Two Layer Coding of HDR Images with Noise Bias Compensation

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Abstract—This paper introduces a noise bias compensation (NBC) to a two layer backward compatible high dynamic range (HDR) image coding to decrease data volume of the compressed data. In this system, dynamic range of the input HDR image is reduced with tone mapping (TM) to generate a low dynamic range (LDR) image. It is encoded to generate a bit stream in the base layer. In the enhancement layer, quantization noise added by lossy coding of the LDR image is amplified by the inverse of TM. It brings about much bit rate in this layer. To cope with this problem, this paper introduces NBC to reduce variance of the noise. It compensates noise bias (NB) according to an observed pixel value. In this paper, NB is defined as the mean of the noise in the same observed pixel value from different original values. It is experimentally observed that the proposed method significantly reduces data volume in the enhancement layer at high quality lossy coding of LDR images.

I. INTRODUCTION

Tone mapping (TM) plays an important role in adjusting image quality. It is also necessary to design a suitable mapping function to convert a high dynamic range (HDR) image to a low dynamic range (LDR) image [1-4]. In a two layer backward compatible HDR image coding system, two kinds of bit streams are produced [5-8]. One is for decoding the LDR image in the base layer, and the other is for the HDR image in the enhancement layer. In the latter layer, it is required to reduce the quantization noise added by lossy coding of the LDR image. However the noise is amplified by the inverse of TM. As a result, the data volume of the bit stream in the enhancement layer is increased.

This paper aims at decreasing the bit rate in the enhancement layer introducing the noise bias compensation (NBC). Note that the noise bias (NB) referred to in this paper does not mean the average of the noise over all pixels in an image. It is defined as the mean over pixel values in a subset in which all elements share the same observed pixel value. Most of the literatures assume zero mean of the noise in designing a filter [9]. So far, numerous attempts have been done in de-noising such as the classical regression filters [10,11]. The photometric distance has been taken into account in the bilateral filters [12,13]. The non-local mean (NLM) filter replaces pixel-wise calculation of the distance with patch-wise one [14-16]. However, little attention has been paid to the NB.

Unlike these previous reports, this paper discusses on the non-zero value of NB. Recently, an idea of noise bias compensation (NBC) was reported to improve quality of a tone mapped noisy image [17]. It makes NB approximately zero by subtracting estimation of NB from the observed pixel value. This paper tries to reduce the data volume of the bit stream of the enhancement layer introducing NBC to the two layer backward compatible HDR image coding. The NBC in this paper is extended from [17] so that it is especially adjusted for the two layer coding [7,8].

II. PROBLEM SETTING

A. Two Layer HDR Image Coding

Figure 1 illustrates the backward compatible two layer HDR image coding [7,8]. The input HDR image \( y_0 \) is tone mapped with a function \( f \) to generate an LDR image \( x_0 \). It is compressed with a lossy encoder to generate a bit stream \( b_{11} \) in the base layer. In decoder, the LDR image \( x_1 \) is reconstructed. The encoder also generates another bit stream \( b_{12} \) in the enhancement layer. Note that this system decodes the HDR image without any loss (lossless). This paper aims at reducing the data volume of \( b_{12} \) introducing NBC to the system.

Procedure inside the system is detailed as below. A pixel value \( y_0 \) of the input HDR image is tone mapped to a pixel value \( x_0 \) of the LDR image as

\[
x_0 = T[y_0] = R_x[f(y_0)], \quad y_0 \in [0,M_y]
\]

Fig.1 Noise Bias Compensation (NBC) is introduced to the two layer backward compatible HDR image coding.
where $R(\cdot)$ denotes rounding to integer and clipping to the range of $[0,M]$. As an example, this paper uses the Hill function defined as

$$f(y_0) = M_x \cdot \left[ 1 + \left( \frac{y_0 - y_0^0}{b \cdot y_0^0} \right)^{\frac{1}{g}} \right]^{-1} \tag{2}$$

where $y_0^0$ denotes the geometric mean of pixel values. The tone mapped image is encoded with a lossy encoder and decoded in the base layer. The decoded pixel value $x_1$ contains the quantization noise added in this coding process. The quantization contributes to compressing the data volume of the base layer. However, it increases the bit rate (the data volume). This paper applies NBC to generate a better approximation of $y_2$ to decrease its data volume.

### B. Model of the Noise Bias Compensation (NBC)

Figure 2 illustrates a model of NBC for analysis. A pixel value $y_0$ is tone mapped to $x_0$ with a function $T$ as described in (1). In the lossy coding process, a scalar $x_0$ is mapped to a random variable $x_1$ by adding the input noise $\epsilon_1$ as

$$x_1 = x_0 + \epsilon_1. \tag{3}$$

It is inversely tone mapped as

$$y_1 = T^{-1}(x_1) = y_0 + \delta_1 \tag{4}$$

where $\delta_1$ is the residual to be reduced and $T^{-1}(\cdot)$ denotes the inverse of the tone mapping in (1). Note that $\epsilon_1$ and $\delta_1$ are supposed to be random variables. This paper applies NBC as

$$y_2 = y_1 - h(y_1) = y_0 + \delta_2 \tag{5}$$

with a newly introduced function $h$. It aims at reducing the variance of $\delta_2$ defined as

$$Var[\delta_2] = \int P(\delta_2) \left( \delta_2 - \overline{\delta_2} \right)^2 d\delta_2 \tag{6}$$

for

$$\overline{\delta_2} = \int P(\delta_2) \delta_2 d\delta_2 \tag{7}$$

where $P(\delta_2)$ denotes the probability density function (PDF) of the output noise $\delta_2$.

Fig. 2 A model of the ‘Noise Bias Compensation’ (NBC).

### C. Noise Bias (NB) after Tone Mapping (TM)

Figure 3(a) illustrates an example of TM. Parameters in (2) are set to $M_x=255, a=b=1$ and $y_0^0=10$, respectively. Fig. 3(b) illustrates the joint PDF $P(x_0, y_1)$ of $x_0$ and $y_1$ in log scale. As an example, PDF of the input noise $\epsilon_1$ is given as

$$P(\epsilon_1 | x_0) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{\epsilon_1^2}{2\sigma^2} \right), & x_0 \in [\epsilon_1, 255 - \epsilon_1] \\ 0, & x_0 \notin [\epsilon_1, 255 - \epsilon_1] \end{cases} \tag{8}$$

with the variance $\sigma^2=4^2$. This is one of the prior information to be used in NBC. An observed or an appropriately modelled PDF is used in practice. Fig. 4(a) illustrates PDF of $y_1$ at $x_0=225$. The mean of $x_1$ is 225.01. Fig. 4(b) illustrates PDF of $y_1$ at $y_0=T^{-1}(225)=75$. The mean of $y_1$ is 76.58. This means that NB ‘after’ TM is non-zero even though NB ‘before’ TM is almost zero. This is due to non-linearity of TM. In NBC, PDF of the input pixel values $x_0$ and that of the input noise $\epsilon_1$ (or their approximations) are used as ‘prior information’ to design the function $h$ in (5).

Fig. 3 An example of TM. (a) Input value $x_0$ is tone mapped to $y_0$. (b) Log-scaled PDF of $x_0, y_1$. Input value $x_0$ is mapped to a random variable $y_1$.

Fig. 4 Probability density function (PDF) of random variables $x_1$ and $y_1$ at $(x_0, y_0)=(225, 75)$. NB after TM is 76.58 -75 =+1.58.

### III. PROPOSED METHOD

#### A. Definition of Noise Bias (NB)

Definition of NB in this paper is not the same as the mean $\delta_1$ in (7). It is defined as

$$\overline{\delta_1}(y_1) = \int P(x_0, y_1) \delta_1 dx_0 \tag{9}$$
where \( P(x_0,y_1) \) denotes the joint probability density function of \( x_0 \) and \( y_1 \). It means the average of the noise \( \delta_1 \) in an observed pixel value \( y_1 \) at different locations from different original values \( x_0 \). Note that \( \delta_1 \) and \( y_1 \) are considered to be random variables. Substituting

\[
P(x_0,y_1) = P(x_0|y_1)P(y_1)
\]

into (9), NB becomes

\[
\bar{\delta}_1(y_1) = \int P(x_0|y_1)\delta_1dx_0
\]

which means the conditional mean of the output noise \( \delta_1 \) for the same observed value \( y_1 \).

\section{NBC in the Proposed Method}

NBC tries to make NB in (11) zero for each of the observed value \( y_1 \) by the compensation in (5) so that the variance in (6) becomes small [17]. In the proposed method, NB is compensated with

\[
h(y_1) = \int P(x_0|y_1)(y_1 - T^{-1}(x_0))dx_0
\]

which is identical to the estimate in (11). Substituting

\[
P(y_1) = \int P(x_0,y_1)dx_0
\]

and (10) into (12), the compensation function is derived as

\[
h(y_1) = \frac{\int P(x_0,y_1)(y_1 - T^{-1}(x_0))dx_0}{\int P(x_0,y_1)dx_0}.
\]

It means the centroid of the output noise \( \delta_1 \) in (4). In this calculation, the prior knowledge \( P(x_0,y_1) \) in Fig. 3(b) is used as weighting of the output noise.

\section{Modelling of PDF}

According to the Bayesian inference, the compensation in (14) is expressed as

\[
h(y_1) = \frac{\int P(y_1|x_0)P(x_0)(y_1 - T^{-1}(x_0))dx_0}{\int P(y_1|x_0)P(x_0)dx_0}.
\]

Since \( P(y_1|x_0) \) is identical to \( P(\delta_1|x_0) \), when

\[
P(\delta_1|x_0) = P(\varepsilon_1|x_0)\left(\frac{df}{dx}\right)^{-1}
\]

holds, the estimate in (15) can be calculated with \( P(x_0) \) and \( P(\varepsilon_1|x_0) \). Namely, the proposed method estimates NB with PDF of the image pixel value \( x_0 \) and that of the noise \( \varepsilon_1 \) under a given TM function \( f \). Note that a mapping table from an integer \( y \) to an integer \( h(y) \) is required to be included into the bit-stream as a side information in implementation.

\section{EXPERIMENTAL RESULTS}

\subsection{Conditions}

Effect of NBC on reducing data volume of the bit stream in the enhancement layer is investigated. Scaled integer pixel values \( y_0 \) of the original pixel values \( w_0 \) in OpenEXR or RGBE images are tested. The scaling was performed as

\[
y_0 = \begin{cases} 
M_y \cdot (w_0 - Mn) & \text{if } w_0 \in [Mn,Mx] \\
Mx - Mn & \text{if } w_0 < Mn \\
0 & \text{if } w_0 > Mx 
\end{cases}
\]

where

\[
Mn = \min\{w_0 | w_0 > 0\} \quad Mx = \min\{w_0 | F(w_0) \geq 0.999\}
\]

and

\[
F(w_0) = \int_{-\infty}^{w_0} P(w)dw.
\]

In the equations above, \( P(w_0) \) denotes PDF of \( w_0 \) and \( F(w_0) \) denotes the cumulative distribution function, respectively. In (17), pixel values are rounded to \( \log_2 M_r \) bit depth integers with a rounding operator [ ] . In the following experiments, the bit stream in the base layer was generated with the JPEG encoder. The dynamic range was set to \( (M_r,M_r) = (255,1023) \).

\subsection{Evaluation of the System for Images}

Figure 5(a) indicates how the variance (measured with PSNR) of the residual in the enhancement layer is reduced by NBC. The horizontal axis indicates the variance (measured with PSNR) of the decoded image in the base layer. It was observed that NBC decreases the variance of the residual at low PSNR of decoded images (at low bit rate coding) of the base layer. The variance to be encoded in the enhancement layer is reduced by NBC decreases the variance of the residual at low PSNR of decoded images (at low bit rate coding) of the base layer. It was observed that NBC decreases the variance of the residual at low PSNR of decoded images (at low bit rate coding) of the base layer. The residual to be encoded in the enhancement layer of the existing method and the proposed method is indicated in Figure 6(c) and 6(d), respectively. The residual to be encoded in the enhancement layer of the existing method and the proposed method is indicated in Figure 6(c) and 6(d), respectively. The variance of \( \delta_2 \) of the proposed method became smaller than that of \( \delta_1 \) of the existing method.

Figure 7 indicates PSNR of the residual in the enhancement layer for various input images at different level of the quantization step size in lossy coding of the base layer. The PSNR of the decoded images in the base layer is 43 (dB) and 30 (dB) in average in Figure 7(a) and 7(b), respectively. It was confirmed that NBC contributes to reduce the variance of the residual in the enhancement layer.
Fig. 5 The variance and the data volume of the enhancement layer.

Fig. 6 Example of processed images for ‘Tree’.

Fig. 7 The variance (in log scale) of the enhancement layer.

V. Conclusions

A simple NBC method for the two layer HDR coding was proposed. It was confirmed that NBC has positive effect on bit rate saving especially at low bit rate lossy coding of LDR images. Since the experiments is limited to lossless coding of HDR images, it should be extended to lossy coding of HDR images. Since the rounding errors are neglected, analysis on effect of the rounding [18] should be also considered in the near future.

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REFERENCES