RVQ-Based Beamforming Design with MMSE Criterion in MISO Interference Channels

Fan-Shuo Tseng+, Jhen-Fong Gu*, and Shou-Sheu Lin†

+*Institute of Communications Engineering, National Sun Yat-sen University, Kaohsiung, Taiwan

†National Kaohsiung First University of Science and Technology, Kaohsiung, Taiwan

E-mail: fs.tseng@mail.nsysu.edu.tw; m993010120@student.nsysu.edu.tw; ssl@nkfust.edu.tw

***Abstract*—This paper proposes robust beamforming designs for a multiple-input single-output (MISO) interference channel (IC). Considering the random vector quantization (RVQ) feedback mechanism, we first derive the closed-form expression for the mean-squared error (MSE) metric by averaging over the noise, the quantization error, and the channel amplitude. With the derived MSE result, we devise the robust beamformer by minimizing total MSEs and by minimizing the maximum per-user MSE. Since the optimizations of both designs are not convex, we then propose iterative methods to find out the solutions. The simulation results verify the robustness of both designs when the quantization error exists.**

***Index Terms*: Multiple-input single-output (MISO), interference channel (IC), robust design, beamformer, minimum mean-squared error (MMSE), random vector quantization (RVQ).**

I. INTRODUCTION

The transceiver designs in multiple-input multiple-output (MIMO) interference channels (IC) channels are mainly categorized in two types. The first adopts the interference alignment (IA) technique, which aligns the interference into a lower dimensional received signal space, the intended signals are in an interference-free subspace [1]. The IA therefore can be regarded as an optimum solution in asymptotical signal-to-noise ratio (SNR) scenario. On the top of the IA technique, the alternative transceiver design for the MIMO IC is to adopt the minimum mean-squared error (MMSE) criterion which can effectively improve the bit-error-rate (BER) performance, especially in the intermediate SNR region [2], [3]. Previous studies propose the min-sum mean-squared error (MSE) design and the min-max per-user MSE design which considers the fairness issue [2]. The min-max per-user stream MSE design is also devised in the MIMO IC to further improve the performance of each user [3]. However, the above MIMO IC transceiver designs mainly rely on the perfect channel state information (CSI) to devise the precoders, which is not practical in wireless communications. The precoder designs based on the limited feedback mechanisms are thus developed [4]-[7] to feed back CSIs, where the random vector quantization (RVQ) limited feedback is attractive due to its simple structure [5]-[7].

In addition to the maximization of the overall capacity criterion, the precoders design with the limited feedback design by the MMSE criterion is also critical for improving the link reliability. To the best of our knowledge, it is not studied yet in the MIMO IC systems. In this paper, we will investigate the MMSE transceiver designs with the RVQ feedback mechanism in MISO IC systems. We first derive the closed-form MSE expression estimated at the transmitter where the MSE is computed by averaging over the noise, the channel amplitude, and the CDI quantization error. Using the derived MSE, we propose two robust design criteria, i.e., minimizing the total MSE and minimizing the maximum per-user MSE, to design the robust beamformers. Since the designed problems are not convex. We then propose two iterative approaches to find out the tractable solutions. With the iterative approach, the beamformers can be derived as the function of the decoding scalars and vice versa. The solutions are obtained as the iterative process is converged. Particularly, for the design with minimizing the maximum per-user MSE, we solve the decoding scalars and the beamformers iteratively by MMSE and second-order cone programming (SOCP). Simulations verify the robustness of our designs and the min-max per-user outperforms the other design due to the inherent worst-case design.

The rest of this paper is organized as follows. The MISO IC system and the MSE computed by the RVQ mechanism are introduced in Section II. In Section III, we formulate the precoders design as two optimizations, and the related solutions are derived. The simulations are illustrated and discussed in Section IV. Finally, we draw the conclusions in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model of MISO Interference Channel

We consider a K-user MISO interference channels, where for the ith transceiver pair, there are N_i antennas at the transmitter and only one antenna at the receiver. Denote the ith transmitted signal be s_i and its beamformer, $\mathbf{f}_i \in^{N_i \times 1}$. The channel between the ith transmitter and the kth receiver is expressed as $\mathbf{h}_{k,i} \in \mathbb{C}^{N_i \times 1}$. The received signal of the kth user, denoted as y_k, can be expressed as

$$y_k = \mathbf{h}_{k,k}^T \mathbf{f}_k s_k + \sum_{i=1, i \neq k}^{K} \mathbf{h}_{k,i}^T \mathbf{f}_i s_i + n_k \,. \tag{1}$$

where n_k is the noise at the kth receiver and follows the complex Gaussian distribution $\mathcal{CN}\left(0, \sigma_{n,k}^2\right)$. The transmit

signal, s_k for all k's , is assumed to be independent and identically distributed (i.i.d.) with zero-mean and variance $\sigma_{s,k}^2$. The power constraint can then be formulated as $\sigma_{s,k}^2 \|\mathbf{f}_k\|^2 \le P_k$ for all k's . The estimated signal at the kth receiver, $\hat{s}_k$, is obtained by the one-tap equalizer a_k , and the resultant MSE is computed as

$$e_k = E\left\{\left|\hat{s}_k - s_k\right|^2\right\} = \sigma_{s,k}^2 \left|a_k\right|^2 \left\|\mathbf{h}_{k,k}^T \mathbf{f}_k\right\|^2 + \left|a_k\right|^2 \left(\sum_{i=1,i\neq k}^{K} \sigma_{s,i}^2 \left\|\mathbf{h}_{k,i}^T \mathbf{f}_i\right\|^2\right) - \sigma_{s,k}^2 a_k \mathbf{h}_{k,k}^T \mathbf{f}_k - \sigma_{s,k}^2 \mathbf{f}_k^H \mathbf{h}_{k,k}^* a_k^* + \sigma_{s,k}^2 + \sigma_{\mathrm{n},k}^2 \left|a_k\right|^2 . \tag{2}$$

B. MSE performance with RVQ

We assume that the receiver can perfectly estimate the channel and the RVQ mechanism is used. Each transmitter can thus know all the quantized channels and utilizes those CSIs to design its beamformer. At the beginning of each transmission link, the kth receiver quantizes the channel direction vector, $\tilde{\mathbf{h}}_{k,i} = \mathbf{h}_{k,i} / \left\|\mathbf{h}_{k,i}\right\|$ and then feeds back the quantized CDI, $\hat{\mathbf{h}}_{k,i}$, by using $B_{k,i}$ bits to the transmitters via an error-free feedback channel. The quantized channel direction vector, $\hat{\mathbf{h}}_{k,i}$, is chosen according to the selection criterion $\hat{\mathbf{h}}_{k,i} = \arg\max_{\mathbf{c}\in C_{k,i}} \left|\left(\tilde{\mathbf{h}}_{k,i}\right)\right|$ where each codeword is isotropically distributed on the N_i-dimensional complex unit sphere. Then, the transmitter uses the quantized CDI, the statistics of the quantization error, and the statistics of channel amplitude to design the beamformers.

Using the RVQ mechanism, the true CDI and the quantized CDI have the following relation [5]

$$\tilde{\mathbf{h}}_{k,i} = \cos\theta_{k,i} \hat{\mathbf{h}}_{k,\mathrm{i}} + \sin\theta_{k,i} \mathbf{g}_{k,i}, \tag{3}$$

where $\mathbf{g}_{k,i}$ is a unit vector orthogonal to $\hat{\mathbf{h}}_{k,\mathrm{i}}$, $\cos^2\theta_{k,i} = \left|\hat{\mathbf{h}}_{k,i}^H \tilde{\mathbf{h}}_{k,i}\right|^2$ and $\sin^2\theta_{k,i} = 1 - \cos^2\theta_{k,i}$, $\cos\theta_{k,i} \ge 0$. Also, at the transmit side, the beamformer can then be designed by considering the quantization error in the MSE performance (2). To proceed, we consider the following property first.

Property 1 [5]-[7]: The random vector has the following results:

$$E[\mathrm{g}_{k,i}] = \mathbf{0}_{N_i\times 1} \tag{4}$$

$$E[\sin^2\theta_{k,i}] = 2^{B_{k,i}} \beta\left(2^{B_{k,i}}, \frac{N_i}{N_i - 1}\right) \tag{5}$$

$$E[\cos\theta_{k,i}] = \sum_{m=1}^{2^{B_{k,i}}} \binom{2^{B_{k,i}}}{m} (-1)^{m+1} m(N_i - 1)\beta\left(m(N_i - 1), \frac{3}{2}\right) \tag{6}$$

$$E\left[\left\|\mathbf{h}_{k,i}\right\|\right] = \frac{\sqrt{\pi} 2^{-N_i} (2N_i - 1)!!}{\Gamma(N_i)} . \tag{7}$$

Considering the RVQ effect in the MSE of Eq. (2), the expectation is taken over the noise, the channel amplitude, and the random vector quantization error. Assuming that each channel element of $\mathbf{h}_{k,i}$ is i.i.d., we can have the following property.

Property2: For each quantized channel direction vector, we have the following statistical results.

$$E\left[\left|\mathbf{h}_{k,i}^T \mathbf{f}_i\right|^2\right] = \left(N_i E\left[\cos^2\theta_{k,i}\right] - \frac{N_i E\left[\sin^2\theta_{k,i}\right]}{(N_i - 1)}\right)\left|\hat{\mathbf{h}}_{k,i}^T \mathbf{f}_i\right|^2 + \frac{N_i E\left[\sin^2\theta_{k,i}\right]}{(N_i - 1)}\left\|\mathbf{f}_i\right\|^2 \tag{8}$$

$$E\left[\left|a_k\right|^2 \mathbf{h}_{k,\mathrm{i}}^* \mathbf{h}_{k,\mathrm{i}}^T\right] = \left|a_k\right|^2 N_i E\left[\cos^2\theta_{k,i}\right] \hat{\mathbf{h}}_{k,\mathrm{i}}^* \hat{\mathbf{h}}_{k,\mathrm{i}}^T + \frac{\left|a_k\right|^2 N_i E\left[\sin^2\theta_{k,i}\right]}{N_i - 1}\left(\mathbf{I} - \hat{\mathbf{h}}_{k,\mathrm{i}}^* \hat{\mathbf{h}}_{k,\mathrm{i}}^T\right). \tag{9}$$

The detailed derivation is ignored due to the limited space. Then we can have the MSE of the kth user as

$$e_k = \left|a_k\right|^2 \left(\sum_{i=1}^{K} \sigma_{s,i}^2 E\left[\left|\mathbf{h}_{k,i}^T \mathbf{f}_i\right|^2\right]\right) - \sigma_{s,k}^2 \left(a_k E\left[\mathbf{h}_{k,k}^T\right] \mathbf{f}_k + \mathbf{f}_k^H E\left[\mathbf{h}_{k,k}^*\right] a_k^*\right) + \sigma_{s,k}^2 + \sigma_{n,k}^2 \left|a_k\right|^2 , \tag{10}$$

where $E\left[\left|\mathbf{h}_{k,i}^T \mathbf{f}_i\right|^2\right]$ and $E[\mathbf{h}_{k,i}]$ can be computed by (4)-(9). In the next section, we will propose two types of MSE design criteria with RVQ to devise the transmit beamformer $\mathbf{f}_i$.

III. MSE-Based Transceiver Designs With RVQ

In this section, we design the beamformers in MISO IC systems with two design criteria, the minimization of all users' MSEs and the minimization of the maximum per-user MSE. The later design considers the fairness issue and consequently improves the BER performance.

A. Min-Sum MSE

In this subsection, we study the transceiver design where a set of transmit beamformers and decoding scalars $\{(\mathbf{f}_k, a_k), k = 1, \cdots, K\}$ are jointly devised to minimize the sum of all users' MSEs and satisfy the individual transmit power constraint. The optimization problem can then be formulated as

$$\min_{\{a_k, \mathbf{f}_k\}} \sum_{k=1}^{K} e_k, \qquad s.t. \quad \sigma_{s,k}^2 \left\|\mathbf{f}_k\right\|^2 \le P_k, \quad \forall k \tag{11}$$

where e_k is expressed in (10). As we can see, the objective function is not jointly convex for all designed parameters. However, it is convex for each of the beamforming vectors (or decoding scalars) when the other design parameters are treated as determined constant terms.

To derive the solution, we express the Lagrangian function related to (12) as:

$$L(a_k, \mathbf{f}_k, \lambda_k) = \sum_{k=1}^{K} e_k + \sum_{k=1}^{K} \lambda_k \left(\sigma_{s,k}^2 \left\|\mathbf{f}_k\right\|^2 - P_k\right) \tag{12}$$

where the parameters of λ_k , for all k, are the Lagrange multiplier associated with the kth user's power constraint. With

the Karush-Kuhn-Tucker (KKT) conditions, we can have the following optimality conditions given by

$$\mathbf{f}_k = a_k^* \left(\sum_{i=1}^{K} |a_i|^2 E\left\{ \mathbf{h}_{i,k}^* \mathbf{h}_{i,k}^T \right\} + \lambda_k \mathbf{I} \right)^{-1} E\left\{ \mathbf{h}_{k,k}^* \right\}, \quad k \in \{1, \cdots, K\} \tag{13}$$

$$a_k = \frac{\mathbf{f}_k^H E\left\{ \mathbf{h}_{k,k}^* \right\}}{\left(\sum_{i=1}^{K} E\left[\left| \mathbf{h}_{k,i}^T \mathbf{f}_i \right|^2 \right] + \sigma_n^2 \right)}, \quad k \in \{1, \cdots, K\} \tag{14}$$

$$\lambda_k \left(\sigma_{s,k}^2 \left\| \mathbf{f}_k \right\|^2 - P_k \right) = 0, \quad k \in \{1, \cdots, K\}. \tag{15}$$

As shown in (13) and (14), the optimal solutions of $\mathbf{f}_k$ and a_k are mutually dependent. It is difficult to simultaneously solve the pair $\{\mathbf{f}_k, a_k\}$ for all k's . Therefore, we adopt the iterative approach to overcome the difficulty. Specifically, whenever the beamformer $\mathbf{f}_k$ is estimated at the kth transmitter, the corresponding decoding scalar a_k can be derived from (14), and then the beamformer, $\mathbf{f}_k$, is updated with (13). If the power constraint (15) related to $\mathbf{f}_k$ is not satisfied, we have to adjust λ_k so that the individual power constraint can be satisfied. Therefore, combining (13) into (15), we find out the updated λ_k so that

$$|a_k|^2 Tr\left\{ E\left\{ \mathbf{h}_{k,k}^T \right\} \sigma_{s,k}^2 \left(\sum_{i=1}^{K} |a_i|^2 E\left\{ \mathbf{h}_{i,k}^* \mathbf{h}_{i,k}^T \right\} + \lambda_k \mathbf{I} \right)^{-2} E\left\{ \mathbf{h}_{k,k}^* \right\} \right\} = P_k. \tag{16}$$

The above solution can be obtained numerically, such as bi-section methods. When the updated λ_k satisfies (15), we then recalculate the beamformer, $\mathbf{f}_k$, according to (13). The above process is repeated until the solutions converge.

B. Min-Max Per-User MSE

The above design, i.e., min-sum MSE, may lead to the case that a certain user has the poor MSE performance even if the sum of MSEs is small. To avoid this shortcoming, we adopt another method called min-max per-user MSE to minimize the maximum of all user's MSE, which is expressed as

$$\begin{aligned} &\min_{\{a_k, \mathbf{f}_k\}, k=1,\cdots,K} \max e_k \\ &s.t. \quad \sigma_{s,k}^2 \left\| \mathbf{f}_k \right\|^2 \le P_k, \quad \forall k. \end{aligned} \tag{17}$$

Like the min-sum MSE problem in Section III-A, the problem (18) is still not a convex problem. We herein iteratively derive the beamforming vectors with the known decoding scalars and then the decoding scalars with given beamforming vectors. Thus, the transmit beamformers are designed via minimizing the maximum of all users' MSEs. With the help of an auxiliary variable t, we can equivalently reformulate the subproblem in (17) as

$$\min_{\{\mathbf{f}_k, t\}} t \quad s.t. \quad \sqrt{e_k} \le t, \quad \sigma_{s,k}^2 \left\| \mathbf{f}_k \right\|^2 \le P_k, \quad \forall k \tag{18}$$

To solve problem (18) effectively, we aim at reformulating (18) as a SOCP. To proceed, we first give some results required in the derivation process. Note that define

$$\Omega_{k,i} = \frac{E\left\{ \left\| \mathbf{h}_{k,i} \right\|^2 \sin^2 \theta_{k,i} \right\}}{N_i - 1} \left(\mathbf{I}_{N_i} - \hat{\mathbf{h}}_{k,i}^* \hat{\mathbf{h}}_{k,i}^T \right). \tag{19}$$

and $\left\| \mathbf{f}_i \right\|^2 - \left| \hat{\mathbf{h}}_{k,i}^T \mathbf{f}_i \right|^2 = \mathbf{f}_i^H \left(\mathbf{I}_{N_i} - \hat{\mathbf{h}}_{k,i}^* \hat{\mathbf{h}}_{k,i}^T \right) \mathbf{f}_i$. , we can further have

$$E\left\{ \left| \mathbf{h}_{k,i}^T f_i \right|^2 \right\} = \mathbf{f}_i^H \Omega_{k,i} \mathbf{f}_i + N_i E\left[\cos^2 \theta_{k,i} \right] \left| \hat{\mathbf{h}}_{k,i}^T \mathbf{f}_i \right|^2. \tag{20}$$

Finally, we have the following cone formulation of e_k, given by

$$e_k = \left\| \begin{bmatrix} \hat{\mathbf{H}}_k^A \mathbf{f}_A \\ \hat{\Omega}_k^A \mathbf{f}_A \\ \sigma_{s,k} \left| a_k \right| \sqrt{\mathrm{var}\left\{ \left\| \mathbf{h}_{k,k} \right\| \cos \theta_{k,k} \right\}} \hat{\mathbf{h}}_{k,k}^T \mathbf{f}_k \\ \sigma_{s,k} \left(1 - a_k E\left\{ \left\| \mathbf{h}_{k,k} \right\| \cos \theta_{k,k} \right\} \hat{\mathbf{h}}_{k,k}^T \mathbf{f}_k \right) \\ \sigma_{n,k} a_k \end{bmatrix} \right\|_2^2 \tag{21}$$

where

$$\mathbf{f}_A = \left[\mathbf{f}_1^T \cdots \mathbf{f}_k^T \cdots \mathbf{f}_K^T \right]^T, \tag{22}$$

$$\hat{\mathbf{H}}_k^A = \begin{bmatrix} |a_k| \sigma_{s,1} \sqrt{N_1 E\left[\cos^2 \theta_{k,1} \right]} \hat{\mathbf{h}}_{k,1}^T & 0 & \cdots & 0 & 0 \\ 0 & \ddots & 0 & \ddots & 0 \\ \vdots & 0 & \mathbf{0}_{1 \times N_i} & 0 & \vdots \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 0 & |a_k| \sigma_{s,K} \sqrt{N_K E\left[\cos^2 \theta_{k,K} \right]} \hat{\mathbf{h}}_{k,K}^T \end{bmatrix} \tag{23}$$

$$\bar{\Omega}_k^A = \begin{bmatrix} \sigma_{s,1} \left| a_1 \right| \Omega_{k,1}^{1/2} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_{s,1} \left| a_K \right| \Omega_{k,K}^{1/2} \end{bmatrix} \tag{24}$$

$$\begin{aligned} &\mathrm{var}\left\{ \left\| \mathbf{h}_{k,k} \right\| \cos \theta_{k,k} \right\} \triangleq E\left[\left\| \mathbf{h}_{k,k} \right\|^2 \cos \theta^2{}_{k,k} \right] - E^2\left[\left\| \mathbf{h}_{k,k} \right\| \cos \theta_{k,k} \right] \\ &= N_k \left(1 - 2^{B_{k,k}} \beta\left(2^{B_{k,k}}, \frac{N_k}{N_k - 1} \right) \right) - \left[\frac{\sqrt{\pi} 2^{-N_k} (2N_k - 1)!!}{\Gamma(N_k)} \times \right. \\ &\left. \sum_{m=1}^{2^{B_{k,k}}} \binom{2^{B_{k,k}}}{m} (-1)^{m+1} m (N_k - 1) \beta\left(m(N_k - 1), \frac{3}{2} \right) \right] \end{aligned} \tag{25}$$

Thus, the min-max optimization can be further formulated as a SOCP problem given by

$$
\begin{aligned}
&\min_{\{\mathbf{f}_k, t\}} t \\
&s.t. \\
&\left\| \begin{bmatrix} \hat{\mathbf{H}}_k^A \mathbf{f}_A \\ \hat{\Omega}_k^A \mathbf{f}_A \\ \sigma_{s,k} \left| a_k \right| \sqrt{\mathrm{var}\left\{ \left\| \mathbf{h}_{k,k} \right\| \cos\theta_{k,k} \right\}} \hat{\mathbf{h}}_{k,k}^T \mathbf{f}_k \\ \sigma_{s,k} \left(1 - a_k E\left\{ \left\| \mathbf{h}_{k,k} \right\| \cos\theta_{k,k} \right\} \hat{\mathbf{h}}_{k,k}^T \mathbf{f}_k \right) \\ \sigma_{n,k} a_k \end{bmatrix} \right\| \le t \\
&\left\| \sigma_{s,k} \mathbf{f}_k \right\| \le \sqrt{P_k}, \forall k.
\end{aligned}
\tag{26}
$$

The optimization in (26) can be effectively solved via the standard SOCP solver, such as SeDuMi [9]. Using the standard SOCP solver, we then obtain the beamformers, $\mathbf{f}_k$'s$(k = 1, \cdots, K)$. After the beamformers being obtained, we then proceed to devise the optimal decoding scalars by (14) The process is terminated until it converges.

IV. SMULATIONS

In the simulations, 2-user MISO interference channels are considered where $N_1 = N_2 = 2$. Each channel element is assumed to be flat-fading and an i.i.d. circularly symmetric complex Gaussian variable with zero mean and unit variance. The number of quantization bits for quantized CDI is 8. The binary phase shift keying (BPSK) modulated symbols are used. We first evaluate the BER performance of the proposed iterative min-sum MSE and min-max per-user MSE methods with different number of iterations. As displayed in Fig. 1, the performance of both methods is gradually improved as the number of iterations increases. This is because the MSE becomes smaller, compared with that in the previous iteration. When the iterative process continues, the multiuser interference eventually diminishes. The min-max per-user MSE design outperforms the min-sum MSE design because it is the worst-case design in each iteration.

We then investigate the BER performance of those designs with the different numbers of quantization bits in Fig. 2. Herein, the beamformers are derived with 4 iterations. As we can see, the designs with a larger number of quantization bits can have better performance since they have a smaller quantization error. When the quantization error is large (i.e. large MSE), the required number of iterations in this case is smaller than the number with a small quantization error. Therefore, 4 iterations are enough for the beamformers with 2 and 4 bits, and the two design criteria have the same performance according to the similar reason provided in Fig. 1.

V. CONCLUSIONS

Considering the statistics of the quantization error and the channel amplitude due to RVQ mechanism, we have derived the closed-form MSE expressions and formulate the robust designs with the min-max per-user MSE and the min-sum MSE criteria respectively. Since the problems are not convex, we proposed two iterative methods to find out the tractable solutions. By the simulations, we can observe that the proposed designs are against quantization error.

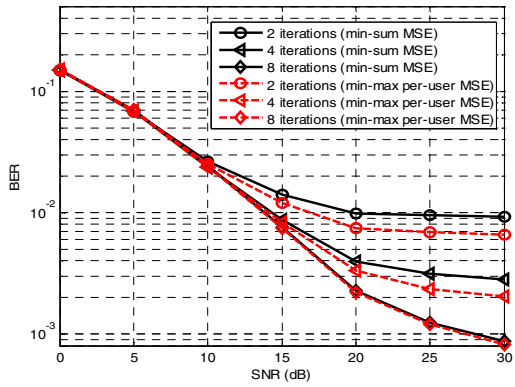

Fig. 1. BER performance of the proposed designs with the different number of iterations.

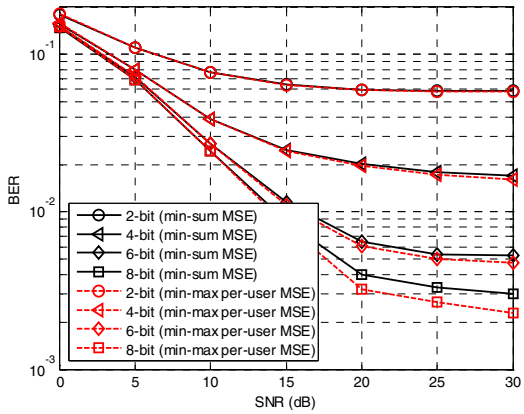

Fig. 2. BER performance of the proposed designs with the different number of quantization bits (Number of iterations = 4).

REFERENCE

[1] S. A. Jafar and M. Fakereddin, "Degrees of freedom for the MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 53, no. 7, pp. 2637-2642, July 2007.

[2] H. Shen, B. Li, M. Tao, and X. Wang, "MSE-based transceiver designs for the MIMO interference channel," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3480-3489, Nov. 2010.

[3] C. E. Chen and W. H. Chung, "An iterative minmax per-stream MSE transceiver design for MIMO interference channel," *IEEE Wireless Commun. Lett.*, vol. 1, no. 3, pp. 229-232, June 2012.

[4] D. J. Love, R. W. Heath Jr., V. K. N. Lau, D. Gesbert, B. D. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE Trans. J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341-1365, 2008.

[5] N. Jindal, "MIMO broadcast channels with ┐nite-rate feedback," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 5045-5060, Nov. 2006.

[6] C. Zhang, W. Xu, and M. Chen, "Robust MMSE beamforming for multiuser MISO systems with limited feedback," *IEEE Signal Processing Lett.*, vol. 16, no. 7, pp. 588-591, July 2009.

[7] F. S. Tseng, Y. C. Wang, C. Y. Hsu, and S. S. Lin, "Robust Tomlinson-Harashima precoder design with random vector quantization in MIMO Systems," *IEEE Comm. Lett.*, vol. 18, no. 2, pp. 265-268, Feb. 2014.

[8] H. Gao, T. Lv, F. Di, S. Yang, C. Yuen, "Limited feedback-based interference alignmment for interfering multi-acess channels," *IEEE Commum. Lett.*, vol. 18, no. 4, pp. 540-543, April 2014.

[9] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB tool for optimization over symmetric cones," *Optim. Methods Softw.*, vol. 11-12, pp. 625-653, 1999.