma_{PR,2} = \begin{cases} 
\frac{\frac{h_{ST,PR}}{\sqrt{P_S}}P_s}{\sigma_{n,PR}^2}, & \text{if PR detects } s_{SU} \text{ successfully} \\
0, & \text{otherwise}
\end{cases}
\]

Here, for simplicity, we only consider the BPSK modulation for \( s_{PU} \) and \( s_{SU} \).

Since SC is adopted at the ST, we have to conduct the BER at the PR with several combinations. Denote \( P_{e_{PR,II}}^{SU} \) as the BER of \( s_{SU} \) and consider equal probability of \( s_{SU} = +1 \) or \(-1\), we have \[6\]

\[
P_{e_{PR,II}}^{SU} (s_{SU} | h_{ST,PR}) = \frac{1}{2} \left( 1 - \frac{\text{SNR}_{PT,PR}}{1 + \text{SNR}_{PT,PR}} \right),
\]

with

\[
P_e(s_{SU} | h_{ST,PR}, s_{PU}) = (1 - \delta^2)^{1/2} \mathcal{Q}\left( \frac{\sqrt{P_s}h_{ST,PR}}{\sigma_{n,PR}} \sqrt{1 - \delta^2} + (1 - \delta)^{1/2} \right).
\]

If \( s_{SU} \) cannot be correctly decoded, the PR merely decodes its data with \( y_{PR,1} \) and its BER, denoted \( P_{e_{PR,DIR}}^{SU} \), is expressed as

\[
P_{e_{PR,DIR}}^{SU} (s_{PU} | h_{PT,PR}) = \mathcal{Q}\left( \frac{2P_Th_{PT,PR}^2}{\sigma_{n,PR}^2} \right),
\]

Otherwise, PR can combine the two-phase received signals with MRC and its BER, denoted \( P_{e_{PR,MRC}}^{SU} \), is computed as

\[
P_{e_{PR,MRC}}^{SU} (s_{PU} | h_{PT,PR}, h_{ST,PR}) = \mathcal{Q}\left( \frac{2P_Th_{PT,PR}^2}{\sigma_{n,PR}^2} + \frac{P_Sh_{ST,PR}^2}{\sigma_{n,PR}^2} \delta^2 \right)
\]

Then, the BER of \( s_{PU} \) in the first case can then be calculated as

\[
P_{e_{PR,II}}^{PU} = \int_0^\infty \int_0^\infty \left( 1 - e_{PR,II}^{SU} (s_{SU} | h_{ST,PR}) \right) P_{e_{PR,MRC}}^{SU} (s_{PU} | h_{PT,PR}, h_{ST,PR}) + e_{PR,II}^{SU} (s_{SU} | h_{ST,PR}) P_{e_{PR,MRC}}^{SU} (s_{PU} | h_{PT,PR}, h_{ST,PR}) e^{-\frac{\text{SNR}_{PT,PR}}{\text{SNR}_{PT,PR}}} d\text{SNR}_{PT,PR} d\text{SNR}_{PT,PR}
\]

In the second case, we study the BER of the \( s_{PU} \) when the ST decodes \( s_{PU} \) incorrectly. Since the ST keeps silence in this case, the PR can merely decode the signal received in the first phase, as described in (8). The BER of \( s_{PU} \) at the PR can then be computed as

\[
P_{e_{PR,II}}^{PU} = \frac{1}{2} \left( 1 - \frac{\text{SNR}_{PT,PR}}{1 + \text{SNR}_{PT,PR}} \right)
\]

where \( \text{SNR}_{PT,PR} = \sigma^2_{PT,PR}P_T / \sigma^2_{n,PR} \). Since the two considered cases depend on the decoding probability of \( s_{PU} \) at the ST, we have to calculate the BER at the ST to decide the average BER of \( s_{PU} \) and compute it as:

\[
P_{e_{PR,II}} = \frac{1}{2} \left( 1 - \frac{\text{SNR}_{PT,ST}}{1 + \text{SNR}_{PT,ST}} \right)
\]

where \( \text{SNR}_{PT,ST} = \sigma^2_{PT,ST}P_T / \sigma^2_{n,ST} \). In light of those two cases, we compute the average BER of \( s_{PU} \) at the PR as:

\[
P_{e_{PR,II}} = P_{e_{PR,II}}^{PT,II} + (1 - P_{e_{PR,II}}^{PT,II}) P_{e_{PR,II}}^{ST,II} + P_{e_{PR,II}}^{ST,II} P_{e_{PR,II}}^{PT,II},
\]
where $P_{e^{ST, II}}$, $P_{e^{PR, II}}$, and $P_{e^{PR, I}}$ are expressed in (12), (11), and (10), respectively.

C. BER Performance at SR

The SU’s BER is also conducted in two cases: the decoded $s_{e^{SU}}$ at the ST is correctly or not. As for $s_{e^{PU}}$ decoded correctly, the ST uses the SC. The $s_{e^{SU}}$ can then be decoded from the composite signal $y_{SR}$. Similar to the derivation of $P_{e^{PR, II}}$, we can have the BER of the $s_{e^{SU}}$ as:

$$P_{e^{SR, I}}(s_{e^{SU}}) = \frac{1}{2} \sum_{i=0}^{1} \left\{ \frac{P_{e} | h_{ST, SR} |^2}{\frac{1}{\sigma_s^{2}} + \sigma_s^{2} (1 - i^2 \delta^2)} \right\}$$

(14)

Averaging (14) with exponential distribution of $|h_{ST, SR}|^2$, we have

$$P_{e^{SR, II}}(s_{e^{SU}}) = \frac{1}{2} \sum_{i=0}^{1} \left\{ \frac{1}{\sigma_s^{2} (1 - i^2 \delta^2)} \right\}$$

(15)

where $SNR_{ST, SR} = \frac{\sigma_s^{2} P_{e} P_{s}}{\sigma_s^{2}}$. Otherwise, the ST merely transmit the $s_{e^{SU}}$ to the SR and its BER can be calculated as

$$P_{e^{SR, II}}(s_{e^{SU}}) = \frac{1}{2} \left( 1 + \frac{SNR_{ST, SR}}{1 + SNR_{ST, SR}} \right),$$

(16)

Finally, the BER is computed by averaging the BER of two cases, given by

$$P_{e^{SR}}(s_{e^{SU}}) = (1 - P_{e^{SR, II}}(s_{e^{SU}})) P_{e^{SR, I}}(s_{e^{SU}}) + P_{e^{SR, II}}(s_{e^{SU}}).$$

(17)

III. OPTIMAL POWER ALLOCATION

A. Optimization of Power Allocation

In this section, we design the SC and the minimum power consumption of each node so that the BER of the PU and that of the PU are guaranteed. Denote $\gamma_{e^{PU}}$ and $\gamma_{e^{SU}}$ are the maximum tolerating BER for the PU and the SR. The optimization can then be formulated as

$$\begin{align*}
(P1) \quad & \min_{P_{e^{PU}}, P_{e^{SU}}} P_{S} + P_{T} \\
\text{s.t.} \quad & P_{e^{PU}}(s_{e^{PU}}) \leq \gamma_{e^{PU}}, \quad P_{e^{SU}}(s_{e^{SU}}) \leq \gamma_{e^{SU}} \\
& P_{S}, P_{T} \geq 0, \quad 0 \leq \delta^2 \leq 0.5.
\end{align*}$$

In light of (13) and (17), we can observe that (18) is not possible to be solved due to the complicated non-convex constraints. To find tractable solution, we first derive approximation expressions of $P_{e^{SPU}}(s_{e^{PU}})$ and $P_{e^{SU}}(s_{e^{SU}})$, denoted $P_{e^{SPU}}(s_{e^{PU}})$ and $P_{e^{SU}}(s_{e^{SU}})$, respectively, and then substitute them into (18), which will facilitate transferring (18) into a convex one. Let’s consider the following two propositions.

Proposition 1: The BER $P_{e^{SPU}}$ expressed in (13) can be approximated as

$$P_{e^{SPU}} \approx P_{e^{SPU}}^A = \left\{ \frac{\sigma_{n, ST}^2}{4\sigma_{n, PR}^2 P_{T}^2} \right\} \times \left\{ \frac{\sigma_{n, PR}^2}{4\sigma_{n, ST}^2 P_{P}^2} \right\}$$

(19)

$$\left\{ \frac{1}{\sigma_{n, ST}^2} \times \left( \frac{1}{\sigma_{n, PR}^2} \times \delta^2 \right) \right\}$$

Proof: Ignored due to the limited space.

Similarly, we can have the following proposition.

Proposition 2: The BER $P_{e^{SU}}$ can be approximated as

$$P_{e^{SU}} \approx P_{e^{SU}}^A = \left\{ \frac{\sigma_{n, ST}^2}{4\sigma_{n, PR}^2 P_{T}^2} \right\} \times \left\{ \frac{\sigma_{n, PR}^2}{4\sigma_{n, ST}^2 P_{P}^2} \right\}$$

(20)

$$\left\{ \frac{1}{\sigma_{n, ST}^2} \times \left( \frac{1}{\sigma_{n, PR}^2} \times \delta^2 \right) \right\}$$

The result can be straightforward extended from Proposition I. We give Fig. 1 to verify Proposition I and II. In this simulation, we let $\sigma_{n, ST}^2 / \sigma_{n, PR}^2 = 5 \text{ dB}$, $\sigma_{n, PR}^2 / \sigma_{n, PR}^2 = 10 \text{ dB}$, $\sigma_{ST, PR}^2 / \sigma_{PR, SR}^2 = 10 \text{ dB}$, $P_{T} = 1$, $\Delta = \delta^2 = 0.1$ and 0.3. As we can see the curves are tightly matched especially in the high SNR region.

Replacing $P_{e^{SPU}}$ and $P_{e^{SU}}$ with (19) and (20) in (18), we can rewrite the optimization as

$$\begin{align*}
(P2) \quad & \min_{P_{e^{PU}}, P_{e^{SU}}} P_{S} + P_{T} \\
\text{s.t.} \quad & P_{e^{SPU}}(s_{e^{SU}}) \leq \gamma_{e^{SU}}, \quad P_{e^{SU}}(s_{e^{SU}}) \leq \gamma_{e^{SU}} \\
& P_{S}, P_{T} \geq 0, \quad 0 \leq \delta^2 \leq 0.5.
\end{align*}$$

For getting concise notations, we further define $\delta^2 = \Delta$, $\eta_{n} = \frac{\sigma_{n, PR}^2 \sigma_{n, ST}^2}{16\sigma_{ST, PR}^2 \sigma_{ST, ST}^2}$, $\eta_{1} = \frac{\sigma_{n, PR}^2 \sigma_{n, ST}^2}{16\sigma_{ST, PR}^2 \sigma_{ST, ST}^2}$, $\eta_{2} = \frac{\sigma_{n, PR}^2 \sigma_{n, ST}^2}{16\sigma_{ST, PR}^2 \sigma_{ST, ST}^2}$, $\eta_{3} = \frac{\sigma_{n, PR}^2 \sigma_{n, ST}^2}{16\sigma_{ST, PR}^2 \sigma_{ST, ST}^2}$.

To find the solution, we resort to the Karush-Kuhn-Tucker (KKT) conditions corresponding to (21) given as follows

$$\begin{align*}
u_{1} P_{e^{PU}} = 0, \quad -u_{2} P_{S} = 0, \quad -u_{2} \Delta = 0, \quad (22-1) \\
u_{1} (\Delta - 0.5) = 0, \quad (22-2) \\
u_{5} (P_{e^{SPU}} - \gamma_{e^{SU}}) = 0, \quad u_{6} (P_{e^{SU}} - \gamma_{e^{SU}}) = 0, \quad (22-3) \\
u_{1} (P_{e^{SPU}} - \gamma_{e^{SU}}) = 0, \quad (22-4) \\
u_{1} \Delta = 0, \quad u_{2} (P_{e^{SU}} - \gamma_{e^{SU}}) = 0, \quad (22-5) \\
u_{1} \Delta = 0, \quad (22-6)
\end{align*}$$

where $u_{i} \geq 0, \quad i=1,..,6$, are the related Lagrangian multipliers. Eqs. (22-1)-(22-2) imply $u_{i}=u_{1}=0, \quad u_{2}=u_{5}=0$. From (22-5), we conclude that $P_{e^{SPU}} - \gamma_{e^{SU}} = 0$ and consequently

$$\frac{\eta_{1} + \eta_{2}}{P_{T} P_{S}} = \frac{1}{P_{T} P_{S}} (1 - 2\Delta)^2 = \gamma_{e^{SU}}.$$
where \( \Delta \) is the solution conflicts with \((i.i.d.)\) and is complex Gaussian distribution. Let \( \sigma^2 = 1 \), \( \sigma_{n,ST}^2 = \sigma_{n,SR}^2 = \sigma_{n,PR}^2 = 1 \), \( \gamma_{PU} = 10^{-4} \) and \( \gamma_{SU} = 5 \times 10^{-4} \). Three kinds of channel quality are compared: \( (\sigma_{PT,PR}^2, \sigma_{ST,PR}^2, \sigma_{ST,SR}^2) \) (dBm) = \((0.5,10,10), (0.5,20,20), (0.1,10,30,30)\), which are referred to as case 1, case 2, and case 3, respectively. The results are shown in Fig. 2. The circles represent the derived solutions. As we can see from those curves, the minimum total power occurs at the circle points, which shows that the conducted optimum \( \Delta \) in (30) can perfectly match the numerical results. We also can observe that the total power consumption of the case 2 is smaller than that of case 1 since the channel quality of case 2 is superior to that of case 1.

### IV. SIMULATIONS

In this section, we evaluate the performance of the proposed SC scheme in the cognitive overlay system. Here, we assume each channel link is identically independent distributed (i.i.d.) and is complex Gaussian distribution. Let \( \sigma^2 = 1 \), \( \sigma_{n,ST}^2 = \sigma_{n,SR}^2 = \sigma_{n,PR}^2 = 1 \), \( \gamma_{PU} = 10^{-4} \) and \( \gamma_{SU} = 5 \times 10^{-4} \). Three kinds of channel quality are compared: \( (\sigma_{PT,PR}^2, \sigma_{ST,PR}^2, \sigma_{ST,SR}^2) \) (dBm) = \((0.5,10,10), (0.5,20,20), (0.1,10,30,30)\), which are referred to as case 1, case 2, and case 3, respectively. The results are shown in Fig. 2. The circles represent the derived solutions. As we can see from

\[
P_T^2 = \frac{\eta_d}{\eta_c} \times \left[ \frac{1 - 2 \Delta^2}{\gamma_{PU} \eta_c} \left[ 1 - \frac{3(1 - 2 \Delta^2)}{\Delta} \right] \right] x P_T^2
\]

\[
+ \left[ \frac{1}{\gamma_{PU} \eta_c} \times \left[ 1 + \frac{3(1 - 2 \Delta^2)}{\Delta} - \frac{\eta_c}{\gamma_{PU}} \right] \right] x P_T + \left[ \frac{\eta_d}{\gamma_{PU} \eta_c} \times \left[ 1 - (1 - 2 \Delta^2) \right] \right] = 0
\]

The related closed-form solutions are expressed as

\[
P_T = \frac{A}{3} + \frac{\sqrt{A^3 + 9B + 9C}}{9}
\]

\[
= \frac{A}{6} \times \left[ \frac{1}{AB} \left[ \frac{1}{2} - \frac{1}{(1 - 2 \Delta)^2} \right] + \frac{AB - A^2 - C}{6} \right] + \frac{B}{3}
\]

where

\[
A = \frac{\eta_d}{\eta_c} \times \left[ \frac{1 - 2 \Delta^2}{\gamma_{PU} \eta_c} \right] x P_T + \left[ \frac{1 - (1 - 2 \Delta^2)}{\gamma_{PU}} \right]
\]

\[
B = \frac{1}{\gamma_{SU} \eta_c} \times \left[ \frac{\eta_c}{\gamma_{PU}} \right]
\]

\[
C = \frac{\eta_d \eta_c}{\gamma_{PU} \eta_c} \times \left[ 1 - (1 - 2 \Delta^2) \right]
\]

Considering the solutions of (25), we can find only one solution is guaranteed to satisfy the constraints since one solution conflicts with \( P_T \geq 0 \) and the other makes \( P_{e,SU} \geq 1 \).

From (23), we can reformulate \( P_S \) as:

\[
P_S = \frac{1}{\gamma_{SU}} \left[ \frac{\eta_d}{\eta_c} x \left( P_T + \frac{\eta_c}{(1 - 2 \Delta)^2} \right) \right] - \frac{\eta_d}{\gamma_{SU}} \left( \frac{1}{(1 - 2 \Delta)^2} \right).
\]

Substituting (25) into (29), we can find the cost function, \( P_S + P_T \), of (21) is a complicated function of \( \Delta \) now. Fortunately, it can be verified to be convex for \( \Delta \in [0,0.5] \) with vector composition method [7]. Thus, considering (25) and (29), we can reformulate P2 of (21) as:

\[
(P3) \quad \min P_S + P_T \quad \text{s.t.} \quad 0 \leq \Delta \leq 0.5
\]

The above optimization is convex and thus can be effectively solved numerically [7].

### REFERENCES


