An ACO Approach for Design of CSD Coefficient FIR Filters

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***Abstract*—In this paper, a design method of FIR (Finite Impulse Response) filters with CSD (Canonic Signed Digit) coefficients using ACO (Ant Colony Optimization) is studied. This design problem is formulated as a combinatorial optimization problem. It requires high computation time to obtain an optimal solution. Instead, several heuristic approaches have been developed for solving this problem in a reasonable computation time. ACO is one of the promising approaches and is appropriate for solving the combinatorial optimization problem. Therefore, we study the design method of CSD coefficient FIR filters using ACO. In the design examples, ACO is compared with the other heuristic approaches. Furthermore, a difference between initial coefficients and optimal coefficients is investigated to reveal differences of algorithm behavior. As a result, it is shown that the ACO approach is effective for the design of CSD coefficient FIR filters.**

I. Introduction

FIR filters are widely used in digital signal processing applications because it can realize a linear phase characteristic and assure a stability. When a high order FIR filter is implemented in a hardware, the circuit scale tends to be large and it causes to increase a power consumption. Thus, many studies are aimed to reduce the circuit scale [1]–[5]. In a hardware implementation, multipliers occupy a majority of the circuit scale of FIR filters. The multiplier is mostly constructed by several shifters. Then, the number of shifters corresponds to the number of nonzero digits involved in each coefficient. Therefore, it is required to reduce the number of nonzero digits of filter coefficients.

CSD representation is one of the effective approaches for reducing the number of nonzero digits [3]. In the CSD representation, an allocation of two adjacent nonzero digits are forbidden and the number of total available nonzero digits is limited in order to reduce the circuit scale. Then, the design problem of CSD coefficient FIR filters can be formulated as a MILP (Mixed Integer Linear Programming) problem [4] which is known as NP hard problem. It can be solved by the branch-and-bound method strictly. Although the optimal solution can be obtained by this method, it requires high computation time.

Instead, several heuristic approaches have been proposed to solve this problem in the reasonable computation time. ACO is one of the approaches applied to a combinatorial optimization problems [6]. It is well-known that ACO is the effective method for solving high dimensional optimization problems [7], [8] and has been also applied to the discrete coefficient filter designs [9], [10]. These design problems are formulated as 0/1 combinatorial problems. By constant, in the CSD coefficient FIR filter design, it falls into 0/1/$\overline{1}$ combinatorial problems and its design problem is more difficult to solve than that of binary coefficient filter because of a complexity of CSD constrains.

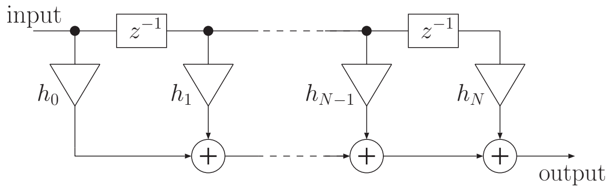

Fig. 1. A structure of N-th order FIR filter

In this paper, we study a design method of CSD coefficient FIR filters using ACO. In the design examples, the ACO is compared with the other heuristic approaches. Furthermore, a difference between initial coefficients and optimal coefficients is investigated to reveal differences of algorithm behavior. As a result, it is shown that the ACO approach is effective for the design of CSD coefficient FIR filters.

II. Design Problem

A. Design of FIR Filters

A structure of N-th order FIR filter is shown in Fig. 1, where h_n, $n = 0, 1, \cdots, N$ are filter coefficients. When N is even number and the impulse response is even symmetric, i.e. $h_n = h_{N-n}$, the magnitude response $H(\omega)$ can be described as,

$$H(\omega) = \sum_{n=0}^{M} a_n \cos n\omega, \tag{1}$$

where $M = N/2$, $a_0 = h_M$, $a_n = 2h_{M-n}$, $n = 1, 2, \cdots, M$ and then the phase characteristic becomes a linear phase characteristic. In a min-max criterion, the design problem is formulated as following,

$$\min_{a_n} \max_{\omega \in \Omega} \left| D(\omega) - \sum_{n=0}^{M} a_n \cos n\omega \right|, \tag{2}$$

where Ω is an approximation frequency band and $D(\omega)$ is a desired magnitude response.

B. Design of CSD Coefficient FIR Filters

In the CSD representation, a_n is expressed as

$$a_n = \sum_{k=0}^{p} x_{n,k} 2^{-k} \ , \ x_{n,k} \in \left\{-1 = \overline{1}, 0, 1\right\}, \tag{3}$$

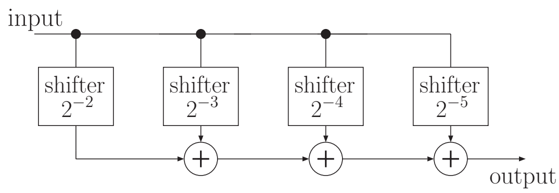

Fig. 2. A structure of multiplier ($a_n = \{0.01111\}_2$)

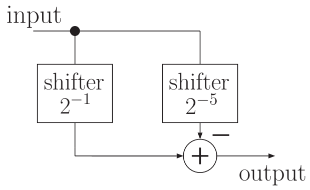

Fig. 3. A structure of multiplier ($a_n = \left\{0.1000\bar{1}\right\}_{\text{CSD}}$)

where p is a word length. The adjacent allocation of two nonzero digits is not be allowed in the CSD representation. For example, the coefficient $a_n = 0.01111$ in a binary representation is represented as $a_n = 0.1000\bar{1}$ in the CSD representation. A structure of the multiplier for the coefficient $a_n = 0.01111$ is shown in Fig. 2 and the coefficient $a_n = 0.1000\bar{1}$ is shown in Fig. 3. From Fig. 2 and Fig. 3, it can be confirmed that the CSD representation can reduce the number of nonzero digits and the circuit scale. The restriction that the allocation of two adjacent nonzero digits is forbidden, can be expressed as

$$|x_{n,k}| + |x_{n,k+1}| \leq 1. \tag{4}$$

Furthermore, the number of total nonzero digits is limited in order to reduce the circuit scale. This restriction is formulated as following,

$$\sum_{n=0}^{M} \sum_{k=0}^{p} |x_{n,k}| \leq \Lambda, \tag{5}$$

where Λ is the maximum number of nonzero digits which is available in total and specified in advance. Therefore, this design problem is formulated as following,

$$\begin{aligned} \min \quad & \delta \\ \text{sub.to} \quad & |D(\omega) - H(\omega)| \leq \delta, \\ & \sum_{n=0}^{M} \sum_{k=0}^{p} |x_{n,k}| \leq \Lambda, \\ & |x_{n,k}| + |x_{n,k+1}| \leq 1, \\ & x_{n,k} \in \left\{\bar{1}, 0, 1\right\}, \end{aligned} \tag{6}$$

where δ is a maximum absolute error between $D(\omega)$ and $H(\omega)$. This problem can be easily transformed into the MILP problem.

III. Design of CSD Coefficient FIR Filters using ACO

A. *Objective Function*

The objective function $F(\boldsymbol{x})$ for ACO is defined by using two penalty functions $\phi_1(\boldsymbol{x})$, $\phi_2(\boldsymbol{x})$ in consideration of the CSD representation as following,

$$F(\boldsymbol{x}) = \delta + \phi_1(\boldsymbol{x}) + \phi_2(\boldsymbol{x}), \tag{7}$$

where

$$\boldsymbol{x} = [x_{0,0}, x_{0,1}, \cdots, x_{N/2,p}]^T. \tag{8}$$

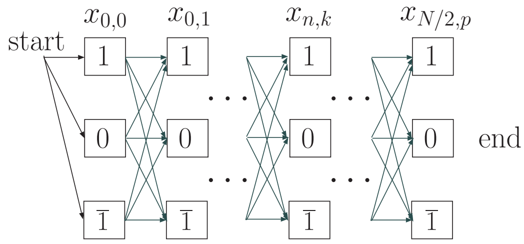

Fig. 4. Search model in design of FIR filters with CSD coefficients

$\phi_1(\boldsymbol{x})$ is the penalty function for limiting the total number of nonzero digits indicated in (4). $\phi_1(\boldsymbol{x})$ is defined using the maximum number of nonzero digits λ as following,

$$\phi_1(\boldsymbol{x}) = \begin{cases} 0, & \lambda \leq \Lambda \\ \lambda - \Lambda, & \text{otherwise} \end{cases}, \tag{9}$$

where

$$\lambda = \sum_{n=0}^{M} \sum_{k=0}^{p} |x_{n,k}|. \tag{10}$$

$\phi_2(\boldsymbol{x})$ is the penalty function for forbidding the allocation of two adjacent nonzero digits indicated in (5). $\phi_2(\boldsymbol{x})$ is defined as

$$\phi_2(\boldsymbol{x}) = \begin{cases} 0, & B_{n,k} \leq 1, \ \forall n,k \\ \displaystyle\sum_{n=0}^{M} \sum_{k=0}^{p-1} B_{n,k}, & \text{otherwise} \end{cases}, \tag{11}$$

where

$$B_{n,k} = |x_{n,k}| + |x_{n,k+1}|. \tag{12}$$

The solutions satisfying these penalty functions are feasible solutions.

B. *Ant Colony Optimization*

ACO is the algorithm inspired by the foraging behavior of ant colonies and one of the multiple search algorithms. The search model in the design of FIR filters with CSD coefficients is shown in Fig. 4. In this study, the pheromone updating based on [8] is applied because it makes possible to relax an intensification of the search. The following formulas to calculate the critical parameter of ACO was cited from [8] and revised by us. A procedure of ACO is carried out as following:

1) Initialization: The initial values of each trail are given based on solutions of (2) solved by a linear programming method over the continuous space and then the coefficients are simply rounded to the nearest CSD number. The pheromone trail of the initial value is defined as τ_{init} and the others are defined as $(1 - \tau_{init})/3$.

2) Transition Probability: In Fig. 4, each agent moves from a current node to the node $\overline{1}$ or 0, or 1. When the agent visits at the node $x_{n,k} = l$ in the t-th iteration, the probability $P_{n,k}^{lm}(t)$ that the agent moves to the node $x_{n,k+1} = m$ is defined as following,

$$P_{n,k}^{lm}(t) = \frac{\tau_{n,k}^{lm}(t)}{\tau_{n,k}^{l1}(t) + \tau_{n,k}^{l0}(t) + \tau_{n,k}^{l\overline{1}}(t)}, \quad (13)$$

where $l, m \in \{\overline{1}, 0, 1\}$, $\tau_{n,k}^{lm}(t)$ is the pheromone trail from the node l to the node m. The agent selects the path according to the probability $P_{n,k}^{lm}(t)$ such as,

$$x_{n,k+1} = \begin{cases} 0, & \text{if } r < P_{n,k}^{l0}(t) \\ 1, & \text{if } P_{n,k}^{l0}(t) \le r < P_{n,k}^{l0}(t) + P_{n,k}^{l1}(t) \\ \overline{1}, & \text{otherwise} \end{cases}, \quad (14)$$

where r is a uniform random value in [0, 1]. When the agent has completed the tour, the tracing path gives one of the candidates of solutions.

3) Pheromone Updating: After all agents have completed the tour, the pheromone is evaporated and deposited. The updating of the pheromone trail $\tau_{lm}^{n,k}(t+1)$ is defined as following,

$$\tau_{n,k}^{lm}(t+1) = \rho\tau_{n,k}^{lm}(t) + \sum_{u=1}^{\sigma-1} \Delta\tau_{n,k}^{lm,u}(t) + \Delta\tau_{n,k}^{lm,*}(t), \quad (15)$$

where ρ is evaporation rate, σ is the number of superior agents. $\Delta\tau_{n,k}^{lm,u}(t)$ is the amount of pheromone deposited by the u-th best agents described as following,

$$\Delta\tau_{n,k}^{lm,u}(t) = \begin{cases} (\sigma - u)\dfrac{Q}{L^u(t)/\delta_{\mathrm{LP}}}, & \text{if the } u\text{-th agent travels on edge } (l,m) \\ 0, & \text{otherwise} \end{cases}, \quad (16)$$

where Q is a constant, $L^u(t)$ is the objective function value of the u-th agent, δ_{LP} is the maximum absolute error calculated by the linear programming method over the continuous space. $\Delta\tau_{n,k}^{lm,*}(t)$ is the amount of pheromone deposited by the best agent described as following,

$$\Delta\tau_{n,k}^{lm,*}(t) = \begin{cases} \sigma\dfrac{Q}{L^*(t)/\delta_{\mathrm{LP}}}, & \text{if the best agent travels on edge } (l,m) \\ 0, & \text{otherwise} \end{cases}. \quad (17)$$

where, $L^*(t)$ is the objective function value of the best agent.

IV. Design Examples

Several design examples are shown to present the efficiency of our method. The desired magnitude response was given as following,

$$D(\omega) = \begin{cases} 1, & 0 \le \omega \le 2\pi f_p \\ 0, & 2\pi f_s \le \omega \le \pi \end{cases}, \quad (18)$$

where f_p is the normalized passband edge frequency and f_s is the normalized stopband edge frequency. The design conditions in each example are listed in Table I. These conditions were set in order to evaluate in various design conditions. ACO parameters were listed as shown in Table II. These are parameter values brought a low value of $f(x)$ in preliminary verifications. In Table II, N_a is the number of agents, I is the number of iterations, T is the number of trials. Our method was compared with GA (Genetic Algorithm) and 0-1PSO (Particle Swarm Optimization) [5]. GA was executed with mutation rate 0.03, rank selection, two-point crossover with crossover rate 0.9, the population size N_a, the number of generations I, the number of trials T. In the 0-1PSO, the number of trials was 1000, the number of particles was 30, and the other parameters of 0-1PSO were set based on the results of preliminary experiments [5]. All the examples are designed using the PC having Intel(R) Core(TM)i3-2130 3.40[GHz] of CPU and 4[GByte] of memories.

The results are listed in Table III. In Table III, $\delta_{\min}$ is a minimum value of δ, δ_{mean} is a mean value of δ. The magnitude response of Ex.1 is shown in Fig. 5, the bassband magnitude response of the Ex.1 is shown in Fig. 6. The magnitude response of Ex.2 is shown in Fig. 7, the bassband

TABLE I
DESIGN CONDITIONS

	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5
N	24	50	100	200	300
p	8	8	16	16	16
f_p	0.225	0.3	0.21	0.19	0.3
f_s	0.275	0.35	0.23	0.20	0.309
Λ	15	30	50	80	100

TABLE II
ACO PARAMETERS

	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5
τ_{init}	0.5	0.55	0.9	0.9	0.9
ρ	0.9	0.9	0.9	0.9	0.9
σ	301	301	361	361	361
Q	10	10	10	10	10
N_a	500	500	600	600	600
I	500	500	600	600	600
T	100	100	100	100	100

TABLE III
DESIGN RESULTS

	method	$\delta_{\min}$	δ_{mean}	variance	Computation time per a trial[s]
Ex.1	0-1PSO	4.745	5.177	0.773	1
	GA	4.689	4.704	0.119	3
	ACO	4.679	4.759	0.994	2
Ex.2	0-1PSO	2.232	2.409	1.428	2
	GA	1.879	2.162	1.300	10
	ACO	1.543	1.760	1.054	5
Ex.3	0-1PSO	3.013	3.988	4.663	6
	GA	2.691	2.983	4.037	52
	ACO	2.709	2.923	1.663	29
Ex.4	0-1PSO	4.075	5.219	39.913	14
	GA	3.506	3.987	1.827	181
	ACO	3.443	3.778	1.288	78
Ex.5	0-1PSO	3.292	5.293	107.625	25
	GA	2.709	3.127	1.978	385
	ACO	2.441	2.670	0.971	189

$\delta_{\min}, \delta_{\mathrm{mean}} : \times 10^{-2}$, variance : $\times 10^{-3}$

TABLE IV
FILTER COEFFICIENTS (EX.1)

	initial	optimal
a_0	0.10000000	0.10000000
a_1	0.10100010	$0.101000\mathbf{01}$
a_2	0.00000000	0.00000000
a_3	$0.0\bar{1}010\bar{1}01$	$0.0\bar{1}010\bar{1}0\mathbf{0}$
a_4	0.00000000	0.00000000
a_5	$0.001000\bar{1}0$	$0.001000\bar{1}0$
a_6	0.00000000	0.00000000
a_7	$0.000\bar{1}0\bar{1}01$	$0.000\bar{1}0\bar{1}0\mathbf{0}$
a_8	0.00000000	0.00000000
a_9	$0.00010\bar{1}00$	$0.00010\bar{1}0\mathbf{1}$
a_{10}	0.00000000	0.00000000
a_{11}	$0.000\bar{1}0000$	$0.000\bar{1}0000$
a_{12}	0.00000000	0.00000000

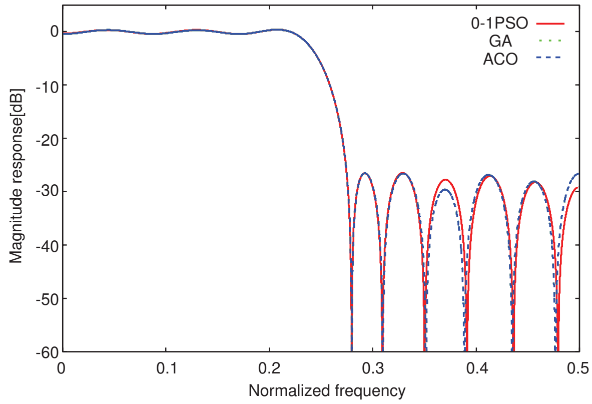

Fig. 5. Magnitude response (Ex.1)

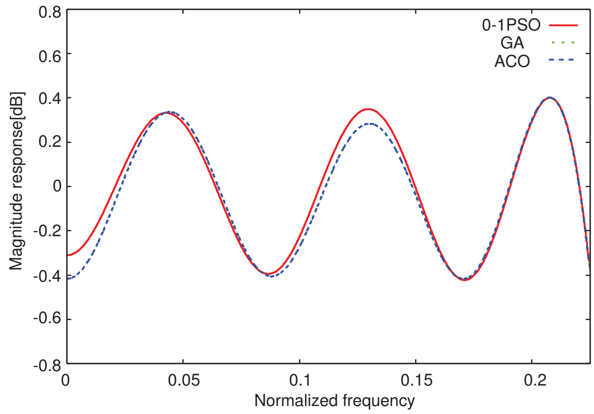

Fig. 6. Passband magnitude response (Ex.1)

magnitude response of the Ex.2 is shown in Fig. 8. The updating curve of the objective function value of Ex.1 is shown in Fig. 9 and its enlarged view is shown in Fig. 10. The updating curve of Ex.3, Ex.5 are shown in Fig. 11, Fig. 13 and its enlarged views are shown in Fig. 12, Fig. 14, respectively. In Ex.1, the mean value of the hamming distance between the initial coefficient and the solution obtained by each heuristic approach are depicted in Fig. 15 and the mean value of the hamming distance in Ex.3, Ex.5 are depicted in Fig. 16, Fig. 17, respectively.

From Table III, ACO required the high computation time in comparison with 0-1PSO because 0-1PSO is simple algorithm and low computation cost. By constant, ACO could reduce the maximum error, the mean error, and the variance in most examples. Comparing with GA, ACO could reduce the maximum error, mean error, and the variance in high order filters. In addition, the computation time of ACO could reduce down to a half of GAs. Furthermore, we confirmed that the solution obtained by ACO is equal to the optimal solution obtained by branch and bound method which required about five minutes in Ex.1. As seen from Fig. 9 to Fig. 14, ACO first found the undesired solutions which does not satisfy the CSD constrains. However, ACO finally obtained the solution which is better than other methods in most examples.

The reason why the better solution can be obtained by ACO is that ACO suits for the design problem of CSD coefficient FIR filters with regard to POP (Proximate Optimality Principle) [11] described as below: In our method, the initial values are given by simply rounding of the continuous coefficients which are calculated by the linear programming method. Therefore, it is assumed that the structure of optimal solution is similar to the initial value and thus some initial coefficients become similar values to optimal solutions. For example, the initial coefficients and optimal coefficients which are obtained by branch-and-bound method are listed in Table IV. From Table IV, the number of different digits between initial coefficients and optimal coefficients at most four. This indicates that POP which is the principle that good solutions possess some similar structures holds in the design problem of CSD coefficients FIR filters. Therefore, it is required that just a few coefficients which are not equal to the optimal coefficients should be changed. As seen from Fig. 15 to Fig. 17, GA changed all coefficients. This means that the search of GA tends to be global search. In constant, 0-1PSO hardly changed coefficients in comparison with other methods. This means that the search of 0-1PSO tends to be significant local search. Thus, GA and 0-1PSO are not able to obtain the better solutions whose structure is similar to the initial coefficients. On the other hand, ACO changed some coefficients and be able to obtain the better solutions which are similar to the initial coefficients. Therefore, it can be considered that ACO is able to work effectively for this design problems.

V. CONCLUSION

In this study, a design of FIR filters with CSD coefficients using ACO was studied. In the design examples, we investigated the difference between initial coefficients and optimal coefficients and revealed that ACO could work for this design problem effectively. Furthermore, it was shown that ACO could reduce the design error and solve in a reasonable computation time in comparison with other methods.

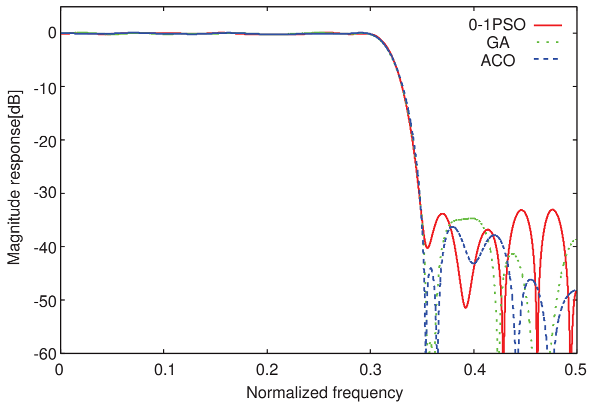

Fig. 7. Magnitude response (Ex.2)

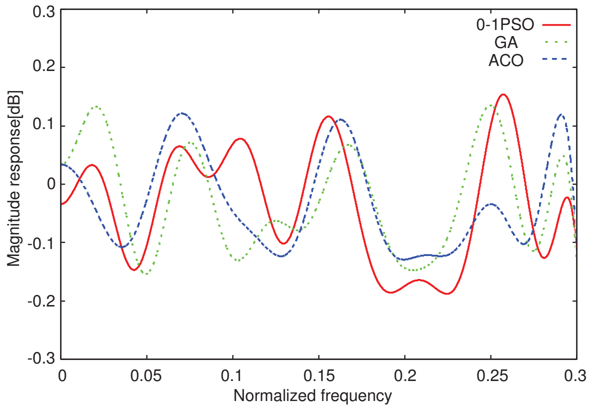

Fig. 8. Passband magnitude response (Ex.2)

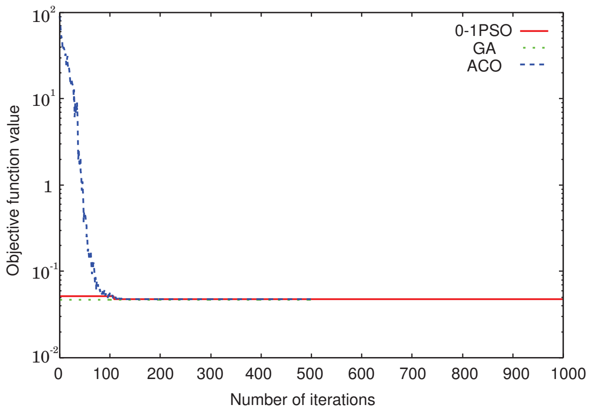

Fig. 9. Updating curve of objective function value (Ex.1)

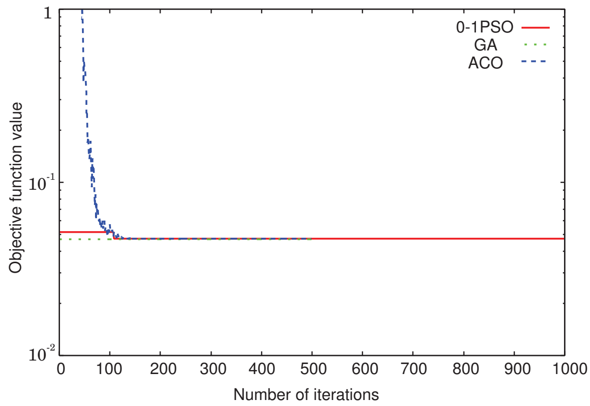

Fig. 10. Enlarged updating curve of objective function value (Ex.1)

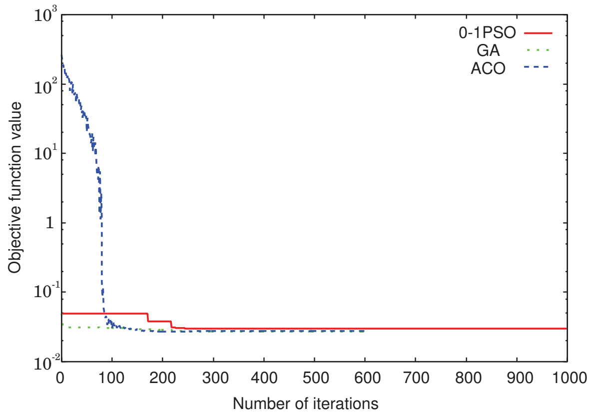

Fig. 11. Updating curve of objective function value (Ex.3)

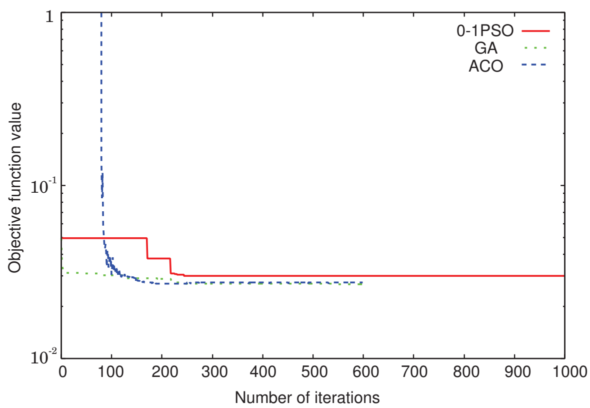

Fig. 12. Enlarged updating curve of objective function value (Ex.3)

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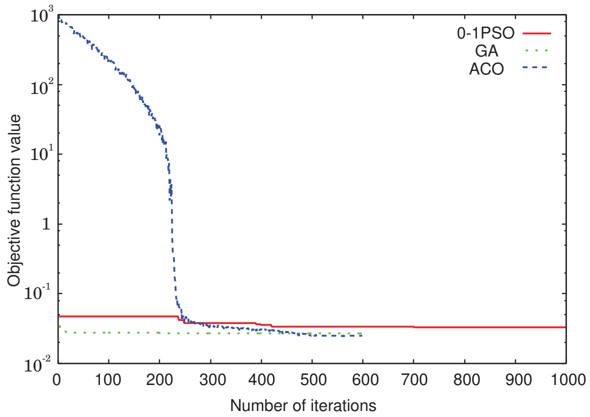

Fig. 13. Updating curve of objective function value (Ex.5)

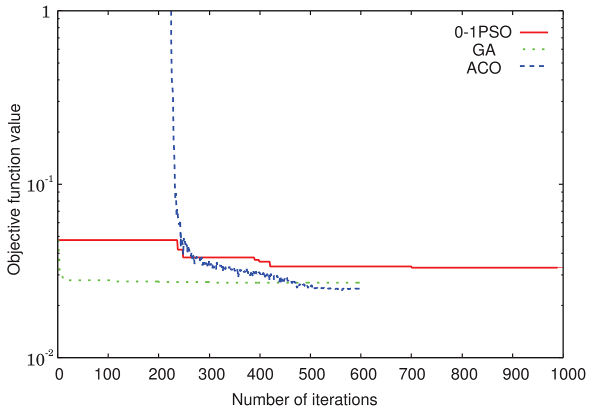

Fig. 14. Enlarged updating curve of objective function value (Ex.5)

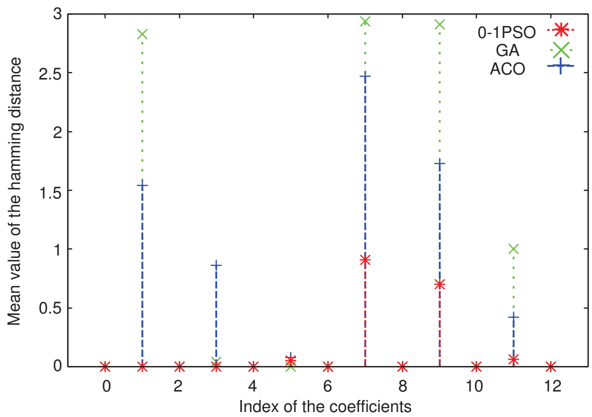

Fig. 15. Mean value of the hamming distance between the initial coefficient and the solution of each heuristic approach (Ex.1)

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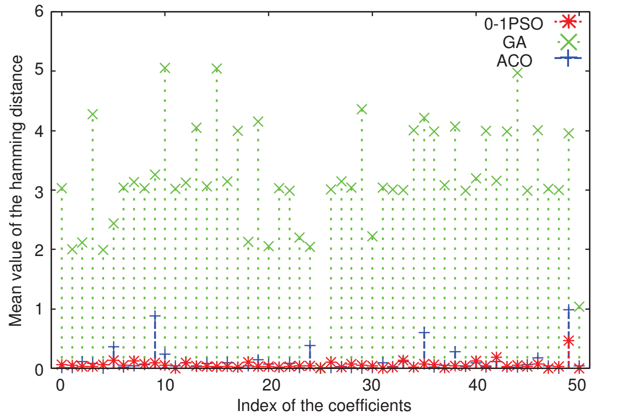

Fig. 16. Mean value of the hamming distance between the initial coefficient and the solution of each heuristic approach (Ex.3)

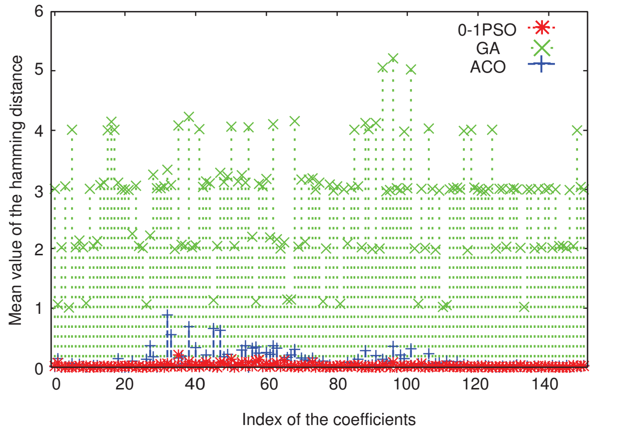

Fig. 17. Mean value of the hamming distance between the initial coefficient and the solution of each heuristic approach (Ex.5)

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