A Narrowband Active Noise Control System with Frequency Mismatch Compensation

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Abstract—Narrowband active noise control (ANC) systems enjoy good performance where sinusoidal signals dominate in the primary noise, on condition that a reference signal of the same frequencies with the primary noise is given. However, frequencies of the reference signal provided by nonacoustic sensors are usually different from that of the primary noise due to temperature changes, aging, etc. Such frequency mismatch (FM) will make the narrowband ANC systems unable to suppress the primary noise effectively, even render them useless. In this paper, we propose a new narrowband ANC system that integrated with a frequency estimation subsystem. The frequency estimation is obtained from a spectrum computation based on an adaptive linear prediction filter. The estimated frequencies are used by the cosine signal generator to produce a more accurate reference signal to the main controller, thus the performance deterioration caused by FM can be mitigated. The effectiveness of the proposed system has been confirmed by numerous simulations.

I. INTRODUCTION

Noise, an unfriendly by-product of vehicles and machines, is harmful to people’s physical and mental health. Traditionally, passive noise control (PNC) systems are employed to absorb or attenuate noise, but such systems are always bulky and weak in dealing with low frequency noise [1]. Active noise control (ANC) systems, on the other hand, are very powerful to suppress low frequency noise [1].

For the special case of narrowband ANC (NANC) systems, since the primary noise are dominated by sinusoids, nonacoustic sensors are usually used to measure the frequency components of the primary noise. The measured frequency components are then utilized by the cosine signal generator to produce reference signals. An adaptive filter is used to process the reference signal to generate the secondary noise, which is of the same amplitude but opposite phase with the primary noise. Obviously, an accurate measurement of the primary noise’s frequencies is crucial for the whole system to fulfill its job.

However, nonacoustic sensors are susceptible to temperature changes, ageing, etc., that unable to provide the exact frequencies of the primary noise. The difference between the true frequencies of the primary noise and the frequencies obtained from the nonacoustic sensors, is referred to as frequency mismatch (FM), which will seriously downgrade the performance of narrowband ANC systems. In the following of this paper, FM is defined as $\left(\frac{\omega_{\text{real}} - \omega_{\text{sensor}}}{\omega_{\text{real}}}\right) \times 100\%$, where $\omega_{\text{real}}$ stands for the real frequency of a particular sinusoid resides in the primary noise, while $\omega_{\text{sensor}}$ stand for the one we get from non-acoustic sensor. We also presume that FM is less than 10%, if not, more accurate nonacoustic sensors should be used.

Recently, the effect of FM in narrowband ANC systems has been analyzed in [2], [3], showing server system performance downgrade even if FM is just 1%. To solve this ill-effect, numbers of compensation methods have been put forward. In [4], adaptive step sizes are suggested to accommodate FM. But the performance improvement of this method is limited by the fact that large step size has to be used to deal with high FM, making the system unstable. Infinite impulse response (IIR) filter is also proposed in [4] to track sinusoids. However, this method only works well when the signal-to-noise ratio (SNR) is high. In [2], [5], and [6], Xiao proposed to use frequency compensators to track the discrete frequencies reside in the error signal. The method has been proved to be effective in suppressing FM, but still unable to deal with large FM quickly. In [7], a minimum variance distortion-less response (MVDR) spectrum is employed to estimate the frequency components of the primary noise. This system has demonstrated fast convergence. Nonetheless, the serious response distortion of the Hilbert transformer, which is used to convert real signals to complex signals for the MVDR spectrum, at low frequency range will make the method incapable of suppressing multiple low frequencies.

In this paper, we propose a new narrowband ANC system that integrated with a frequency estimation subsystem. In the subsystem an adaptive linear prediction filter [8], [9], is used to identify discrete frequencies of the source noise. The discerned frequencies are used to produce more accurate reference signals to the main controller to mollify the performance deterioration caused by FM. The proposed system enjoys faster convergence compared with Xiao’s method [2].

The rest of the paper is organized as follows. Conventional NANC system is first introduced in section II. In section III,
Xiao’s frequency compensators method and our novel method are presented. Numbers of simulations are conducted in section IV, confirmed the effectiveness of the new method. The paper is summarized by section V.

II. CONVENTIONAL NANC SYSTEM

A typical conventional NANC system in [1] is depicted in Fig. 1. The primary noise can be expressed as:

\[ d(n) = \sum_{i=1}^{q} \{a_{p,i} \cos(\omega_{p,i}n) + b_{p,i} \sin(\omega_{p,i}n)\} + v_d(n) \]  

(1)

where \( q \) is the number of sinusoids in the primary noise, \( \omega_{p,i} \) is frequency of \( i \)-th component, \( v_d(n) \) is a zero-mean white noise with variance of \( \sigma_d^2 \), \( \{a_{p,i}, b_{p,i}\} \) stand for the amplitudes of cos- and sin-part of the \( i \)-th frequency component.

Fig. 1. \( i \)-th channel of conventional NANC system

The reference signal of \( i \)-th channel is generated by cosine signal generator, and can be expressed as:

\[ x_i(n) = a_{r,i} \cos(\omega_{r,i}n) + b_{r,i} \sin(\omega_{r,i}n) \]  

(2)

where \( \{a_{r,i}, b_{r,i}\} \) are the amplitude of the cos- and sin-part of the reference signal, \( \omega_{r,i} \) is \( i \)-th frequency measured by non-acoustic sensor. The secondary noise \( y(n) \) is formed by summing up the output of each channel:

\[ y(n) = \sum_{i=1}^{q} y_i(n) \]

\[ = \sum_{i=1}^{q} \{h_{i,0}(n)x_i(n) + h_{i,1}(n)x_i(n-1)\} \]

(3)

where the output \( i \)-th channel of formed by coefficients \( \{h_{i,0}(n), h_{i,1}(n)\} \) and the reference signal of that channel. The filter-x least mean square (FxLMS) algorithm are used to update the coefficients \( \{h_{i,0}(n), h_{i,1}(n)\} \) to the end that the error signal \( e(n) \) is at its minimum possible, and the error signal is given by:

\[ e(n) = d(n) - y_s(n) \]

(4)

where \( y_s(n) \) is the version of secondary noise \( y(n) \) passed through secondary path \( S(z) \). Here, \( y_s(n) \) can be expressed as follows:

\[ y_s(n) = \sum_{j=0}^{M-1} s_j y(n-j) \]  

(5)

where \( M \) and \( \{s_j\}_{j=0}^{M} \) are the length and coefficients of the secondary path, which is considered as a finite impulse response (FIR) filter. The update equation for coefficients \( \{h_{i,0}(n), h_{i,1}(n)\} \) can be expressed as:

\[ h_{i,0}(n+1) = h_{i,0}(n) + \mu e(n)x_{s,1,i}(n) \]

(6)

\[ h_{i,1}(n+1) = h_{i,1}(n) + \mu e(n)x_{s,2,i}(n-1) \]

(7)

where \( x_{s,1,i}(n) \) is the version of reference signal filtered by the estimated secondary path \( \hat{S}(z) \). \( \mu \) is the step size parameter of \( i \)-th channel. And \( x_{s,2,i}(n) \) is given by:

\[ x_{s,2,i}(n) = \sum_{j=0}^{M-1} \hat{s}_j x_{s,1,i}(n-j) \]

(8)

where \( M \) and \( \{\hat{s}_j\}_{j=0}^{M} \) are the length and coefficients of the estimated secondary path.

III. XIAO’S METHOD AND THE PROPOSED METHOD

In [2], Xiao proposed to use frequency compensator to track sinusoids in the primary noise. The basic diagram for Xiao’s method is depicted in Fig.2. The reference signal of \( i \)-th channel is generated by frequency compensator according to the following equation:

\[ x_i(n) = -c_i x_i(n-1) - x_i(n-2), n \geq 2 \]

(9)

The initial value of reference signal is set up according to \( \omega_{r,i}(0) \), \( i \)-th reference frequency obtained from nonacoustic sensors:

\[ x_i(0) = a_{r,i} \cos(\omega_{r,i}(0)) + b_{r,i} \sin(\omega_{r,i}(0)) \]

(10)

In order to tracking sinusoids reside in the primary noise, the frequency related coefficient \( c_i(n) \) is updated by minimizing residual error signal \( e(n) \) with function:

\[ \frac{\partial e^2(n)}{\partial c_i(n)} = -e(n) \frac{\partial}{\partial c_i(n)} \sum_{j=0}^{M-1} f_j y_i(n-j) \]

\[ = -e(n) f_0 \frac{\partial y_i(n)}{\partial x_i(n)} \frac{\partial x_i(n)}{\partial c_i(n)} \]

\[ = f_0 e(n) h_{i,0}(n) x_i(n-1) \]

(11)
The update recursions for $c_i(n)$ can be expressed:

$$c_i(n+1) = c_i(n) - u_{i,c}(n) h_{i,c}(n) x_i(n-1), \quad i = 1, 2, ..., q.$$  \hspace{1cm} (12)

Such algorithms have been proved in [2], [4] and [5] to be effective to mitigate FM of 1% and 5%, even up to 10%. Nonetheless, this frequency compensator method suffers from slow convergence, especially when FM is high.

Our novel NANC system is depicted in Fig. 3. Here, the source noise $x(n)$ is first sensed, using a reference microphone close to the source noise, before it pass downstream in the duct. The source noise can be expressed as:

$$x(n) = \sum_{i=1}^{q} \{a_i \cos(\omega_{p,i} n) + b_i \sin(\omega_{p,i} n)\} + v_s(n) \quad \text{where} \quad \{a_i, b_i\}_{i=1}^{q} \text{ are the amplitudes of sin- and cos- part of the sinusoids in the source noise, } v_s(n) \text{ is a zero mean white noise with variance } \sigma_v^2.$$  \hspace{1cm} (13)

Second, the spectrum of the source noise is given by:

$$Q_\omega(\omega) = \frac{1}{1 - \sum_{i=1}^{L} w_i \exp(-j\omega i)}. \quad \text{ (16)}$$

A sinusoid of frequency $\omega^*$ in the primary noise will result in an extremely sharp peak in the spectrum plot, for that $Q_\omega(\omega^*) \rightarrow \infty$ [9]. Such the sinusoids of the source noise can be discerned by finding the peaks of $Q_\omega(\omega^*)$.

Obviously, numerous computations have to be made to discern all the sinusoids from the spectrum estimation function $Q_\omega(\omega)$. But the computational burden can be elevated by the premise that FM is no more than 10%, so that we are granted to search the peaks of the spectrum within the vicinity of the frequencies obtained from the nonacoustic sensor, rather than the entire frequency range. The burden can be alleviated even further if we find the zeros of the denominator of the spectrum computation $Q_\omega(\omega)$ by golden search process.

For any specific frequency $\omega_{p,i}$, on average, $10(L+1)$ multiplications and 20 adds are needed to find an satisfying value $\omega^*_{p,i}$, which will guarantee that $|\omega_{p,i} - \omega^*_{p,i}|/\omega_{p,i} \times 100\%$ is less than 0.1% that NANC system are able to suppress the primary noise effectively.
To demonstrate the effectiveness of the novel narrowband ANC system, numerous simulations have been carried out. For three different cases the FM is set to be 1%, 5% and 10%, respectively. In these cases, the primary noise is the sum of three sinusoids with normalized frequencies of 0.1\pi, 0.2\pi, and 0.3\pi. The amplitude coefficients of the primary noise are \( a_{p,1} = 3, b_{p,1} = 1, a_{p,2} = 2, b_{p,2} = 1, a_{p,3} = 1, b_{p,3} = 1 \).

The step size parameters \( \mu_i \) for main controller of all narrowband ANC system is 0.05. The step sizes \( \nu_{i,j} \) for Xiao’s frequency compensator are 0.0001, 0.0025 and 0.001 for \( i = 1, 2, \) and 3, accordingly. The step size parameter \( \mu_i \) for our adaptive linear prediction filter is 0.015. Length of the adaptive linear prediction filter is 12. These step size parameters are selected out by numerous trying to find the best. The additive noise in the primary noise is a zero mean white noise with \( v_p = 0.33 \), the measurement noise of the acoustic sensor is a zero mean white noise with \( v_s = 0.02 \). The amplitude of the cos- and sin- part of the source noise are \( a_i = 1, b_i = 1 \) for all three frequencies. The secondary path is set up as a low pass filter with cutoff frequency of 0.4\pi and of length 11. An off-line LMS algorithm is employed to estimate the secondary path with length of 25. Figures listed at the end of this paper show performance comparison between Xiao’s method and the proposed method.

It is clearly showed that the proposed system are able to deal with the problem of FM effectively. In fact, the steady state error \( e(n) \) can’t be zero all the time even in steady state, such that the error sensitive coefficient \( c_i(n) \) of Xiao’s method has a hard time to converge to stable value in case of high FM. So that the frequency compensator method can’t provide accurate reference signals to the main controller, making the power of steady state error \( e(n) \) relatively high compared with the proposed method, for the frequencies obtained from spectrum computation is independent of the error signal. It is also shown by the results that the proposed method is un-sensitive to the degree of frequency mismatch, because the spectrum computation is based solely on the source noise.

The amplitude and power of error signal \( x_i(n) \) of the adaptive linear prediction filter in case of FM 10% is depicted in the Fig. 6. The fast convergence of the adaptive linear prediction filter is taken advantage by the frequencies estimation subsystem to provide the reference signal generator with more accurate reference frequencies well before it reaches the steady state, further accelerate convergence of the main controller. Nonetheless, one drawback of the proposed system is that the spectrum computation is sensitive to SNR of the source noise, that the proposed method works well when SNR is high. Also the acoustic feedback effect is neglected in the simulations, and this factors needs to be taken into consideration in the future research.

In this paper, we proposed to integrate the conventional NANC system with a frequency estimation subsystem, which is based on adaptive linear prediction filter. More accurate reference frequencies are obtained by discerning sinusoids reside in the spectrum of the source noise. The estimated frequencies are used to produce reference signals for the main controller, and in this way the problem of FM can be dealt effectively. Numerous simulation has proved the effectiveness of the proposed method.

**REFERENCES**


**Fig. 3.** System performance with FM of 1%  
(a) Xiao’s method  
(b) proposed method
Fig. 4. System performance with FM of 5%.
(a) Xiao’s method  (b) proposed method

Fig. 5. System performance with FM of 10%.
(a) Xiao’s method  (b) proposed method

Fig. 6. Linear prediction error signal in case of 10% FM,
(a) error signal amplitude  (b) power of error signal