Abstract—Parametric loudspeakers have attracted intensive interest due to its application in directional sound reproduction. Recently, Volterra filters have been adopted for compensating the nonlinear distortions in the parametric loudspeaker. The harmonic distortions are effectively reduced at one single control point using a pth-order inverse filter. However, in practical use, a lower total harmonic distortion (THD) level over a larger geometrical area using multiple control points can be more interesting. In this study, an exact linearization method using Volterra filters for two positions based on the multiple-input/multiple output inverse theorem (MINT) is proposed with one parametric loudspeaker and one moving coil loudspeaker. Two inverse filters based on the established Volterra filter models are designed by which input signals are pre-distorted before being fed to those two loudspeakers respectively. To validate this proposed approach, experiments have been conducted. The experimental results show that with these two inverse Volterra filters, THD levels of the parametric loudspeaker over an area defined by at least two points can be reduced.

I. INTRODUCTION

Parametric loudspeakers can generate audible sounds with high directivity, through the nonlinear interaction of two primary high-frequency sound waves in the air. They have attracted intensive interest since almost half century ago [1]. A great deal of achievements have been made in theory and practice [2, 3]. Yoneyama et al. [4] first modulated ultrasonic signals with audio signals using double sided band amplitude modulation (DSBAM) and fed to parametric loudspeakers to produce audio sounds through the demodulation process provided by the nonlinearity of the air, which however, also generates harmonic distortions degrading the sound quality of the desired sounds. To eliminate those undesired harmonic distortions, several improved modulation methods including square-root amplitude modulation (SRAM) [5], single sideband amplitude modulation (SSBAM) [6], and modified amplitude modulation (MAM)[7], etc. have been proposed according to the Berktay’s far-field solution [8]. And yet, those Berkty-based pre-processing methods mentioned above cannot reduce the harmonic distortions effectively when the levels of the primary waves are higher than a certain level due to the limitation of the Berkty’s far-field solution [9]. Therefore, a better approach is desired to reduce the nonlinear distortions.

Recently, a Volterra filter has been introduced to model the nonlinearity of parametric loudspeakers [10, 11]. The nonlinearity can be modeled with accuracy using a low-order, truncated Volterra filter. The coefficients of the Volterra filter were obtained from the Burger’s equation-based numerical simulations. Furthermore, a more accurate result of the Volterra filter model derived from the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation was introduced [12]. Based on the Volterra filter model, a 2nd-order inverse has been designed to reduce the 2nd harmonic distortion [13]. Moreover, a 3rd-order inverse filter has been proposed to compensate for the 2nd and 3rd harmonic distortion in the demodulated signal [14].

Though a Volterra filter-based inverse shows good performance on compensating for the nonlinearity of parametric loudspeakers, the distortion reduction is mainly concentrated at the control point. That is due to the fact that the nonlinearity of the parametric loudspeaker varies along the distance as illustrated in [12]. The Volterra filter model at one position is not able to model the nonlinearity at other positions. Hence, the Volterra filter-based inverse cannot effectively eliminate the distortion at the other positions. Consider a parametric loudspeaker in an exhibition hall illustrated in Fig. 1. Visitors of all heights are expected to get audible signals with low distortion. That means linearization of the parametric loudspeaker over a larger area is necessary. To meet this requirement, more control points covering a larger area have to be considered, and the multiple-input/multiple output inverse theorem (MINT) [15], which has also been adopted in linearization of multiple loudspeaker systems in [16,17], is introduced in this paper.

The rest of this paper is organized as follows. The review
of the conventional inverse filter is presented in Section II. The proposed inverse using the MINT methods is derived in Section III. Experiments to evaluate the performance of the proposed method are given in Section IV. Finally, the conclusion and future work are discussed in Section V.

II. REVIEW OF CONVENTIONAL INVERSE-FILTERING

The block diagram of the conventional Volterra filter-based compensation method in a single-input single-output parametric loudspeaker is shown in Fig. 2. $H_1$ and $H_2$ are linear and nonlinear operations of the 2nd-order Volterra model for the nonlinear system. Here, a 2nd-order model is considered since the 2nd harmonic is the dominant distortion in the experimental measurements. Moreover, discussions in this study can be extended to the cases of the system with higher order distortion. An inverse filter is constructed to eliminate the 2nd harmonic distortion according to the model kernels. $H_1^{-1}$ is the inverse of the linear response of the nonlinear system. It would appear that the output of the nonlinear system in cascaded with the inverse filter is

$$Y = H_1 \cdot (X − H_1^{-1} \cdot H_1[X]) + H_2[X − H_1^{-1} \cdot H_1[X]],$$

where $R$ is the extra higher order distortion. This equation indicates that the inverse filter can effectively reduce the 2nd harmonic distortion of the nonlinear system.

However, as demonstrated by the simulation and experimental results in [18], the linear and harmonic responses of the difference frequency wave vary with distance from the parametric loudspeaker source. That means, the Volterra filter model estimated at one observation point is not able to model the nonlinearity of the system at the other point. Assuming the linear and nonlinear responses at one position is modeled with a 2nd-order Volterra filter as $H_1 + H_2$, the responses at the other position (position B as shown in Fig. 3) is $(H_1 + \Delta H_1) + (H_2 + \Delta H_2)$, where $\Delta H_1$ and $\Delta H_2$ are the model errors of the linear and 2nd harmonic responses, respectively, between the models at two positions. Hence, the output of the system at position B compensated with the inverse filter, which is designed based on the model kernels at position A is

$$Y_2 = H_1 \cdot Q + \Delta H_1 \cdot Q + H_2[Q + \Delta H_2[Q] + R.$$

where

$$Q = X − H_1^{-1} \cdot H_1[X]$$

$$H'_1[X] = \Delta H_1[X] − \Delta H_2 \cdot H_2[X]$$

$H'_1[.]$ is the linear response, $H'_2[.]$ is the 2nd harmonic, and $R$ is the 3rd or higher-order harmonics after compensation at position B. It can been seen from (2) that the inverse filter cannot eliminate the 2nd harmonic distortion at position B as large as the case for position A.

III. PRINCIPLE OF PROPOSED MINT INVERSE-FILTERING

To reduce the nonlinearities at two or more positions, the inverse filter should be redesigned. In this paper linearization at two positions have been considered, and linearization at more positions can be derived similarly. Consider a one-input two-output system shown in Fig. 4. The kernels $Q_1$ and $Q_2$ of the 2nd-order inverse to eliminate the 2nd harmonic distortion should be designed such that the outputs $Y_1$ and $Y_2$ at two positions satisfying

$$H_{ai} \cdot Q_1 + H_{ai} \cdot Q_2 + H_{ai}[\cdot \cdot] = H_{ai} \cdot X,$$

$$H_{ai} \cdot Q_1 + H_{ai} \cdot Q_2 + H_{ai}[\cdot \cdot] = H_{ai} \cdot X.$$

When $Q_i$ is set equal to $X$, (4) can be expressed in matrix form as

$$\begin{bmatrix}
H_{ai} \\
H_{ai}
\end{bmatrix}Q_2 = \begin{bmatrix}
H_{ai}[X] \\
H_{ai}[X]
\end{bmatrix}.$$

Here, there is no exact solution for (5) considering the number of the columns is less than that of the rows. Accordingly, only approximate inverse of the system can be obtained through LMS method.

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Consider a two-input two-output nonlinear system shown in Fig. 5. This system can be obtained by adding an extra signal-transmission channels of this system in Fig. 4. The nonlinearity at two positions are combined results from both signal-transmission channels of this system in Fig. 4. The inverse filters have been designed according to (8).

If the kernels \( Q_{y1} \) and \( Q_{y2} \) are set as \( X \), the kernels \( Q_{M1} \) and \( Q_{M2} \) can be derived from the following equation without considering the harmonic components higher than 2nd order

\[
\begin{align*}
H_{MB1} \cdot Q_{M1} + H_{MB2} \cdot Q_{M2} + H_{MB1} \cdot Q_{M1} + H_{MB2} \cdot Q_{M2} + H_{MB1} \cdot Q_{M1} + H_{MB2} \cdot Q_{M2} + H_{NB1} \cdot Q_{N1} + H_{NB2} \cdot Q_{N2} &= H_{MB1} \cdot X + H_{MB2} \cdot X \\
H_{MB1} \cdot Q_{M1} + H_{NB1} \cdot Q_{N1} + H_{NB2} \cdot Q_{N2} &= H_{MB1} \cdot X + H_{NB1} \cdot X.
\end{align*}
\]

(6a)

(6b)

Hence, (7) can be simplified as

\[
\begin{align*}
\begin{bmatrix} H_{MB1} & H_{MB2} \\ H_{NB1} & H_{NB2} \end{bmatrix} \begin{bmatrix} Q_{M1} \\ Q_{M2} \end{bmatrix} &= -\begin{bmatrix} H_{MB1} \cdot X + H_{MB2} \cdot X \\ H_{MB1} \cdot X + H_{NB1} \cdot X \end{bmatrix}.
\end{align*}
\]

(7)

Actually, this adding signal transmission channel \( N \) in Fig. 5 can be chosen as a linear system, and effects of the nonlinear operations \( H_{MB2} \) and \( H_{NB2} \) are insignificant. Hence,

(8)

IV. EXPERIMENTS

To evaluate the performance of the above proposed method, experiments have been conducted in a normal room with dimensions of 6 m long, 4 m wide and 3.5 m high. The ambient temperature and relative humidity are 20°C and 40%, respectively. In the experiments, the test signal is generated and modulated with the SSBAM method with a measurement system NI-PXIe1075. The SSBAM method (as shown in Fig. 6) is experimented since it yields the lowest THD level among all the improved amplitude modulation methods [9]. The carrier frequency is 40 kHz. The modulated signal is then fed to the ultrasonic emitter array through a high performance Class-D power amplifier with uniform gain and low phase shift within its operating frequency band 0-100 kHz. The ultrasonic emitter array consists of 147 piezoelectric transducers in an almost circular area as shown in Fig. 7. Synchronized with the output of the modulated signal, the signals are also reproduced by a moving coil loudspeaker. The audible sound from the parametric loudspeaker and the moving coil loudspeaker is captured by a B&K type 4189 microphone at a distance of 1.2 m and 2.3 m, respectively, from the emitter array. The signal flow of the measurement is shown in Fig. 8, and the experimental setup is given in Fig. 9. The sound pressure level at the carrier frequency at 2.3 m is 132 dB (measured with the GRAS 46BE microphone). The microphones and the emitter are 1.5 m above the ground level. Microphones are covered with acoustic filters with pass band 0–8 kHz to attenuate the primary waves [19], and the frequency response of the acoustic filter to the audio signal has been compensated before the system identification.

The moving coil loudspeaker is 1.4 m above the ground. The transfer functions of the parametric loudspeaker and the moving coil loudspeaker have been characterized with Volterra filter models. The kernels of the Volterra filter models were estimated using the NLMS adaptive algorithm through the inputs of the speakers and outputs at different positions.

The conventional inverse has been designed (as shown in Fig. 3) firstly. The inverse kernels are based on the Volterra filter model of the parametric loudspeaker at the observation point 2.3 m. A signal of single frequency 1 kHz has been used as the input. The spectra of the output of the nonlinear system at 2.3 m before and after compensation using the conventional inverse are shown in Fig. 10 and Fig. 11, respectively. It can be found from the figure that the inverse filter can effectively reduce the 2nd harmonic distortion by more than 20 dB. The spectra of the output at position 1.2 m before and after compensation using the conventional inverse are shown in Fig. 12 and Fig. 13. It is found that the 2nd harmonic distortion of the system at this position has not been reduced by the inverse filter as the case at the position 2.3 m. This result complies with the theoretical analysis above.

Hence, an extra loudspeaker employed is also illustrated in Fig. 5. The inverse filters have been designed according to (8). The spectra of the output of the nonlinear system after compensated using the proposed method at 2.3 m are given in Fig. 14. It can be seen from the figure that the proposed method can reduce the 2nd harmonic distortion of the nonlinear system at 2.3 m as the conventional inverse can do. Furthermore, Fig. 15 gives the spectra of the output of the overall system at 1.2 m after compensation using the proposed method. It can be seen that the proposed method can eliminate the 2nd harmonic distortion by about 15 dB at this position.
The performances of the conventional inverse and the proposed method at other positions have also been evaluated. The 2nd harmonic distortion reduction given by the conventional inverse are shown by the solid line in Fig. 16. The harmonic distortion reduction at the control point 2.3 m is as high as 20 dB. As the distance changes, the reduction reduces. That is due to the fact that the nonlinear model established at the point 2.3 m cannot well model the nonlinear characters at the other points since the nonlinearity of the parametric loudspeaker differs with the distances. Hence, the inverse filter based on the model kernels would not effectively reduce the harmonic distortion at other points. It is also to be noted that the distortion reduction at the far field remains almost at the level of 10 dB, whereas, the reduction in the near field drops off rapidly with distance away from the controlled point at 2.3 m. Especially within the distance 1.5 m, the reductions are negative values, which means the distortion is increased by the inverse filter. That is because the nonlinear interaction of the parametric loudspeaker is mainly in the near field, whereas, the parametric loudspeaker system acts as an ordinary speaker whose responses is attenuated linearly with distances in the far field. The 2nd harmonic distortion reduction given by the proposed method is shown as the dashed line in Fig. 16. With the proposed method, the reductions at these two control points (1.2 m and 2.3 m) are higher than 15 dB. Besides, the reductions at the positions between these two points (from 1.2 m to 2.3 m, as marked by box) are mostly larger than 10 dB. That means, low harmonic distortion level over a larger geometrical area is realized.

V. CONCLUSIONS

The conventional Volterra filter-based inverse of the parametric loudspeaker system can only reduce the harmonic distortion at the control point. To enlarge the affected area of the linearization method, one more transmission channel has been employed preliminarily to realize a multiple input multiple output system, and multiple-input/multiple output inverse theorem (MINT) has been introduced to compensate simultaneously for the nonlinearity at multiple positions. The experiments have been conducted to evaluate the performance of the proposed MINT method. Experimental results reveal that compared with the conventional inverse design, the MINT method can effectively reduce the 2nd harmonic distortion at two positions and enlarge the area of low harmonic distortion. This is proved to be of practical use for the parametric loudspeaker in sound reproduction applications. Actually, the directivity of the moving coil loudspeaker is not as good as that of the parametric loudspeaker. Multiple parametric loudspeaker systems using MINT is a possible alternative solution for the future study but consideration on the nonlinear interactions between them is indispensable.

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Fig. 10 Spectra of the output of the parametric loudspeaker at 2.3 m

Fig. 11 Spectra of the output of the parametric loudspeaker after compensation using the conventional inverse at 1.2 m

Fig. 12 Spectra of the output of the parametric loudspeaker at 1.2 m

Fig. 13 Spectra of the output of the parametric loudspeaker after compensation using the conventional inverse at 1.2 m

Fig. 14 Spectra of the output of the parametric loudspeaker after compensation using the proposes method at 2.3 m

Fig. 15 Spectra of the output of the parametric loudspeaker after compensation using the proposes method at 1.2 m
Fig. 16 2nd harmonic distortion of the parametric loudspeaker along the distance using the conventional inverse and the proposed method.

REFERENCES