Performance Evaluation of IIR Filter Design Using Multi-Swarm PSO

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***Abstract*—In this paper, we study a method to avoid a premature convergence in the design of IIR (Infinite Impulse Response) filters using PSO (Particle Swarm Optimization). Although PSO is applicable to solve nonlinear optimization problems, it is reported that a local minimum stagnation often occurs due to a strong directivity during the searching. To avoid the local minimum stagnation, the particle reallocation method has been proposed in our previous work. In this method, a reallocation space is determined by using the stagnation swarm and the another two swarms selected randomly. In this paper, a design performance is evaluated though several design examples.**

I. Introduction

The design problem of IIR filters is difficult to solve because it is generally a nonlinear optimization problem [1]. Recently, several design methods using heuristic methods have been developed for IIR filter design [2]–[4]. Especially, PSO [5] can enumerate the solution candidates in low computational cost because its updating scheme is described as a simple model. Therefore, there are some reports applied to the design problems of IIR filters.

PSO is the multi-point searching algorithm inspired by social behavior of animals like a flock of birds. PSO has a swarm consists of some particles. Then, the particle is specified by a position vector and a velocity vector, and its position is updated toward to the best solution of the swarm. Although PSO has many advantages for solving the nonlinear optimization problems, it often occurs the local minimum stagnation because of the strong directivity toward a local minimum. In a lot of previous works, the stagnation due to the strong directivity is seemed to be a drawback of PSO and several methods to prevent the stagnation have been developed [4], [6]–[11]. On the other hand, we consider that the strong directivity is an advantage of PSO because it contributes to the enumeration of local minimums. It goes without saying that this ability is derived from the simple updating model of PSO. Based on this concept, in our assertion, aggressive stagnations are prompted while keeping the simple updating model as it is and an escape from the stagnation point is attempted every the stagnation. Therefore, the escape strategy plays an important role in our method.

A particle reallocation strategy has been studied as a direct escape method for the local minimum stagnation [3], [8]. The particle reallocation prompts the updating by shifting particles or swarms to the another space from the stagnation position. In [3], particles having high fitness are moved around the area which is the neighborhood of $gbest$ and the others are initialized. However, there is possibility that such particles fall into the same local minimum. In [8], all of particles are reinitialized if the local minimum stagnation has occurred.

As the another approaches to avoid the local minimum stagnation, multi-swarm PSO [9]–[11] are developed. Those methods focused on just the relationship among swarms and intended to improve a diversity of searching by regrouping the particles or changing the updating procedure.

On the other hand, in our previous work [12], the reallocation of particles to the space that includes the local minimum which has not searched yet is attempted. In [12], the updating procedure is kept as it is and just a decision method of good reallocation space is focused. Then, multi-swarm PSO is used, and some $gbests$ are used for determining the reallocation space. In general, a lot of local minimums are included in the design problem of IIR filters. Therefore, the space specified by some $gbests$ is non-convex, and thus an existence of the another local minimum can be expected within such a space, then particles are reallocated there.

In [12], a stagnated swarm and the another two swarms chosen randomly are selected among all swarms for determining the reallocation space. It was shown that such a strategy is effective through several design examples. However, only the fixed number of swarms was discussed and a suitable number of swarms is still unknown. Thus, in this paper, the suitable number of swarms is revealed based the performance evaluation of IIR filter design.

II. Design Problems

The frequency response of IIR filters is described as follow,

$$H(\omega) = a_0 \frac{\prod_{n=1}^{N}(1 - z_n e^{-j\omega})}{\prod_{m=1}^{M}(1 - p_m e^{-j\omega})}, \tag{1}$$

where a_0 is a coefficient of IIR filters, N is a numerator order and M is a denominator order, z_n $(n = 1, 2, \cdots, N)$ are zeros, p_m $(m = 1, 2, \cdots, M)$ are poles, and ω is an angular frequency. In this paper, real coefficient filters are designed, thus z_n and p_m are complex conjugates or real numbers. The design problem of IIR filters based on the Chebyshev approximation criteria can be described as

$$\min_{\boldsymbol{x}} \max_{\omega \in \Omega} W(\omega)|D(\omega) - H(\omega)|, \tag{2}$$

where $\boldsymbol{x} = [a_0, z_1, \cdots, z_N, p_1, \cdots, p_M]^T$ is the design parameter vector, Ω is an approximation frequency band, $D(\omega)$

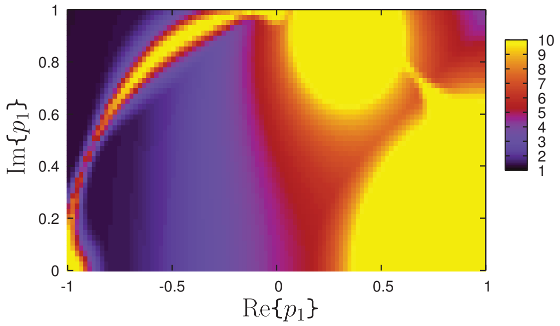

Fig. 1. Objective function of IIR filter design problem

is a desired frequency response, $H(\omega)$ is an obtained frequency response. Fig.1 shows the objective function depicted over p_1. The vertical axis denotes the imaginary part of p_1 and the horizontal axis denotes the real part of p_1. The depth of color in this figure means the value of the objective function. The frequency response of IIR filters is the rational function, and all of poles must exist within the unit circle on z-plane. These conditions make the objective function a nonlinear form. Thus, the objective function of the design problem of IIR filters is a multi-modal function and is difficult to obtain the optimal solution.

III. Particle Swarm Optimization

A. Normal-PSO

PSO is the multi-point searching method based on the social behavior of animals. PSO has many advantages such as simplicity, low computational cost, and little variable parameters. The best advantage of PSO is rapid intensification. PSO has multiple particles, and the swarm is consisted of them. The particles share the information and enumerate the local minimums of the objective function. The i-th particle is defined by the position vector $\boldsymbol{x}_i$ and the velocity vector $\boldsymbol{v}_i$, where i $(i = 1, \cdots, N_p)$ is the particle number and N_p is the number of particles. The searching is carried out by updating $\boldsymbol{x}_i$ and $\boldsymbol{v}_i$. Updating of the position and the velocity of the particle i in the t-th iteration are carried out as follows,

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \boldsymbol{v}_i^{t+1}, \quad (3)$$
$$\boldsymbol{v}_i^{t+1} = w^t \boldsymbol{v}_i^t + c_1 r_1 (\boldsymbol{p}_i^t - \boldsymbol{x}_i^t) + c_2 r_2 (\boldsymbol{g}^t - \boldsymbol{x}_i^t), \quad (4)$$

where $\boldsymbol{p}_i^t$ is the best solution which the i-th particle has searched before, $\boldsymbol{g}^t$ is the best solution among all particles up to the t-th iteration. w^t is the inertia weight parameter, c_1 is a weight parameter toward $\boldsymbol{p}_i^t$, c_2 is a weight parameter toward $\boldsymbol{g}^t$, r_1 and r_2 are uniform random numbers in the interval of $[0, 1]$. w^t is linearly decreased using a following equation,

$$w^t = w_{max} - \frac{t}{I_{max}}(w_{max} - w_{min}), \quad (5)$$

where w_{max} is an upper bound of w, w_{min} is a lower bound of w. Equation (5) means that PSO changes from the global search to the local search, thus particles are gathered around the local minimum. As a result, PSO can enumerate the candidates of solution rapidly. Although PSO has many advantages, due to this characteristic, particles tend to stagnate around the local minimum. There are many local minimums in the design problem of IIR filters, thus that tendency is remarkable.

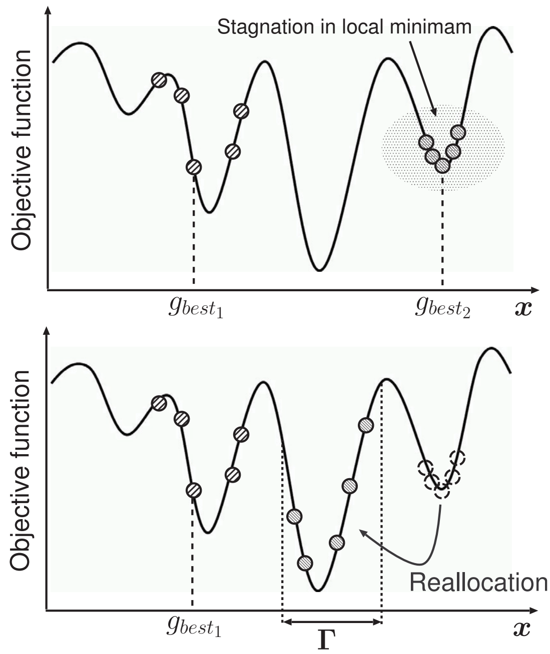

Fig. 2. An example of the avoidance of the local minimum stagnation by particle reallocation

B. Avoidance of Stagnation of Searching by Particle Reallocation

In [12], the method that use many advantages of PSO is proposed. It aimed to realize a successive search by moving particles to the reallocation space when the searching has stagnated without changing the updating procedure. This means that the intensification of particles is prompted the good characteristic of PSO is kept. Furthermore, this method prompts it by adopting a large value of c_2 intentionally.

In this method, multiple swarms and some local minimums obtained by the intensification of particles are used for avoiding the local minimum stagnation. In [12], all of particles belonging to the stagnation swarm are reallocated to the reallocation space $\boldsymbol{\Gamma}$ as shown in Fig.2. For the decision of good reallocation space, a convex combination of $gbests$ of each swarm was used as follow,

$$\boldsymbol{\gamma} = \sum_{k=1}^{K} \lambda_k \boldsymbol{g_{best}}_k, \quad (6)$$

where K is the number of selected swarms, $\boldsymbol{g_{best}}_k$ is the best solution of the k-th swarm, and λ_k $(\sum_{k=1}^{K} \lambda_k = 1,$ $\lambda_k \geq 0 (k = 1, \cdots, K))$ is a weight parameter. $\boldsymbol{\gamma}$ is the center of $\boldsymbol{\Gamma}$, and the range of $\boldsymbol{\Gamma}$ is defined as $[\boldsymbol{\gamma} - h\boldsymbol{e}, \boldsymbol{\gamma} + h\boldsymbol{e}]$,

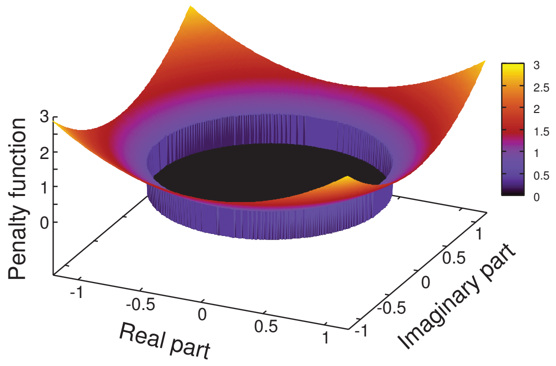

Fig. 3. Penalty function $\phi(\boldsymbol{x})(R = 0.9)$

where h is a width of perturbation for the reallocation, $\boldsymbol{e} = [1, 1, \cdots, 1]^T$. To ensure the stability of IIR filters after the particle reallocation, h needs to set a small value less than 1.0. In [12], it is not suitable to decrease w gradually like (5) because particles repeat the diversification and the intensification. Therefore, w should be set to a constant value.

IV. Objective Function for the Design Problem of IIR Filters

A. Definition of the Objective Function

The objective function is defined as follow to apply PSO to the design problem of IIR filters,

$$F(\boldsymbol{x}) = W(\omega_u)|D(\omega_u) - H(\omega_u)|, \tag{7}$$

where $\omega_u (u = 1, \cdots, S)$ is the discrete angular frequency, S is the number of frequency samples.

B. Penalty Function

A necessary and sufficient condition of stability of IIR filters is that all of poles exist within the unit circle on z-plane. To ensure the stability of IIR filters and decrease an excess magnitude ripple around a transition band, a penalty function $\phi(\boldsymbol{x})$ is defined as follows,

$$\phi(\boldsymbol{x}) = \begin{cases} {p_{max}}^2, & p_{max} \geq R \\ 0, & p_{max} < R \end{cases}, \tag{8}$$

where p_{max} is a maximum pole radius, R ($R < 1.0$) is a maximum pole radius specified in advance. Fig.3 shows the penalty function when R is set to 0.9. Adding the penalty function to (7), the objective function is redefined as follow,

$$F'(\boldsymbol{x}) = w_f F(\boldsymbol{x}) + c_s \phi(\boldsymbol{x}), \tag{9}$$

where w_f and c_s are weight parameters. If c_s is set to be large adequately, the stability of IIR filter can be ensured. The design problem of IIR filters in this paper is to determine $\boldsymbol{x}$ that minimizes $F'(\boldsymbol{x})$.

C. Design Procedure

The design procedure of IIR filters using multi-swarm PSO is described in **Algorithm1**. In this method, the reallocation space $\boldsymbol{\Gamma}$ is set within $[\boldsymbol{\gamma} - h\boldsymbol{e}, \boldsymbol{\gamma} + h\boldsymbol{e}]$.

Algorithm 1 proposed method

N : numerator order
M : denominator order
S : the number of division of frequency
N_p : the number of particle
L : the number of swarms
I_{max} : the limit of iterations
α : the maximum number of stagnation judgment
Initialize the position and the velocity of each particle
$t \leftarrow 1$
Divide all particles into L swarms randomly
for t=1 to I_{max} **do**
  Calculate the objective function value
  Update $\boldsymbol{p}_i^t$
  Update $\boldsymbol{g}^t$ of each swarm
  Update the position and the velocity of all particles
  if there is a swarm that does not update $\boldsymbol{g}^t$ α times **then**
    Select K swarms randomly including a stagnated swarm
    Set the reallocation space $\boldsymbol{\Gamma}$
    Shift all of particles belonging to a stagnated swarm into $\boldsymbol{\Gamma}$
  end if
end for

V. Performance Evaluation and Investigation of Suitable Number of Swarms

In this chapter, four examples are shown to find the relationship between the number of swarms and the best value of error. $D(\omega)$ was given as

$$D(\omega) = \begin{cases} e^{-j\omega\tau_d}, & 0 \leq \omega \leq 2\pi f_p \\ 0, & 2\pi f_s \leq \omega \leq \pi \end{cases}, \tag{10}$$

where τ_d is a desired group delay, f_p is a normalized pass band edge frequency, f_s is a normalized stop band edge frequency.

Design conditions are listed in Table I. I_{max} in Table I denotes the maximum number of iteration. For all examples, parameters were set to $S = 100$, $W(\omega) = 1$, $w_f = 1$, $c_s = 100$. Each initial value was set to a random number. Initial value of a_0 was set to $[-0.01, 0.01]$, the modulus and the angle of zeros were set to $[0, 3]$ and $[-\pi, \pi]$, the modulus and the angle of poles were set to $[0.3, 0.3 + R]$ and $[-\pi, \pi]$. The width of perturbation h was set to 0.01, the number of stagnation α was set to 10. The parameter of PSO w was set to 0.4, c_1 was set to 1.0, and c_2 was set to 3.0. In all design examples, the number of swarms L was set to from 3 to 7 and the number of selected swarms K was set to 3. The PC having CPU: Intel(R) Core(TM) i3-2130 3.40[GHz], memory: 4[GByte] was used in all design examples. The specifications of Ex.1 and Ex.2 are design problems used in [12]. The design of Ex.3 means that the pass band is narrow. The design of Ex.4 means that the transition band is narrow. These are difficult design examples so that particles are easy to stagnate to local minimums. The design results are listed in Table II, Table III, Table IV, and Table V.

TABLE I
DESIGN CONDITIONS

	Ex.1	Ex.2	Ex.3	Ex.4
N	4	8	4	8
M	4	6	4	6
τ_d	3	5	3	5
f_p	0.3	0.175	0.05	0.2
f_s	0.4	0.25	0.1	0.22
R	0.85	0.92	0.9	0.92
N_p/swarm	20	30	20	30
I_{max}	5000			
The number of trial	50			

TABLE II
DESIGN RESULTS OF EX.1

the number of swarm	3	4	5	6	7
Best ($\times 10^{-2}$)	7.5398	7.5347	7.5335	7.5314	7.5308
Average ($\times 10^{-2}$)	8.3556	8.1672	8.3253	8.2349	8.3650
Deviation ($\times 10^{-2}$)	0.8427	0.3837	1.1764	0.9235	1.0475
Time [s]	176.7	230.6	286.3	348.5	407.4

TABLE III
DESIGN RESULTS OF EX.2

the number of swarm	3	4	5	6	7
Best ($\times 10^{-2}$)	2.2285	2.2070	2.1951	2.1381	2.1323
Average ($\times 10^{-2}$)	5.2586	2.7803	2.7803	2.7403	3.2353
Deviation ($\times 10^{-2}$)	0.5244	0.6062	0.5670	0.4379	1.5698
Time [s]	351.3	475.3	593.6	713.5	832.3

TABLE IV
DESIGN RESULTS OF EX.3

the number of swarm	3	4	5	6	7
Best ($\times 10^{-2}$)	8.1953	8.1614	8.1578	8.1535	8.1503
Average ($\times 10^{-2}$)	9.1261	9.0416	8.8527	8.9574	9.2831
Deviation ($\times 10^{-2}$)	1.1885	1.0075	0.9380	0.8881	1.4242
Time [s]	164.8	219.9	273.4	327.5	387.3

TABLE V
DESIGN RESULTS OF EX.4

the number of swarm	3	4	5	6	7
Best ($\times 10^{-2}$)	15.347	15.335	15.287	15.188	15.163
Average ($\times 10^{-2}$)	18.175	18.291	16.912	17.767	17.209
Deviation ($\times 10^{-2}$)	4.3315	5.0072	2.7813	3.9576	3.2217
Time [s]	357.1	467.6	583.7	710.0	815.3

A. Design Results

Table II shows that the best value of error was improved by increasing of the searching swarms. Computational time was about from 1.17 to 1.30 times with increase of one swarm.

Table III also shows that the best value of error was improved by increasing of the searching swarms. In the case of 7 swarms, the best value of error was improved about 4.3 percent compared with the case of 3 swarms. Computational time was about from 1.16 to 1.35 times with increase of one swarm.

From the design results of Ex.3 and Ex.4, the difficult design examples also could improve the best value of design error because of adding the searching swarms. Computational time was about from 1.18 to 1.33 times in Ex.3 and about from 1.14 to 1.30 times in Ex.4 with increase of one swarm.

In Ex.1, Ex.2, and Ex.3, when the searching swarms were added, the standard deviation of the obtained solution tended to be large. It is because the number of particles increased together with the increment of the searching swarms. In order to decrease the standard deviation, especially in the difficult design examples, it may be considered that more iterations are required. On the other hand, it was confirmed that the standard deviation of the method proposed in [12] is smaller than normal-PSO. For example, in [12], that of normal PSO for ex.1 is 3.5289×10^{-2}. It reveals that the method in [12] can enumerate a similar local minimum up to the end of iterations and it does not depend on the trial. That is, the particles almost always enumerate desirable local minimums independent of trials. According to these results, the particle reallocation method is effective for the design problem of IIR filters. However, computational time is simply concerned with the number of particles. When the searching swarms are added, computational time increases almost linearly.

From Fig.4 to Fig.19, they show the magnitude response, the group delay, and the design error of the latter half of the best trial in 3 swarms and 7 swarms in each design example. In Fig.7, Fig.11, Fig.15, and Fig.19, it was found that the case of 7 swarms could realize the successive updating and calculate better solution than the case of 3 swarms. It is clear that the error of magnitude response in the case of 7 swarms is smaller than that in the case of 3 swarms in all design examples.

VI. CONCLUSIONS

In this paper, the performance of IIR filter design using our method was evaluated through several design examples. The design error and computational time against the number of swarms were compared in the same design conditions. From design results, it was revealed the relationship between the number of swarms and the best value of error.

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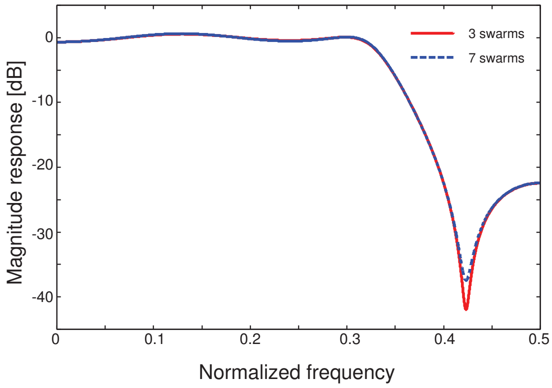

Fig. 4. Magnitude response (Ex.1)

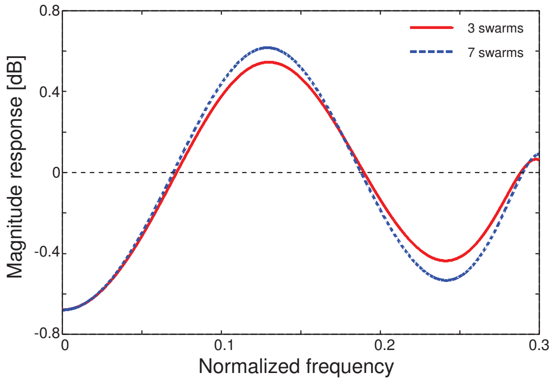

Fig. 5. Pass band of magnitude response (Ex.1)

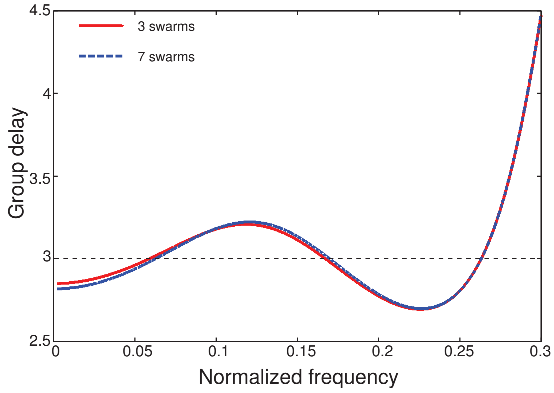

Fig. 6. Group delay (Ex.1)

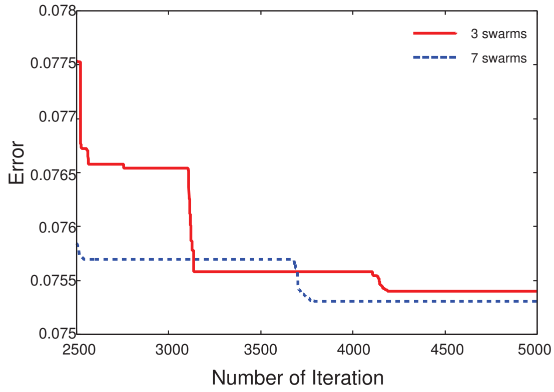

Fig. 7. Updating curve (Ex.1)

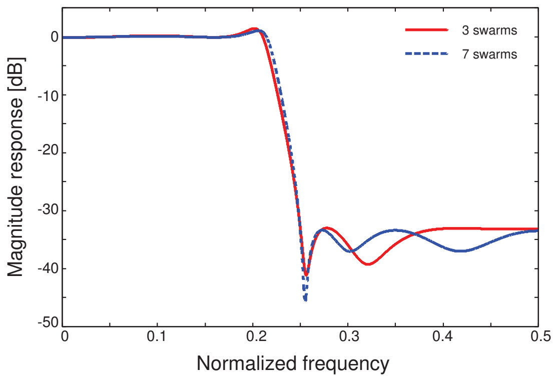

Fig. 8. Magnitude response (Ex.2)

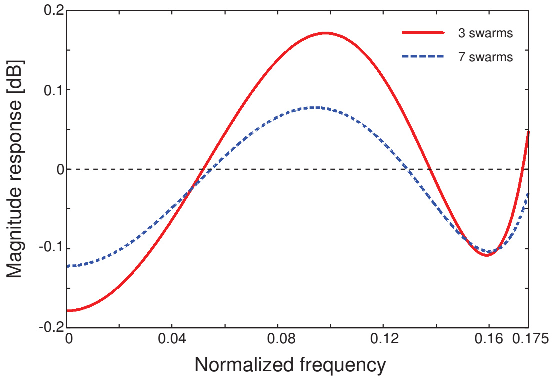

Fig. 9. Pass band of magnitude response (Ex.2)

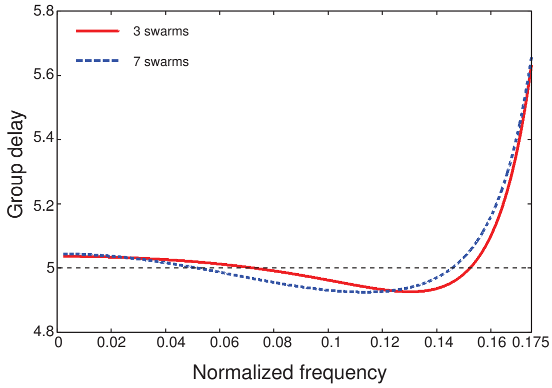

Fig. 10. Group delay (Ex.2)

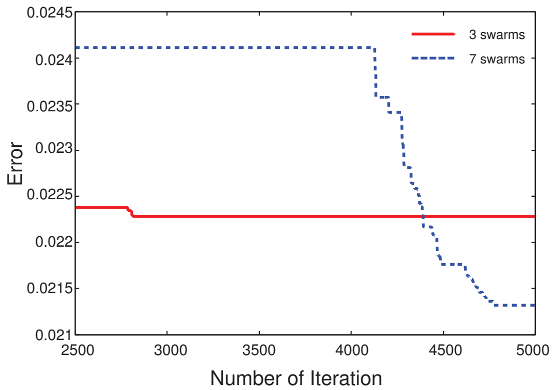

Fig. 11. Updating curve (Ex.2)

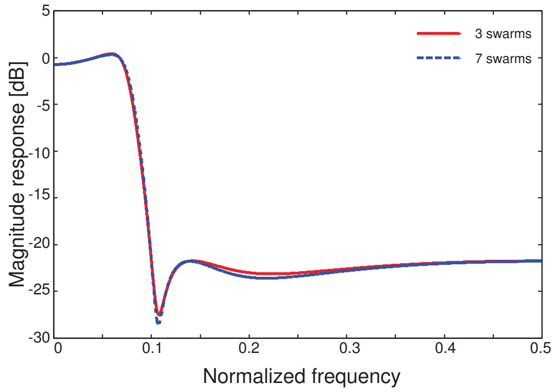

Fig. 12. Magnitude response (Ex.3)

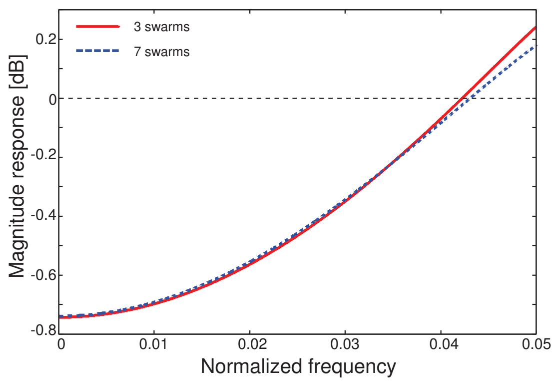

Fig. 13. Pass band of magnitude response (Ex.3)

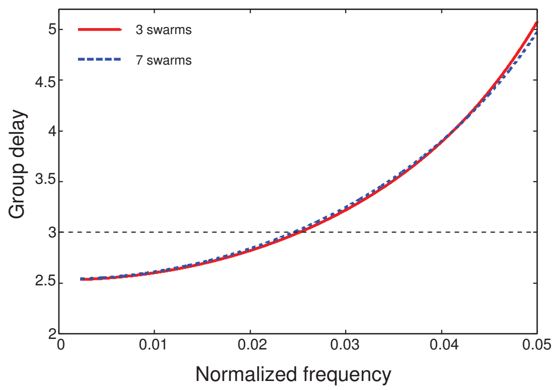

Fig. 14. Group delay (Ex.3)

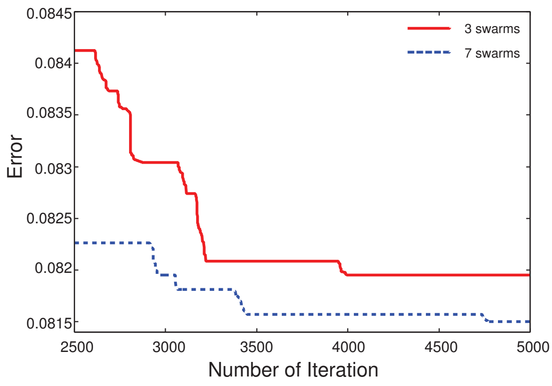

Fig. 15. Updating curve (Ex.3)

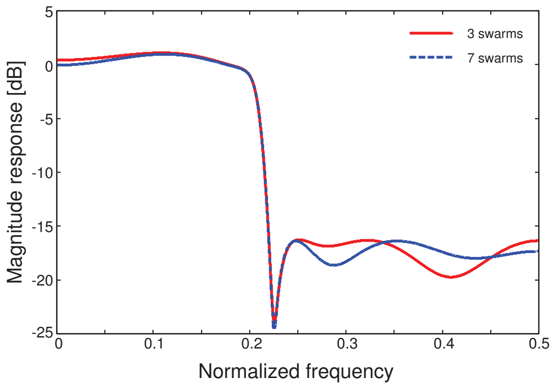

Fig. 16. Magnitude response (Ex.4)

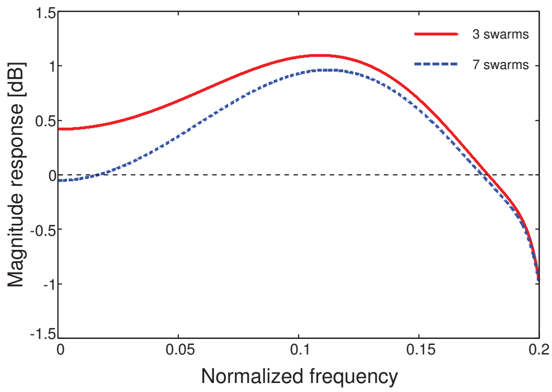

Fig. 17. Pass band of magnitude response (Ex.4)

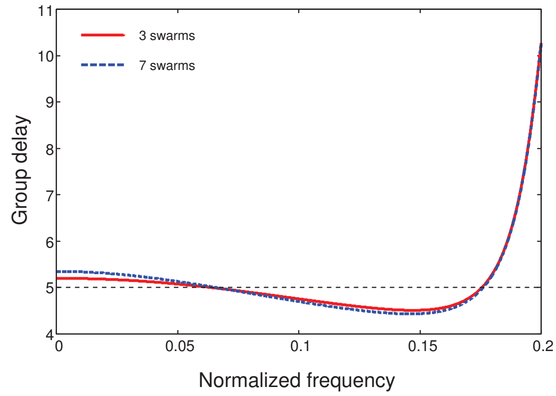

Fig. 18. Group delay (Ex.4)

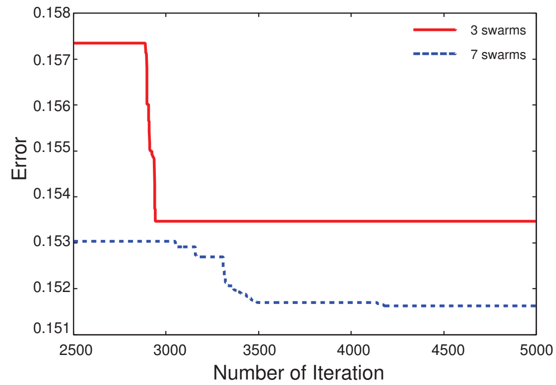

Fig. 19. Updating curve (Ex.4)