Noise Bias Compensation based on Bayesian Inference for Tone Mapped Noisy Image

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***Abstract*— This paper introduces a noise bias compensation to a tone mapped noisy image so that the variance of the noise is reduced. Although the noise bias is assumed to be zero before tone mapping (TM), it becomes non-zero value after TM. The reason includes some factors such as the non-linearity of TM and the asymmetry of the probability density function of the noise. In this paper, pixels in the noisy image are classified into several subsets according to the observed pixel value, and compensates the pixel value in each subset with a preliminary determined compensation value (CV). In this paper, CV is determined from the histogram of pixel values in the image and that of the noise before TM or their modeled versions based on the Bayesian inference deterministically. As a result of experiments, it is observed that the peak-signal to noise ratio is improved by the proposed method.**

I. INTRODUCTION

Although a large number of studies have been made on de-noising, what seems to be lacking, however, is effect of the noise bias (mean of noise) and its compensation. In regression filters, a convolution kernel was determined based on the spatial distance between pixels [1,2]. Those were extended to bilateral filters introducing the photometric distance [3,4]. However, little attention has been given to the noise bias (NB).

Recently, the concern with non-local mean (NLM) filters has been growing [5-10]. This class of filters replaces the pixel-wise calculation of the distance with the patch-wise one [7,8]. In such situations, it deserves careful attention to NB because NB builds up when noisy pixel values are weighted and added repeatedly. However, most of previous reports assumed NB to be zero in designing filters [9,10].

Unlike these previous reports, this paper deals with the case where NB becomes non-zero value. Even though NB is initially given as zero, it becomes non-zero value after applying a tone mapping (TM) function to noisy pixel values. This is due to the non-linearity of TM such as the power function, the logarithmic function and the Hill function, etc. [11-13]. The asymmetry of the probability density function of the noise after TM can be also conceivable.

In this paper, pixels in the noisy image are classified into several subsets according to the observed pixel value, and compensates the pixel value in each subset with a preliminary determined compensation value (CV). This is the noise bias compensation (NBC). Although a primitive idea was reported in [14], is contains a hyper-parameter to be determined in a heuristic way. Extending the idea, CV is determined from the histogram of pixel values in the image and that of the noise before TM based on the Bayesian inference theory. Some experiments show that the variance of the noise is reduced by the proposed method alone and also in combination with the non-local mean (NLM) filter [10].

II. PROBLEM SETTING

A. Noise Bias Compensation

Figure 1 illustrates the noise bias compensation (NBC) introduced in this paper. For an input image expressed as

$$x_0(\mathbf{n}) = x_0(n_1, n_2) \tag{1}$$

where $x_0(\mathbf{n})$ denotes a pixel at location $\mathbf{n}=[n_1,n_2]$, a pixel value x_0 is tone mapped with a function f as

$$y_0 = f(x_0), \quad x_0 \in [0, T_x], \quad y_0 \in [0, T_y]\,. \tag{2}$$

Figure 1(a) illustrates this ideal case. This paper investigates a practical case where a noise

$$\varepsilon_1(\mathbf{n}) = \varepsilon_1(n_1, n_2) \tag{3}$$

is added to the input image $x_0(\mathbf{n})$ and then tone mapped. It is described as

$$y_1 = f(x_1) = f(x_0 + \varepsilon_1) = y_0 + \delta_1 \tag{4}$$

as illustrated in Fig. 1(b). The question is how to statistically recover the ideal output y_0 from y_1. Note that ε_1 and δ_1 are considered to be random variables.

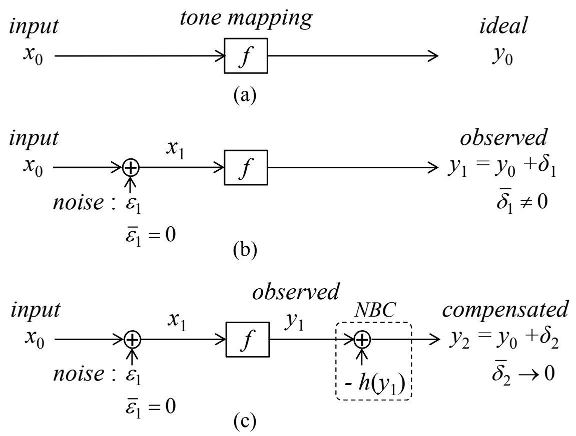


Fig.1 Noise bias compensation (NBC) is introduced to reduce the variance of the output noise δ_2.

Figure 1(c) illustrates the NBC. Although each value of the input noise ε_1 is unknown, it is assumed that its probability density function (PDF) or its modelled version is given beforehand. It also requires PDF of x_0 or its modelled version. Using these 'prier information', NBC tries to recover the ideal tone mapped value y_0 from an observed value y_1. Figure 1(c) illustrates NBC which subtracts a compensation value (CV) from a pixel value as

$$y_2 = y_1 - h(y_1) = y_0 + \delta_2 \,. \tag{5}$$

The compensation function h maps an observed value y_1 to CV. It is designed so that the variance of δ_2 is reduced.

B. Noise Bias after Tone Mapping

Figure 2(a) illustrates an example of TM. In this case, the function f is set to

$$f(x) = \begin{cases} T_y \cdot (T_x^{-1} \cdot x)^{1/\gamma} & for \quad x \in [0, T_x] \\ 0 & for \quad x < 0 \\ T_y & for \quad x > T_x \end{cases} \tag{6}$$

where T_x=T_y=255 and γ=3. Figure 2(b) illustrates the joint PDF $P(x_0, y_1)$ of x_0 and y_1. In this example, PDF of the noise ε_1 on x_0 is given as the Gaussian function

$$P(\varepsilon_1 \,|\, x_0) = \frac{1}{\sqrt{2\pi}\sigma} \exp \frac{-\varepsilon_1{}^2}{2\sigma^2} \tag{7}$$

where σ^2 denotes the variance. Note that the observed PDF or an appropriately modeled PDF such as the general GM in [13] can be used. Although PDF in (7) is symmetric before TM, it becomes asymmetric due to the nonlinearity of TM. Figure.3 illustrates an example. PDF of x_1 at x_0=30 in Fig.3(a) is altered through TM to PDF of y_1 in Fig.3(b). As a result, the bias (the arithmetic mean) of the noise δ_1 becomes non-zero value. In NBC, the bias is compensated to be zero according to an observed value y_1 using CV which is determined from PDF of ε_1 and PDF of x_0. (or their modeled versions)

Note that NB in this paper does not mean the ensemble average of the noise over 'all' pixels. It is defined as

$$E_{\delta_1}[\delta_1(\mathbf{n})] = \frac{1}{\#\mathrm{N}} \sum_{\forall \mathbf{n} \in \mathrm{N}} \delta_1(\mathbf{n}), \quad \mathrm{N} = \{\forall \mathbf{n} \in image\} \tag{8}$$

where #N denotes the total number of pixels which belong to the given image $x_0(\mathbf{n})$. Instead, NB in this paper is defined as the conditional mean

$$E_{\delta_1|y_1}[\delta_1(\mathbf{n})] = \frac{1}{\#\mathrm{M}} \sum_{\forall \mathbf{m} \in \mathrm{M}} \delta_1(\mathbf{m}), \quad \mathrm{M} = \{\mathbf{m} \,|\, y_1(\mathbf{m}) = y_1\} \tag{9}$$

where M denotes a subset of N. All pixels in M has the same pixel value y_1. Using this expression, it is described as

$$\bar{\varepsilon}_1 = E_{\varepsilon_1}[\varepsilon_1(\mathbf{n})] = 0 \quad \overset{TM}{\Rightarrow} \quad \bar{\delta}_1 = E_{\delta_1|y_1}[\delta_1(\mathbf{n})] \neq 0 \tag{10}$$

that NB after TM is not zero even though NB is zero before TM. This paper aims at minimizing the variance

$$Var[\delta_2(\mathbf{n})] \rightarrow \min \tag{11}$$

by NBC

$$\bar{\delta}_2 = E_{\delta_2|y_1}[\delta_2(\mathbf{n})] \rightarrow 0 \,. \tag{12}$$

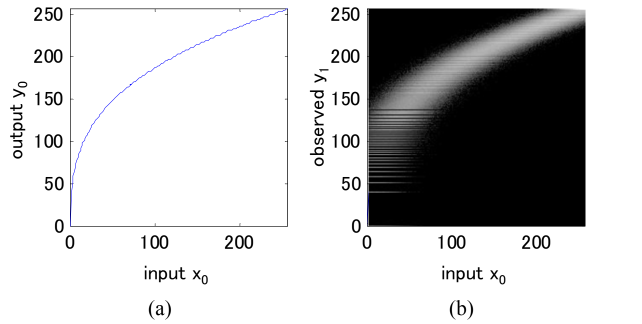


Fig.2 An example of tone mapping. (a) Input value x_0 is tone mapped to y_0. (b) Log-scaled $P(x_0,y_1)$ is illustrated.

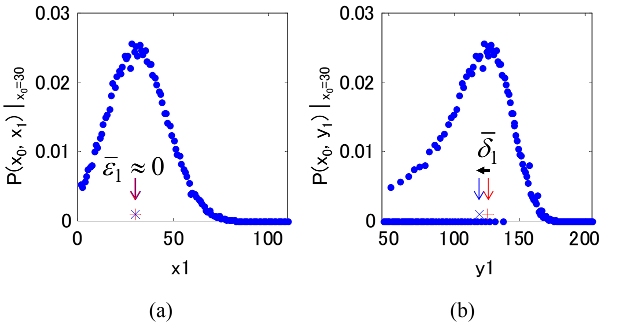


Fig.3 Probability density function (PDF) of random variables x_1 and y_1. (a) $P(x_0, x_1)$ at x_0=30. (b) $P(x_0, y_1)$ at x_0=30.

III. Noise Bias Compensation

A. Previous NBC

In the previous NBC in [14], the compensation function h in (5) is set to

$$h(y_1) = \alpha \cdot \sum_{\forall \varepsilon_1} P(\varepsilon_1 \,|\, x_0) \{ f(x_0 + \varepsilon_1) - f(x_0) \} \tag{13}$$

using PDF of the input noise ε_1 such as (7) for

$$x_0 \leftarrow f^{-1}(y_1) \,. \tag{14}$$

The hyper parameter α in (13) is searched so that the variance of δ_2 in (11) becomes minimum. Namely,

$$\alpha_{opt} = \min_{\alpha} \arg Var[\delta_2(\mathbf{n})] \,. \tag{15}$$

However, it has a problem that the hyper parameter α should be searched lacking a rational as the best solution.

B. Proposed NBC

Using the conditional PDF $P(\delta_1|y_1)$ of δ_1 for given y_1, NB of δ_1 in (10) is expressed as

$$\bar{\delta}_1 = E_{\delta_1|y_1}[\delta_1(\mathbf{n})] = \sum_{\forall \delta_1} P(\delta_1 \mid y_1)\delta_1 . \quad (16)$$

Substituting (4), it becomes

$$\bar{\delta}_1 = \sum_{\forall (y_1 - y_0)} P((y_1 - y_0) \mid y_1)(y_1 - y_0) \quad (17)$$

Unlike (13) in the previous NBC, we calculated it as

$$h(y_1) = \sum_{\forall x_0} P(x_0 \mid y_1)(y_1 - y_0) \quad (18)$$

for

$$y_0 \leftarrow f(x_0) \quad (19)$$

in the proposed NBC. It is not necessary to search the hyper parameter α in (15). According to

$$P(y_1) = \sum_{\forall x_0} P(y_1 \mid x_0)P(x_0) \quad (20)$$

and

$$P(x_0, y_1) = P(y_1 \mid x_0)P(x_0) = P(x_0 \mid y_1)P(y_1) , \quad (21)$$

the function in (18) is identical to

$$h(y_1) = \frac{\sum_{\forall x_0} P(x_0, y_1)\{y_1 - f(x_0)\}}{\sum_{\forall x_0} P(x_0, y_1)} . \quad (22)$$

The compensation function in (22) means the centroid of the noise δ_1. Figure 4 illustrates an example for x_0={0,1,2}. The input value x_0=0 becomes an observed value y_1 at the probability of $P(y_1|\,x_0$=0). Other input values x_0=1 and x_0=2 also become the same observed value y_1 at the probability of $P(y_1|x_0$=1) and $P(y_1|x_0$=2), respectively. Therefore (22) means the weighted summation of the difference between the ideal value y_0 and the observed values y_1. Namely it is the centroid. In this calculation, the prior knowledge $P(x_0,y_1)$ in Fig. 2(b) is used as the weight.

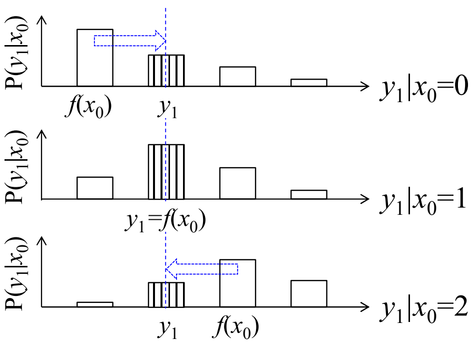


Fig.4 The mean of the output noise δ_1 is estimated as the centroid of the probabilistic relation between x_0 and y_1.

C. Modelling of PDF

According to the Bayesian inference, the compensation in (22) is expressed as

$$h(y_1) = \frac{\sum_{\forall x_0} P(y_1 \mid x_0)P(x_0)(y_1 - f(x_0))}{\sum_{\forall x_0} P(y_1 \mid x_0)P(x_0)} . \quad (23)$$

Since $P(y_1 \mid x_0)$ is essentially identical to $P(\delta_1 \mid x_0)$ and

$$P(\delta_1 \mid x_0)\cdot \Delta y = P(\varepsilon_1 \mid x_0)\cdot \Delta x \quad for \quad \Delta y = f(x_0 + \Delta x) - f(x_0) \quad (24)$$

holds, the compensation function in the proposed NBC can be calculated with $P(x_0)$ and $P(\varepsilon_1|x_0)$. In conclusion, it is clarified that the proposed method estimates NB with the histogram of the image x_0 and that of the noise ε_1 (or their approximations) under a given TM function f.

IV. Experimental Results

A. Quality of Compensated Images

Quality of the compensated image y_2 is evaluated with the peak signal to noise ratio defined as

$$PSNR = 20\log_{10} T_y - 10\log_{10} Var[\delta_2(\mathbf{n})] . \quad (25)$$

Pixel values normalized to [0, T_x] in each image are used as the input image x_0 to utilize full range of TM.

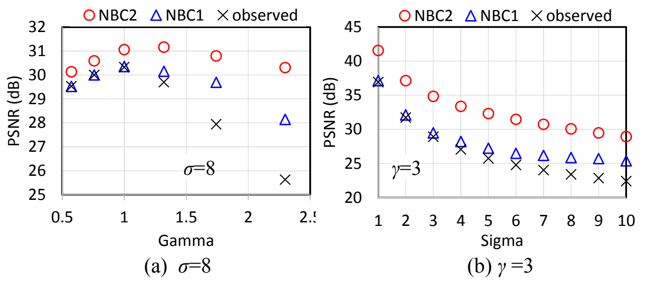
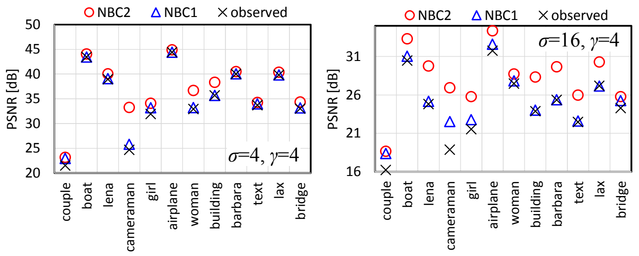


Fig.5 PSNR of the compensated image for ‘Cameraman’ in various TM.

Fig. 5(a) investigates effect of Gamma γ in (6). It is observed that both of the existing NBC (NBC1) and the proposed NBC (NBC2) are effective for high value of γ. At γ=0, NBC1 does not improve PSNR. However, NBC2 improves it from 30.3 to 31.6 (dB). This is because NBC2 utilizes non-uniformness of the histogram of x_0. Fig. 5(b) investigates effect of Sigma σ in (7). It is observed that NBCs are more effective for lower PSNR (heavily degraded) cases.

Fig. 6(a) illustrates PSNR for various input images. For ‘Boat’, ‘Lena’, ‘Lax’ and ‘Barbara’, NBCs have no significant effect at σ=3. However, NBC2 increases PSNR for all tested images at σ=16 as illustrated in Fig. 6(b). The highest improvement was observed for ‘Cameraman’. Fig7 (a)-(d) illustrate images in this case.



(a) σ=4, γ =4 (b) σ=16, γ =4

Fig.6 PSNR of the compensated images for various input images.

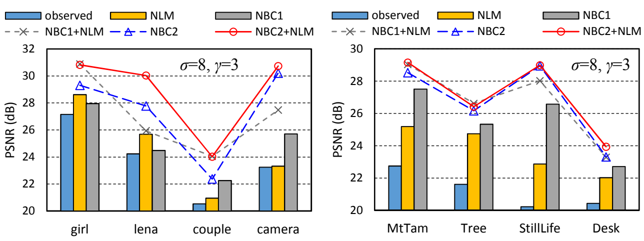
B. Combination with Non Local Mean Filter

Figure 8(a) investigates combination of NBC and a non-local mean (NLM) filter [10] for SIDBA images. For 'Girl' image, PSNR is improved from 27.2 to 28.6 and 28.0 (dB) by NLM and NBC1, respectively as illustrated in bar graphs. In combination with NBC and NLM, it is improved to 30.8 and 30.9 (dB) by NBC1+NLM and NBC2+NLM, respectively. For other images, NBC2+NLM was observed to be the best. Fig. 8(b) summarizes results for OpenEXR images which were normalized to 12 bit depth integer pixel values. In this parameter setting, superiority of NBC2 +NLM, NBC2 and NBC1 +NLM is confirmed. At least, those are better than the existing method NLM or NBC1.

(a) Reference y_0 (b) Observed y_1, 23.25(dB)

(c) y_2 in NBC1, 25.74(dB) (d) y_2 in NBC2, 30.21(dB)

Fig.7 Result of TM and NBC for the input image 'Cameraman'



(a) SIDBA (b) OpenEXR

Fig.8 PSNR of the compensated and filtered images.

V. CONCLUSIONS

NB after TM is compensated using the histogram of the image and that of the noise. Estimation of NB is calculated according to an observed noisy pixel value after TM. Superiority of the proposed method (NBC2 or NBC2 +NLM) over the existing method (NBC1 or NLM without NBC2) was experimentally confirmed for several images. Modelling of the PDF (prior knowledge) and its effect on performance should be investigated in the near future.

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