A New Detector of LSB Matching Steganography Based On Likelihood Ratio Test for Multivariate Gaussian Covers

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Abstract—Recently, steganalysis based on hypothesis test theory becomes a focus. However, the correlation between adjacent pixels is not exploited and this is obviously not reasonable. In this paper, a new detector for least significant bit matching (LSBM) steganography is proposed with the consideration of pixel correlation. The cover pixels are modeled by multivariate Gaussian distribution and a new detector for LSBM is derived based on likelihood ratio test. Moreover, we propose to calculate the detector by considering only the smooth image pixels and this is very helpful for improving the detection performance.

I. INTRODUCTION

Least significant bit matching (LSBM, also known as ±1 embedding) is a widely used steganographic algorithm. Moreover, compared with least significant bit replacement (LSBR) algorithm, it is harder to detect, especially in the case of low embedding rate since LSBR method exists inherent asymmetry problem which is avoided by LSBM [2].

There are many methods proposed to improve the detection performance for LSBM. In [1], Harmsen and Pearlman notice that the histogram of stego contains less high-frequency power than the histogram of the corresponding cover image, thus the center of mass (COM) of the histogram is used in detection as a feature. In [2], Ker et al. point out that the method in [1] performs quite well in cover of RGB images. To extend the method to grayscale LSBM detection, the method is improved from two aspects. The COM is computed in the downsampled version of the image and the COM of the adjacency histogram characteristic function (HCF) is defined to gain the new detector. To improve the detecting performance of the method in [2], Li et al. [3] suggest using calibration on difference image which is defined as the difference of the adjacent pixels. Later, Peng et al. [7] extend the HCF-COM to the second order difference images and choose the Gaussian filter to do the calibration. Besides HCF-COM, steganalysts observe some other features of the HCF. Instead of considering the COM, Zhang et al. [15] choose to focus on the local extrema of the histogram. Cancelli et al. in [4] extend the idea in [15] to two dimensional histogram. This kind of method is further improved by Gao et al. [5] and Cai et al. [6] to obtain a better classification.

Recently, some work [9-13] based on hypothesis test are proposed to detect the steganography. In [10], Remi et al. believe that although the use of HCF is effective, the statistical properties are evaluated just by simulation and analytically unknown. Thus in [10], an asymptotic upper-bound for the detection power of any test is given based on hypothesis test theory. In paper [10], the pixels of the cover image are assumed to be independent and satisfy the Gaussian distribution. However, this assumption is of limited interests in the real world since redundancy exists in natural images. In this light, to exploit the correlation of the adjacent pixels, a new detector for LSBM is proposed based on a more appropriate model. In our method, we first divide the image into non-lapping and equal-sized blocks. Then each block is modeled as a multivariate Gaussian distribution (MGD) which apparently contains the correlation of the pixels in a block. Using likelihood ratio test and formula derivation, a new detector for LSBM is derived. Furthermore, an improved approach of calculating the detector is suggested in this paper. Rather than simply summing up the detectors among all blocks in the image, we do the Pixel Selection (PS) by adaptively selecting the blocks whose noisiness is the smallest. The detector values in these blocks are significantly different between cover and stego. Experimental results show that our new detector outperforms the previous work [10] with a higher accuracy.

The rest of the paper is organized as follows. Some related work are introduced in Section 2. In Section 3, we show the proposed detector and PS in details. Experimental result is showed in Section 4. Finally, the conclusion is drawn in the last section.

II. RELATED WORKS

We introduce the previous work [10] briefly in this section. Let \( x = (x_1, ..., x_N) \) be a cover image with \( N \) pixels and \( y = (y_1, ..., y_N) \) be the corresponding stego image after LSBM embedding with an embedding rate \( R \). For LSBM, the cover image is first randomly scrambled and then \( RN \) pixels are selected randomly for embedding. For a to-be-embedded cover pixel \( x_n \), if the message bit \( m \) equals to the LSB of \( x_n \), then the corresponding stego pixel stays unchanged as \( y_n = x_n \). Otherwise the cover pixel is randomly added or subtracted by 1, thus \( y_n = x_n + 1 \) or \( y_n = x_n - 1 \).

Consequently, according to the embedding rate \( R \), there are
$\frac{RN}{4}$ pixels selected to be added and other $\frac{RN}{4}$ to be subtracted by 1, while other pixels remain unchanged.

In [10], the pixel $x_n$ in cover image is assumed to follow a distribution denoted as $p_{\theta_n}$, and characterized by the parameters $\theta_n = (\mu_n, \sigma_n)$ where $\mu_n$ is the mean and $\sigma_n$ is the variance of pixel $x_n$. The $p_{\theta_n}$ is supposed to satisfy the standard Gaussian distribution as:

$$p_{\theta_n}(x) = \frac{1}{\sqrt{2\pi \sigma_n^2}} \exp \left( -\frac{(x - \mu_n)^2}{2\sigma_n^2} \right).$$

Due to the impact of LSBM steganography, it can be noted that

$$\begin{cases} 
P[y_n = x_n] = 1 - R/2 \\
|P[y_n = x_n + 1] = P[y_n = x_n - 1] = R/4. 
\end{cases}$$

Hence the distribution of the pixel in stego image $q_{\theta_n}$ can be written as

$$q_{\theta_n}(x) = \frac{R}{4} (p_{\theta_n}(x - 1) + p_{\theta_n}(x + 1)) + (1 - \frac{R}{2}) p_{\theta_n}(x).$$

When analyzing the observed image $z = (z_1, \ldots, z_N)$, the goal of LSBM steganalysis is to distinguish the distribution of image $Z$ from one of the following two hypotheses: $H_0 = \{z \sim P\}$ and $H_1 = \{z \sim Q\}$, where $P$ and $Q$ represent the distributions of cover and stego image, respectively. According to the Neyman-Pearson lemma, the most powerful test is the likelihood ratio test (LRT) given by the following rule

$$\delta = \begin{cases} 
H_0, & \text{if } \Lambda(z) < \tau \\
H_1, & \text{if } \Lambda(z) \geq \tau 
\end{cases}$$

where $\Lambda$ is the likelihood ratio (LR), and $\tau$ is the threshold to make the classification decision. The LR is designed as follows

$$\Lambda(z) = \log \frac{Q(z)}{P(z)}.$$  

The advantage of using LRT is that it can be used for any detection problem where both the distributions are supposed to be provided. In [10], the pixels of images are supposed to be modeled independently by Gaussian distribution, so $P(x) = \prod_{n=1}^{N} p_{\theta_n}(x_n)$ and $Q(x) = \prod_{n=1}^{N} q_{\theta_n}(x_n)$. Based on Eq. (3) and the distributions of both cover and stego, LR can then be expressed as

$$\Lambda(z) = \sum_{n=1}^{N} \Lambda_n(z_n),$$

where

$$\Lambda_n(z_n) = \log \frac{q_{\theta_n}(z_n)}{p_{\theta_n}(z_n)}.$$  

With Eq. (1) and Eq. (2), the LR of each pixel (4) can be written as

$$\Lambda_n(z_n) = \log \left( \frac{R}{4} \frac{p_{\theta_n}(z_n - 1) + p_{\theta_n}(z_n + 1)}{p_{\theta_n}(z_n)} + (1 - \frac{R}{2}) \right)$$

$$= \log \left( 1 + \frac{R}{2} \frac{(p_{\theta_n}(z_n - 1) + p_{\theta_n}(z_n + 1) - 2p_{\theta_n}(z_n))}{2p_{\theta_n}(z_n)} \right).$$

Here we can note that $\Lambda_n(z_n)$ depends mainly on the form

$$\frac{1}{2} \frac{p_{\theta_n}(z_n - 1) + p_{\theta_n}(z_n + 1)}{p_{\theta_n}(z_n)}.$$

From Eq. (1), we have

$$\frac{p_{\theta_n}(z_n - 1)}{p_{\theta_n}(z_n)} = \exp(-\frac{1}{2\sigma_n^2}(\frac{z_n - \mu_n}{\sigma_n^2} + o(\sigma_n^{-2}))$$

and

$$\frac{p_{\theta_n}(z_n + 1)}{p_{\theta_n}(z_n)} = \exp(-\frac{1}{2\sigma_n^2}(\frac{\mu_n - z_n}{\sigma_n^2} + o(\sigma_n^{-2})).$$

Thus, the Eq. (5) can be written as

$$\frac{1}{2} \exp(-\frac{1}{2\sigma_n^2}(\frac{z_n - \mu_n}{\sigma_n^2} + \exp(\frac{\mu_n - z_n}{\sigma_n^2})).$$

It’s easy to observe that

$$\frac{1}{2} \frac{p_{\theta_n}(z_n - 1) + p_{\theta_n}(z_n + 1)}{p_{\theta_n}(z_n)} - 1 \rightarrow 0.$$

Hence, the Taylor expansion of function $\log(1 + x)$ can be used and accompany with the Taylor expansion of $e^x$, LR can be written as

$$\Lambda_n(z_n) = \frac{R}{4\sigma_n^4} \left( \frac{(z_n - \mu_n)^2 - \sigma_n^2 + \frac{1}{4}}{4} \right) - \frac{R^2}{32\sigma_n^4} + o(\sigma_n^{-4}).$$

Here, the embedding rate $R$ is just assumed to be known. Finally, the ultimate detector is computed as follow

$$\Lambda_n(z_n) \approx \frac{(z_n - \mu_n)^2}{\sigma_n^4}.$$  

where the parameters will be discussed in detail in the following section.

### III. Proposed Method

Although the use of LRT makes the detector quite effective, the independent Gaussian distribution model is unreasonable to some extend. It apparently ignores the redundancy existed in natural images. Based on this, we decide to use the MGD model which considers the correlation of adjacent pixels. In the case of MGD, the cover image $x = (x_1, \ldots, x_N)$ is divided into $K$ blocks $\{x_1, \ldots, x_K\}$, each block is of length $S$, and $K = N/S$. Each block follows a distribution as

$$P_k(x_k) = \frac{1}{C_k} \exp(-x_k^T \Sigma_k^{-1} x_k)$$

in which $\Sigma_k$ is the covariance matrix of the image block and $C_k$ is the normalization constant. Due to the property of LSBM steganography, the distribution of the corresponding stego image block $x_k$ can be expressed as

$$Q_k(x_k) = P_k \otimes f(x_k) = \int P_k(x_k + M)f(M)\,dM$$

where $M = (M_1, \ldots, M_S)$ is the splitted messages of size $S$. The corresponding stego block can be written as $x_k + M$. 


The function $f$ is dependent of the embedding rate $R$, where
$$f(M) = \sum_{j=1}^{\infty} \rho_j(M_j)$$
and
$$\rho_j(M_j) = \begin{cases} 1 - 2\beta_j, & \text{if } M_j = 0 \\ \beta_j, & \text{if } M_j = \pm 1 \end{cases}$$
where for each pixel $x, \beta_j = R/4$. Then, we can describe LR as
$$\Lambda(x) = \sum_{k=1}^{K} \Lambda_k(x_k) = \sum_{k=1}^{K} \frac{Q_k(x_k)}{P_k(x_k)} = \sum_{k=1}^{K} \frac{f(M)P_k(x_k + M)}{P_k(x_k)},$$
As the cover model satisfies MGD, based on (7) we can learn that
$$P_k(x_k + M) = e^{-(x_k + M)^T \Sigma_k^{-1} (x_k + M) + x_k^T \Sigma_k^{-1} x_k}$$
then $\Lambda(x)$ can be written as
$$\Lambda(x) = \sum_{k=1}^{K} f(M)e^{-2M^T \Sigma_k^{-1} x_k - M^T \Sigma_k^{-1} M}. \tag{8}$$
Now, for each block $x_k$, we will discuss Eq. (8) under three different situations. When $M = 0$,
$$\Lambda_k(x_k) = f(0) = \sum_{j=1}^{S} (1 - 2\beta_j)$$
It is easy to find that this value only depends on the embedding rate. When $M = \pm e_i$,\n$$\Lambda_k(x_k) = \beta_i \prod_{j \neq i} (1 - 2\beta_j)e^{\pm 2 \sum_{j=1}^{S} a_{ij} x_j - a_{ii}} \tag{9}$$
in which $a_{ij}$ is the element of the matrix $\Sigma_k^{-1}$. If there are at least two nonzero values in $M$, then $\Lambda(x_k) = o(|\beta|^2)$. Notice that, we do not consider higher order terms in our method.

After the discussion, we find the second situation needs to be researched precisely. Due to Eq. (9), we can get that,
$$\Lambda_k(x_k) = 1 - 2 \sum_i \beta_i + \sum_i \beta_i (e^{-2 \sum a_{ij} x_j - a_{ii}} + o(|\beta|))$$
Ignoring the higher order term which is not considered in our method
$$\Lambda_k(x_k) = 1 + \sum_i \beta_i (e^{-2 \sum a_{ij} x_j - a_{ii}} + \sum a_{ij} x_j - a_{ii} - 2) \tag{10}$$
Using the Taylor expansion of $e^x$,
$$e^{-2 \sum a_{ij} x_j - a_{ii}} + e^{2 \sum a_{ij} x_j - a_{ii}} - 2 \text{ can be replaced by}$$
$$-2a_{ii} + \left(2 \sum a_{ij} x_j + a_{ii}\right)^2 + \left(2 \sum a_{ij} x_j - a_{ii}\right)^2$$
Then, simplifying Eq. (10) we can derive that
$$\Lambda_k(x_k) \approx 1 + \sum_i \beta_i \left(a_{ii}^2 - 2a_{ii} + 4\left(\sum a_{ij} x_j\right)^2\right)$$
Finally, assuming the embedding rate is known by the steganalyst,
$$\Lambda \propto \sum_{i,j} (a_{ij} (x_j - \mu_j))^2$$
Here we rewrite our proposed detector as $D$ for better understanding. It can be finally computed as
$$D = \sum_{k=1}^{K} D_k = \sum_{k=1}^{K} (x_k - \mu_k)^T (\Sigma_k^{-1})^2 (x_k - \mu_k) \tag{11}$$
where $D_k$ is the detector for $k$-th block, $\mu_k = (\mu_{k1}^2, \cdots, \mu_{kS}^2)$ is the mean of block $x_k$.

As we know in steganography, embedding efficiency and embedding distortion are two important measurements. Messages is preferred to be embedded in noisy areas, which are full of textures. On the contrary, in steganalysis, detectors in flat areas perform more effective. This reminds us that the sum of detectors can appear to be far more different among the pixels in flat areas between cover and stego. Inspired by this idea, in this paper, in order to improve the performance of the detector and do a better classification, we utilize the PS operation to enlarge the distinction in detectors between cover and stego which means to select the pixels in the flattest areas to get the optimized detector. Specifically, rather than simply summing up the detectors of all blocks like (11), the detector $D$ is summed up only by $B$ blocks whose noisiness is the smallest among all blocks of the image. We first calculate the noisiness set $V = \{v_1, \cdots, v_K\}$ where $v_k$ is the $k$-th block’s noisiness. For each block noisiness, it is measured by the sum of the noisiness all pixels in the block, so $v_k = \sum_{x_i=1}^{S} v_i$, where $v_i$ means the noisiness of pixel $x_i$. Then we pick up the first $B$ blocks whose noisiness is the smallest and get an optimized block set $V$. Finally, the optimized detector $\hat{D}$ is derived as
$$\hat{D} = \sum_{k=1}^{B} D_k, v_k \in \hat{V} \tag{12}$$
We now discuss how to determine the parameters in this paper and the first one is the block size $S$. For $S \geq 3$, the experimental result performs not quite well. We think the reason is that the block size is too large to contain the correlation and hence it affects the precision. On the other hand, large block size brings difficulty in estimating the inverse matrix $\Sigma_k^{-1}$. We finally choose to set $S = 2$ for above reasons. Then the observed image $z$ is divided into $N/2$ non-overlapping blocks $\{z_1, \cdots, z_{2K}\}$ with $z_k = (z_{2k-1}, z_{2k})$. The two dimensional inverse matrix $\Sigma_k^{-1}$ of the covariance matrix $\Sigma_k$ is measured as
$$\Sigma_k^{-1} = \frac{1}{\Delta_k} \begin{pmatrix} \sigma_{2k}^2 & -\sigma_{2k-1,2k} \\ -\sigma_{2k-1,2k} & \sigma_{2k-1,2k} \end{pmatrix}$$
where $\sigma_k^2$ is the variance of the $k$-th pixel, $\sigma_{2k,j}$ is the covariance between the pixel $x_j$ and $x_j$ and $\Delta_k$ is the determinant of $\Sigma_k$. Here are some parameters need to be estimated, we adopt the same way as [14] do for effectiveness and convenience in further comparison. Specifically, the variance of the $i$-th
The value of detector in these areas is more different between cover and stego and it appears to perform better. However, more improvements can be done in the future work. The way to estimate the mean and variance of a pixel (13) can be improved, the samplings of current pixels in a small surrounding area can be used to get a good estimation. Besides, we only propose a one-dimension detector, and it can be expanded in different directions of the image in the future to derive a high dimensional detector.

REFERENCES