A spectrum sensing method based on fractional lower order moments in weakly correlated Laplace noise

Yaping Bao
College of Electronics
and Information Engineering,
Nanjing Tech University
Nanjing, China 211816

Yingdong Zhu
College of Electronics
and Information Engineering,
Nanjing Tech University
Nanjing, China 211816

Xiaomei Zhu
Nanjing Tech University
Nanjing university of posts
and telecommunications,
Nanjing, China 211816

Yuzhi Chu
Nanjing Tech University
Nanjing university of posts
and telecommunications,
Nanjing, China 211816

***Abstract*—In the actual wireless communication systems, non-Gaussian noise often has a negative effect on the signals which cognitive users finally receive. In addition, due to the high frequency of collecting samples, there may be a certain correlation among the noise components. A detection method based on fractional lower order moments (FLOM) is applied in this paper to solve spectrum sensing in weakly correlated Laplace distributed noise environment. Different from the traditional detector, this detector does not require the priori knowledge of the primary user signal, noise and communication channels. By computer simulating, the detection performance versus the signal-to-noise ratio, the scale parameter b, the order p and the correlation parameter τ are studied both in non-fading communication channel and Rayleigh fading communication channel in this paper with the comparison to the traditional energy detector. Simulation results show that, in the weakly correlated Laplace distributed noise environment, the FLOM-based detector has better performance than the traditional energy detector.**

I. Introduction

Wireless spectrum is the key to achieve wireless communication. In recent years, with the rapid development of wireless communication, the demand for the wireless spectrum is growing. However, the wireless spectrum resources cannot be utilized efficiently due to the traditional fixed allocation principle [1]. Therefore, cognitive radio (CR) is proposed to alleviate the shortage of radio spectrum resources and improve the utilization rate of spectrum [2][3].

Spectrum sensing is the most critical technology in cognitive radios. Cognitive users (CU) detect the spectrum holes by observing the change of the communication environment intelligently. Cognitive users can access to use the spectrum as soon as they find the spectrum holes. However, once the CUs detect the existence of the primary users(PU) again, they must quit within a specified time to avoid communication interference to the primary users.

In the actual wireless communication systems, non-Gaussian noise often has a negative effect on the signals which cognitive users finally receive. Besides, the noise components could be correlated when the sampling frequency gets higher and higher. In this case, the detection performance of the traditional signal detectors (such as energy detector, which is based on second order statistics) will be degraded or even failed. In the literature [4], the locally optimal detector was put forward to detect the PU signal in weakly correlated noise. But this detector needs to require the noise distribution which is usually difficult to be known in advance. The optimal detector based on generalized likelihood ratio test was proposed in the literature [5] to detect the PU signal in non-Gaussian noise. However, this detector needs to make maximum likelihood estimation both in both H_0 and H_1 hypotheses which requires a great deal of complex calculation. Compared to the GLRT detector, Rao test based detector in the literature [6] only needs to estimate unknown parameters in H_0 hypothesis which means less calculation. However, these two detectors only assume all the noise is independent. Thus, it is significant and valuable to research how to detect primary users efficiently without suffering from communication interference in weakly correlated non-Gaussian noise environment.

In this paper, a statistical method based on fractional lower order moments (FLOM) is applied to solve spectrum sensing in weakly correlated non-Gaussian noise environment. One of the highlighted advantages of FLOM detector is that there is no need to obtain a priori knowledge of the primary user signal, the noise and the communication channel. FLOM detector was proposed in the literature [7] to solve spectrum sensing in α-stable distributed noise. But it was limited to independent α-stable distributed noise. According to numerous simulations, we find FLOM detector can also achieve good detection performance in other uncorrelated or weakly correlated Non-Gaussian noise. Non-Gaussian noise is usually modeled as generalized Gaussian distribution (GGD), mixed Gaussian distribution (MGD) and alpha stable distribution, etc. In this paper, the non-Gaussian noise is modeled as Laplace distribution which is one of the most typical distribution in GGD. That is because this distribution can simulate the actual noises with different degrees of tailing by adjusting the scale parameter values. Correlated noise is usually modeled as φ-mixed noise model [8], m-correlated noise model [9] and transformed noise model [10]. When the noise components are weakly correlated, independent random process of first order moving average (First-Order Moving Average, FOMA) value is often used to simulate the weak correlation noise model [11][12].

The reminder of this paper is organized as follows: in section II, the system model is introduced. The FLOM statistic is derived under weakly correlated Laplace noises in section III. Simulation results and performance analysis are presented in section IV. At last, conclusions of the paper are shown in section V.

II. Problem Formulation

A. System Model

Here we assume that a cognitive radio network is composed of a primary user, M cognitive users and a fusion center. In the wireless channel, each cognitive user detects the primary user signal within a specified time interval. Then spectrum sensing problem can be formulated into the two hypotheses test model: H_0 and H_1. H_0 represents there is no primary user and H_1 represents the primary user exists. The length of samples is N under two hypotheses. The spectrum sensing model is as follows:

$$\begin{cases} H_0: & z_m(n) = w_m(n) \\ H_1: & z_m(n) = h_m s(n) + w_m(n) \end{cases} \tag{1}$$

In equation (1), $z_m(n)$ is the observed value of the m-th cognitive user at the n-th moment. $w_m(n)$ is background non-Gaussian noise under two hypotheses. And in the paper, $w_m(n)$ is modeled as weakly correlated Laplace distribution. $s(n)$ is the PU signal at n-th moment with zero mean and variance $\sigma_s^2 = E[|s(n)|^2]$. h_m is the channel gain under both non-fading channel and Rayleigh fading channel. h_m is equal to one under non-fading channel. Under Rayleigh fading channel, the mean of h_m is zero, and the variance is $\sigma_{h_m}^2 = E[|h_m|^2]$. The probability density function (PDF) of Rayleigh distribution h_m is

$$f(h_m) = \frac{h_m}{\sigma_{h_m}^2} * e^{-\frac{h_m}{2\sigma_{h_m}^2}}. \tag{2}$$

III. Spectrum sensing based on FLOM under weakly correlated Laplace noises

A. Noise model

1) weak correlation model: In the paper, independent random process of first order moving average (First-Order Moving Average, FOMA) value is used to simulate the weak correlation noise model. The noise sequence $\{w(n)\}_{n=1}^{N}$ is denoted as follows:

$$\begin{cases} w(1) = e(1) \\ w(n) = e(n) + \tau e(n-1) \ for \ n \in [2, N] \end{cases} \tag{3}$$

In equation (3), $e(n)$ is subjected to independent Laplace distribution, τ is the correlation coefficient and $|\tau| < 1$.

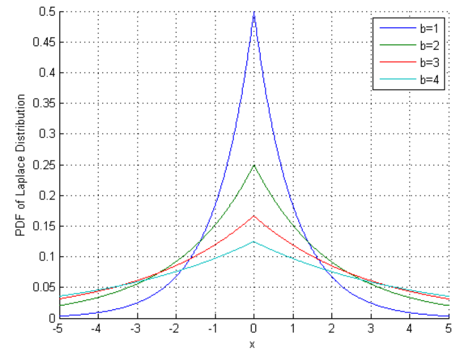

Fig. 1. PDF of Laplace Distribution with different scale parameter b and $\mu = 0$

2) Laplace distribution model: Laplace distribution is a continuous probability distribution and its probability density function is expressed by the absolute value of the difference with respect to the mean value. The PDF of the random variable $e(n)$ in (3) can be expressed as

$$f(e(n)|\mu, b) = \frac{1}{2b} e^{-\frac{|e(n)-\mu|}{b}} \tag{4}$$

In equation (4), μ is the location parameter, b is the scale parameter and its value is always positive. Fig. 1 is the PDF figure of the Laplace distribution. It can be seen that by varying the scale parameter b, different tail behaviours can be obtained. The higher the value of b is, the more slowly the tail decays when the location parameter is fixed. Therefore, with the heavy tail, the Laplace can be used to fit the non-Gaussian noise in practical CR system. In non-Gaussian noise, spectrum sensing needs to account for these large magnitude samples with heavy tail to reduce the probability of false alarm.

A good detector for weakly correlated non-Gaussian noise usually utilizes the nonlinear method to reduce the noise spikes. As will be seen below for FLOM-based detector.

B. Spectrum sensing based on FLOM

According to the characteristics of Laplace distribution, the traditional Lp paradigm algorithm [13] is extended to fractional lower order moments, and the method based on FLOM is studied. M cognitive users transmit the received signals to the fusion center, and the fusion center based on the fractional lower order

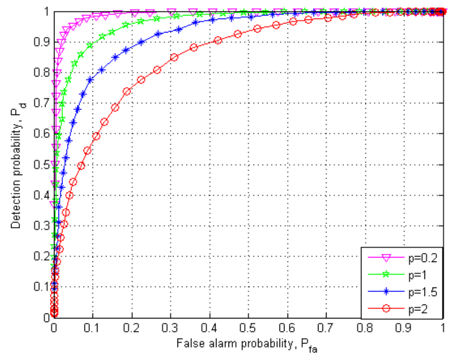

Fig. 2. ROC curve with different orders p under non-fading channel

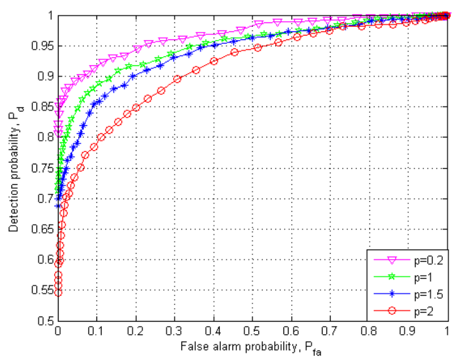

Fig. 3. ROC curve with different orders p under Rayleigh fading channel

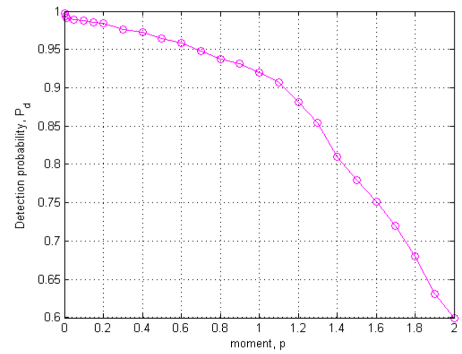

Fig. 4. Diagram of different detection probabilities with different orders p under non-fading channel

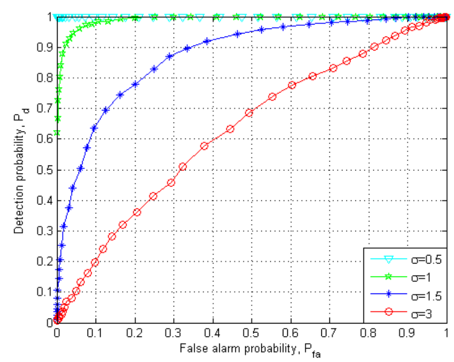

Fig. 5. ROC curve with different scale parameters b under non-fading channel

moments obtains the suboptimal perceptual statistics as follows:

$$Y_{flom} = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} |z_m(n)|^{p(m)} \; for \; p \in (0,2) \tag{5}$$

It can be seen from (5) that FLOM statistics only needs the observed signal $z_m(n)$ and the order p without the knowledge of the primary user signal or the non-Gaussian noise PDF. Therefore, it can effectively achieve spectrum sensing under weakly correlated Laplace noise with unknown prior knowledge of primary user signal and noise. When the value of Y_{flom} is calculated, we compare Y_{flom} with the specific threshold η to determine whether the primary user exists. If $Y_{flom} > \eta$, it means the PU signal exists. If $Y_{flom} < \eta$, it means the PU signal does not exist.

IV. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

This section is the simulation results under both non-fading channel and Rayleigh fading channel. In following simulations, the length of observed signal sequence is 500 and Monte Carlo runs 1000 times.

Fig. 2 and Fig. 3 are the diagrams of the detection probability and false alarm probability with different orders p ($p = 0.2, 1, 1.5, 2$) with the condition of $SNR = -8dB$, $b = 1/\sqrt{2}$, $\tau = 0.1$, $M = 1$ under both non-fading channel and Rayleigh fading channel. It can be seen that when the order p decreases, the detection probability increases, which means the detection performance gets better. In particular, when p is equal to 2, it is the traditional energy detector. It is clear from figures that the detection performance of FLOM detector is better than that of the traditional energy detector. For example, in Fig. 2, the detection probability of FLOM detector is approximately 98% when the false alarm probability is equal to 0.1 and the order p is equal to 0.2. However, when the false alarm probability is equal to 0.1 and the order p is equal to 2, the detection probability of energy detector is approximately 60%. Notice that for FLOM detector, the range of p is (0,2). We also do the simulations when p is smaller than 0.2, as shown in Fig. 4. From the simulation results, we find that the detection performance gets much better if the value of p is more close to 0. Here the order cannot be 0 because when p is equal to 0, the value of Y_{flom} is equal to 1, which means the detector will be failed.

Fig. 5 and Fig. 6 are the diagrams of the detection probability and false alarm probability with different

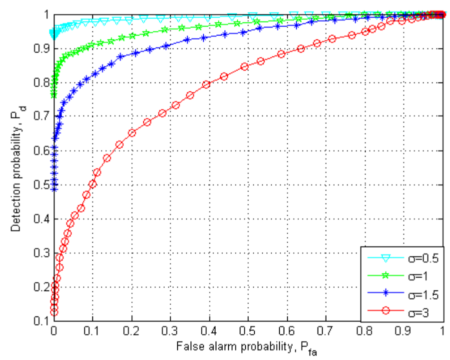

Fig. 6. ROC curve with different scale parameters b under Rayleigh fading channel

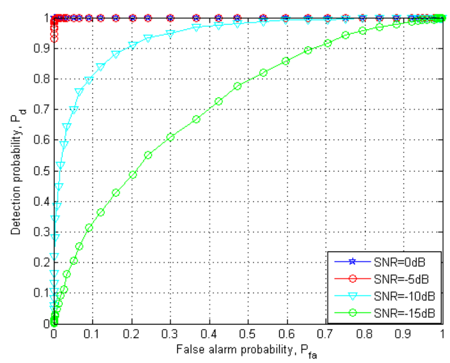

Fig. 7. ROC curve with different SNR under non-fading channel

scale parameters b ($b = 0.5/\sqrt{2}$, $1/\sqrt{2}$, $1.5/\sqrt{2}$, $3/\sqrt{2}$) with the condition of $SNR = -8dB$, $p = 0.4$, $\tau = 0.1$, $M = 1$ under both non-fading channel and Rayleigh fading channel. Here, σ is standard deviation and the relation between the scale parameter and the standard deviation is that $b = \sigma/\sqrt{2}$. We can see that the higher the scale parameter b, the lower the detection probability. That is because when the scale parameter b increases, the tailing of the Laplace noise gets heavier, which will be more likely to cause false alarm and lead to the decline in detection performance.

Fig. 7 and Fig. 8 are the diagrams of the detection probability and false alarm probability with different SNR ($SNR = 0dB, -5dB, -10dB, -15dB$) with the condition of $b = 1/\sqrt{2}$, $p = 0.4$, $\tau = 0.1$, $M = 1$ under both non-fading channel and Rayleigh fading channel. As can be seen from figures, with the decrease of the SNR, the detection performance is declining. In Fig. 7, when the false alarm probability is equal to 0.1, $SNR = -5dB$, the corresponding detection probability is 100%, but when the false alarm probability is equal to 0.1, the signal to noise ratio of $SNR = -15dB$, the corresponding detection probability is 38%, the detection probability is decreased by about 60%. This is because when the SNR is lower, the noise environment gets worse, which means it is more difficult to detect

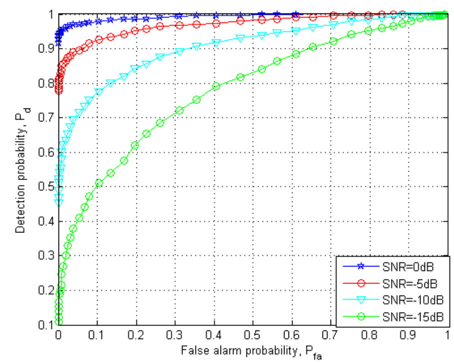

Fig. 8. ROC curve with different SNR under Rayleigh fading channel

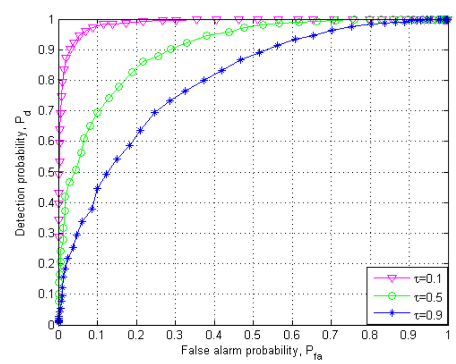

Fig. 9. ROC curve with different correlation parameters under non-fading channel

primary user signal.

Fig. 9 and Fig. 10 are the diagrams of the detection probability and false alarm probability with different correlation parameters τ (τ=0.1, 0.5, 0.9) with the condition of $b = 1/\sqrt{2}$, $p = 0.4$, $SNR = -8dB$, $M = 1$ under both non-fading channel and Rayleigh fading channel. As can be seen from figures, when the

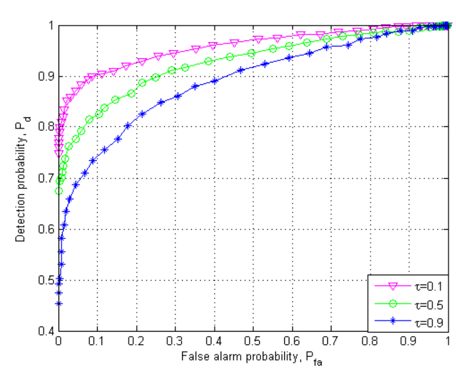

Fig. 10. ROC curve with different correlation parameters under Rayleigh fading channel

correlation coefficient is smaller, the detection performance based on fractional lower order moment detector gets better.

V. Conclusion

In this paper, a new algorithm based on FLOM is used to detect the primary user signal under weakly correlated Laplace noise. The detector based on fractional lower order moments does not require a priori knowledge about the primary user signal, noises and communication channels. We investigate the detection performance with different parameters (SNR, the moment p, scale parameter, correlation coefficient) under both non-fading channel and Rayleigh fading channel by Monte Carlo simulation. Simulation results show that, whether it is under non-fading channel or Rayleigh fading channel, the detection performance of the detector based on fractional lower order moments is significantly higher than the conventional energy detector in weakly correlated Laplace noise environment.

VI. Acknowledgment

This work was supported by the National Natural Science Foundation of China under Grant 61501223 and the Open research foundation of key lab of broadband wireless communication and sensor network technology (Nanjing university of posts and telecommunications) under Grant NYKL201505.

References

[1] Wang Yingwei, "Research on the technology of cooperative spectrum sharing in cognitive radio [D],"Beijing: Beijing University of Posts and Telecommunications, 2010.

[2] Zheng Baoyu, Wang Lei, Li Lei, "Cooperative spectrum sensing based on random matrix theory [J]," *Electronic and information technology*, 2009, 31 (8):1925-1929.

[3] J. Mitola, III and G. Maguire, Jr, Cognitive radio: making software radios more personal, *IEEE Personal Communications*,vol.6, no.4, pp.13 C18, Aug.1999.

[4] F. Shayegh and F. Labeau, "On signal detection in the presence of weakly correlated noise over fading channels," *IEEE Trans. on Communications*,vol. 62, no. 3, pp.797C809, Mar.2014.

[5] K. Hyun Gu, S. Iickho, Y. Seokho, and K. Yun Hee, "A Class of Spectrum-Sensing Schemes for Cognitive Radio Under Impulsive Noise Circumstances: Structure and Performance in Nonfading and Fading Environments," *Vehicular Technology, IEEE Transactions on*,vol.59, pp.4322-4339, 2011.

[6] Xiaomei Zhu,Champagne Benoit and Weiping Zhu, "Rao test based cooperative spectrum sensing for cognitive radios in non-Gaussian noise," *Signal Processing*,vol.97,pp.183-194, Apr.2014.

[7] Xiaomei Zhu,Champagne Benoit and Weiping Zhu, "Spectrum sensing based on fractional lower order moments for cognitive radios in alpha-stable distributed noise," *Signal Processing*,vol.111, pp.94-105, Jun.2015.

[8] Cheung,J.,Kurz, L, "Asymptotically optimum finite-memory detectors in φ-mixing dependent processes," *IEEE Transactions on Signal Processing*, 1994, 42(9):2344-2354.

[9] Kokkinos, EA, Maras, AM, "Narrowband incoherent threshold detection in non-additive Markov noise," *Signal Processing*, Jan 1999, 72(1):39-45.

[10] Moon,J,Park,J, "Pattern-dependent noise prediction in signal-dependent noise," *IEEE Journal on Selected Areas in Communications*, Apr 2001, 19(4):730-743.

[11] Moustakides G V and Thomas J B, "Min-max detection of weak signals in φ-mixing noise," *IEEE Transactions on Information Theory*, 1984, 30(3):529-537.

[12] Yang X, Poor H V and Petropulu A P, "Memoryless discrete-time signal detection in long-range dependent noise," *IEEE Transactions on Signal Processing*, 2004, 52(6):1607-1619.

[13] F. Moghimi, A. Nasri and R. Schober, "Adaptive Lp-Norm Spectrum Sensing for Cognitive Radio Networks," *IEEE Transactions on Communications*, vol.59, pp.1934-1945, 2011.