Linearization of the Parametric Array Loudspeaker upon Varying Input Amplitudes

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Abstract—In spite of being a compact directional sound device, the parametric array loudspeaker (PAL) has often been criticized for its severe nonlinear distortion. In some preliminary studies, the nonlinear distortion is able to be reduced by using the linearization system based on Volterra filter identification, which models the nonlinear sound process of the PAL. However, the nonlinearity of the PAL changes with the input amplitude. When the input amplitude is high enough, there is strong highorder nonlinearity and the 2nd–order Volterra kernel cannot be accurately identified. Therefore, an improved identification method of the 2nd–order Volterra kernel is proposed to exclude the interference from the 4th–order nonlinearity.

I. INTRODUCTION

For daily sounds, such as voices, musics, and noises, the linear acoustic model generally holds in air and is easy to apply. However, when a large amplitude ultrasound is propagating, the nonlinear acoustic effect becomes noticeable. Such a large amplitude sound wave is often referred to as a finite amplitude wave in literature [1], [2]. When two finite amplitude waves at close frequencies coincide, intermodulation frequencies, such as the sum and difference frequencies, are generated because of the nonlinear acoustic effects. It is of interest to note that the sum and difference frequencies travel in a similarly narrow beam as the finite amplitude waves [3].

Making use of the nonlinear acoustic effect, the PAL was invented as shown in Fig. 1 [4]. The input is necessarily modulated on an ultrasonic carrier. The sideband provides one finite amplitude wave and the ultrasonic carrier provides another finite amplitude wave. Therefore, the difference frequency recover the input with some distortion. This self-demodulation process is described by the Berktay's far-field solution [5].

The modulated input $p_1(t)$ is written as

$$p_1(t) = E(t) \sin\omega_c t,\tag{1}$$

where E(t) is the envelope function, also known as the preprocessed input; and ω_c is the angular frequency of the ultrasonic carrier. When the ultrasonic pressure is below the threshold for shock formation, the Berktay's equation is reasonably accurate and provides the self-demodulated level as

$$p_2(t) = \text{const} \times \frac{\partial^2}{\partial t^2} E^2(t),$$
 (2)

where the const is determined by the medium and carrier frequency. However, when shock formation occurs, the Berktay's equation has to be modified to the form of

$$p_2(t) = \text{const} \times \frac{\partial^2}{\partial t^2} |E(t)|,$$
 (3)



Fig. 1. Block diagram of the PAL.

where the square operation has been replaced by the absolute value [6].

Both the square operation and absolute value are nonlinear functions. They introduce undesired nonlinear distortion. Previous works are mostly developed based on the Berktay's equation in the form of (2). Since 1983, Yoneyama et al. proposed to use the double sideband amplitude modulation (DSBAM) in the PAL, where the envelope function was given by E(t) = 1 + mg(t) [4]. m and g(t) denoted the modulation index and the input, respectively. In 1984, Kamakura et al. proposed the square root amplitude modulation (SRAM), of which the envelop function became $E(t) = \sqrt{1 + mg(t)}$ [7]. In the next year, Kamakura et al. also proposed to carry out the single sideband amplitude modulation (SSBAM) [8]. The SSBAM led to the encouraging low harmonic distortion. In 2013, Ikefuji et al. attempted to combine the DSBAM and SSBAM in order that the harmonic distortion at the high frequency band could be reduced and the sound pressure level at the low frequency band could be improved [9]. Furthermore, Shi et al. adopted the psychoacoustic approach to retain some harmonic distortion for reproducing the perceptual bass by the missing fundamental effect [10].

However, when (3) becomes the acoustic equation instead of (2), the aforementioned preprocessing methods may not be so effective as when they were proposed. For this reason, the linearization system based on Volterra filter identification is more favorable [11]–[17]. The Volterra filter identification that models the self-demodulation process of the PAL is able to adapt to different environments. In general, there are two identification approaches. They are the adaptive Volterra filter and the frequency response method. For example, Ji *et al.* used the normalized least mean squares (NLMS) algorithm to identify the 2nd–order Volterra kernel [12], and Shi *et al.* applied the sparse NLMS algorithm in the ultrasound-toultrasound Volterra filter [13], [14].

On the other hand, using the frequency response method, we have demonstrated the effectiveness of the linearization system to compensate for both the 2nd–order harmonic and intermodulation distortion [15]. The implementation of the Volterra filter requires high computational power. Hence, Mu *et al.* introduced the one-dimension Volterra filter that had very low computational complexity, but the effectiveness for the intermodulation distortion was likely to be limited [16]. Recently, the parallel cascade structure of the 2nd–order Volterra kernel was examined by the authors. Reduced computational complexity and competitive performance were achieved at the same time [17].

However, the nonlinearity of the PAL changes with the input amplitude. In our previous experiments, when the input amplitude is high, high–order nonlinearity becomes strong [18]. Particularly, the 4th–order nonlinearity can introduce significant errors in the identified 2nd–order Volterra kernel. These errors in turn degrade the performance of the linearization system. In this paper, an improved identification method of the 2nd– order Volterra kernel is proposed to exclude the interference from the 4th–order nonlinearity.

II. LINEARIZATION SYSTEM BASED ON VOLTERRA FILTER IDENTIFICATION

The Volterra series is an effective mathematical tool to model the nonlinear behavior of an unknown system. In signal processing, the use of the Volterra series is also known as the Volterra filter [19]. When the Volterra filter is used to model the self-demodulation process of the PAL, it is often truncated at the 2nd–order and adopts a finite memory length. Hence, the nonlinear response of the PAL is written as

$$y(n) = \sum_{k_1=0}^{N-1} h_1(k_1)x(n-k_1) + \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} h_2(k_1,k_2)x(n-k_1)x(n-k_2), \quad (4)$$

where N is the memory length; x(n) and y(n) are the discrete input and output signals in the audible frequency range; h_1 and h_2 are the coefficients of the 1st–order and 2nd–order Volterra kernels, respectively.

Figure 2 shows the linearization system of the PAL based on Volterra filter identification. In Fig. 2, $\hat{H}_2(z_1, z_2)$ is the identified 2nd–order Volterra kernel and $H_1^{-1}(z)$ is the inverse filter of the identified 1st–order Volterra kernel. The overall response of the linearization system and the PAL is thus



Fig. 2. Block diagram of the linearization system based on Volterra filter identification.

 $H_1^{-1}(z)$: Linear Inverse Filter

 $z^{-\Delta}$: Delay

written as

$$\begin{aligned} \left[z^{-\Delta} - \hat{H}_{2}(z_{1}, z_{2})H_{1}^{-1}(z)\right] \left[H_{1}(z) + H_{2}(z_{1}, z_{2})\right] & (5) \\ = z^{-\Delta}H_{1}(z) & (1st\text{-order component}) \\ + z^{-\Delta}H_{2}(z_{1}, z_{2}) - \hat{H}_{2}(z_{1}, z_{2})H_{1}^{-1}(z)H_{1}(z) \\ & (2nd\text{-order component}) \\ \hat{H}_{1}(z_{1}, z_{2})H_{1}^{-1}(z)H_{1}(z_{2}, z_{2}) & (recidual component) \end{aligned}$$

$$-H_2(z_1, z_2)H_1(z)H_2(z_1, z_2).$$
 (residual component)

The 2nd-order component of the overall response is further written as

$$z^{-\Delta}H_2(z_1, z_2) - \hat{H}_2(z_1, z_2)H_1^{-1}(z)H_1(z)$$

= $z^{-\Delta}H_2(z_1, z_2) - \hat{H}_2(z_1, z_2)z^{-\Delta}$
= $z^{-\Delta}\{H_2(z_1, z_2) - \hat{H}_2(z_1, z_2)\}.$ (6)

When the nonlinear response of the PAL contains only the 2nd-order nonlinearity, there is no discrepancy between the 2nd-order nonlinearity of the PAL and the 2nd-order Volterra kernel identified by the frequency response method, *i.e.* $\hat{H}_2(z_1, z_2) = H_2(z_1, z_2)$. In this case, the 2nd-order nonlinear distortion of the PAL can be completely removed by the linearization system, as implied by (6). However, when the identified 2nd-order Volterra kernel incurs errors, the performance of the linearization system is expected to be degraded.

III. NONLINEAR SYSTEM IDENTIFICATION

A. Frequency Response Method

The frequency response method computes the coefficients of Volterra kernels from the inverse Fourier transform of the respective frequency responses [20]. The frequency response of the 1st–order Volterra kernel is obtained from the ratio of the output spectrum $Y(\omega)$ and the input spectrum $X(\omega)$, which is written as

$$\hat{H}_1(\omega) = \frac{Y(\omega)}{X(\omega)}.$$
(7)

Subsequently, two sine sweep signals are applied to the PAL to measure the frequency response of the 2nd–order Volterra

kernel. The output spectrum $Y(\omega_1 + \omega_2)$ is divided by the input spectra $X(\omega_1)$ and $X(\omega_2)$, which is expressed by

$$\hat{H}_2(\omega_1, \omega_2) = \frac{Y(\omega_1 + \omega_2)}{X(\omega_1)X(\omega_2)} \frac{N}{\alpha_2},\tag{8}$$

where α_2 denotes the number of symmetries in the frequency response of the 2nd–order Volterra kernel.

B. Proposed Method

When the input amplitude of the PAL is large enough, the nonlinear response of the PAL contains more than 2nd– order nonlinearity. During the identification of the 2nd– order Volterra kernel, intermodulation frequencies resultant from high–order nonlinearity overlap in the output spectrum $Y(\omega_1 + \omega_2)$. This introduces errors in the identified 2nd–order Volterra kernel.

For instance, the 4th-order Volterra kernel is written as

$$H_4(\omega_1, \omega_2, \omega_3, \omega_4) = \frac{Y(\omega_1 + \omega_2 + \omega_3 + \omega_4)}{X(\omega_1) X(\omega_2) X(\omega_3) X(\omega_4)} \frac{N^3}{\alpha_4},$$
(9)

where $Y(\omega_1 + \omega_2 + \omega_3 + \omega_4)$ is the output spectrum; $X(\omega_1)$, $X(\omega_2)$, $X(\omega_3)$, and $X(\omega_4)$ are the input spectra; and α_4 denotes the number of symmetries in the frequency response of the 4th–order Volterra kernel.

When the four input spectra are provided by the same sinusoid wave, a part of the 4th–order intermodulation distortion is mistaken as the 2nd–order harmonic distortion, *i.e.*

$$Y(\omega + \omega + \omega - \omega) = \alpha_4 H_4(\omega, \omega, \omega, -\omega) X(\omega)^3 X(-\omega) / N^3,$$
(10)

where ω denotes the frequency of the sinusoid wave; and $\alpha_4 = 4$ in this specific case. Hence, the output spectra of the 2nd–order harmonic distortion $Y(\omega + \omega)$ and the 4th–order intermodulation distortion $Y(\omega + \omega + \omega - \omega)$ are overlapped as

$$Y(2\omega) = H_2(\omega, \omega) X(\omega)^2 / N + 4H_4(\omega, \omega, \omega, -\omega) X(\omega) X(\omega) X(\omega) X(-\omega) / N^3.$$
(11)

An improved identification method is proposed to separate the 2nd–order harmonic distortion and the 4th–order intermodulation distortion. It is assumed that when there is perturbation in the input amplitude, the change in the nonlinearity of the PAL is negligible. It is validated in the later experiment results that this assumption holds when the input amplitude is relatively large.

Two sine sweep signals with different amplitudes are input to the PAL in sequence. Based on (11), two simultaneous equations are obtained as

$$\begin{bmatrix} X(\omega)^2/N & 4X(\omega)^3 X(-\omega)/N^3 \\ X'(\omega)^2/N & 4X'(\omega)^3 X'(-\omega)/N^3 \end{bmatrix} \begin{bmatrix} H_2(\omega,\omega) \\ H_4(\omega,\omega,\omega) \\ H_4(\omega,\omega,\omega,-\omega) \end{bmatrix} = \begin{bmatrix} Y(2\omega) \\ Y'(2\omega) \end{bmatrix}, \quad (12)$$

where $X(\omega)$ and $X'(\omega)$ are the input spectra with different power; $Y(2\omega)$ and $Y'(2\omega)$ are the corresponding output spectra. The solution to the simultaneous equations in (12) leads



Fig. 3. Experiment setup.

to the separation between the 2nd–order harmonic distortion and the 4th–order intermodulation distortion. Moreover, the amplitude difference between the two sine sweep signals needs to be selected carefully. It has to be sufficiently large to ensure (12) a nonsingular problem and sufficiently small to keep the aforementioned perturbation assumption valid.

IV. EXPERIMENT RESULTS

Experiments were carried out to examine the performance of the linearization system based on Volterra filter identification upon varying input amplitudes. The input voltages of the PAL ranged from 0.05 V to 0.25 V with an interval of 0.05 V. The experiment setup was shown in Fig. 3. The PAL was placed 3.0 m from the microphone. The sampling frequency was chosen at 16000 Hz. The sine sweeps were generated from 31.25 to 2000 Hz to identify the Volterra kernels. Both the 1st–order and 2nd–order Volterra kernels were obtained by the conventional frequency response method in the first place. Then, the proposed method was carried out to refine the 2nd–order Volterra kernel. The tap length of 1st–order Volterra kernel was 512 and the dimension of the 2nd–order Volterra kernel was 512 × 512.

Identification accuracies of the frequency response method and the proposed method are compared by the performance of the linearization system. The compensation amounts for the 2nd–order harmonic distortion are plotted in Fig. 4. By using the frequency response method for Volterra filter identification, the performance of the linearization system degrades when the input amplitude becomes large, as shown in Fig. 4(f). This is because that the large input amplitude leads to strong high– order nonlinearity and yields the identified 2nd–order Volterra kernel with noticeable errors. However, when the input amplitude is small, the perturbation in the input amplitude may



Fig. 4. Experiment results of the linearization system upon different input amplitudes.

change the nonlinearity of the PAL. Therefore, the proposed method cannot lead to better performance of the linearization system when the input voltage is 0.05 V.

Moreover, Figs. 5 and 6 show the total harmonic distortion (THD) and second harmonic ratio (SHR) curves before and after compensation. The THD level and SHR are defined as

THD =
$$\sqrt{\frac{T_2^2 + T_3^2 + T_4^2}{T_1^2}} \times 100 \ [\%]$$
 (13)

and

SHR =
$$\frac{T_2}{T_1} \times 100 ~[\%],$$
 (14)

where T_i represents the amplitude of the *i*th-order harmonic component. The THD level is approximated by calculating up to the 4th-order harmonic component in this paper.

In Fig. 5, the THD level before compensation increases with the input amplitude. This is similar to the observation when the modulation index increases [21]. The proposed method is able to reduce the THD level by 10% more than the frequency response method, when the input voltage is higher than 0.05 V. But when the input amplitude is very low, the proposed method is relatively not effective. In Fig. 6, the SHR before compensation increases slower with respect to the input amplitude, as compared to the THD level before compensation in Fig. 5. This validates the existence of more than 2nd–order nonlinearity in the PAL when the input amplitude is

large. The proposed method helps the linearization system based on Volterra filter identification to reduce the 2nd–order harmonic distortion of the PAL effectively, when the high– order nonlinearity is not negligible.

V. CONCLUSION

In this paper, the 2nd-order harmonic distortion of the PAL were compensated for by the linearization system based on Volterra filter identification. When the input amplitude of the PAL was relatively large, the Volterra filter identified by the frequency response method incurred errors. This was explained by the existence of high-order nonlinearity that overlapped the 4th-order intermodulation distortion into the identified 2nd-order Volterra kernel. An improved method was therefore proposed. Experiment results demonstrated that the proposed method was very effective to deal with large input amplitudes result in large output sound pressure levels. If the nonlinear distortion can be well controlled by the proposed method, a breakthrough of the PAL is expected to make loud sound output while still achieve good sound quality.

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Fig. 5. THD of before and after compensation.

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Fig. 6. SHR of before and after compensation.

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