Fast NLMF-Type Algorithms for Adaptive Sparse System Identifications

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Abstract—**Adaptive sparse system identification (ASIDE) techniques have been successfully applied in many applications, such as sparse channel estimation and radar target detection. Normalized least mean fourth (NLMF)-type algorithms are considered as one of the stable ASIDE techniques even at low signal-to-noise ratio (SNR). However, the convergence capability of sparse NLMF algorithms is severely decreased by initial mean square error (MSE) and input variance in the high SNR regimes. To improve the convergence speed of the sparse NLMF algorithms in all SNR regions, in this paper, we propose a kind of non-constraint fast sparse NLMF-type algorithms for applying in ASIDE. Unlike the conventional methods, the proposed algorithms provides an alternative way to get rid of the restriction of SNR-dependent initial MSE and input variance. The proposed fast sparse NLMF-type algorithms are validated via computer simulations.**

Keywords—Normalized least mean fourth (NLMF), adaptive sparse system identification (ASIDE), fast convergence speed, sparse constraints.

I. INTRODUCTION

A. Background and motivation

Adaptive system identification, as shown in Fig. 1, has been applied in many applications, such as channel estimation [1] and echo cancellation [2]. Least mean square (LMS) algorithm is one of popular identification methods. Due to its sensitivity to random scaling of input signal, normalized least mean square (NLMS) was also proposed to improve identification performance [3]. In many scenarios, finite impulse response (FIR) vector of unknown system are often modelled as sparse, consisting of only a few nonzero coefficients. A typical example of the sparse system is shown in Fig. 2, the length of the system is 256 while the number of nonzero coefficients is only 4. Taking advantage of such sparse prior information can improve the system identification performance [1], where sparse penalty functions are introduced to standard NLMS to exploit system sparsity. However, NLMS-type algorithm cannot achieve high identification performance, especially in the low signal-to-noise ratio (SNR) case.

Normalized least mean fourth (NLMF) algorithm [4] outperforms the NLMS algorithm [3] in achieving a better steady-state performances especially in the low SNR scenario. Recent years, sparse NLMF algorithms have been intensely investigated. By introducing different sparse penalty functions, i.e., zero-attracting (ZA), reweighted zero-attracting (RZA), reweighted ℓ_1-norm (RL1), ℓ_p-norm (LP), and ℓ_0-norm (L0), various sparse NLMF-type algorithms have been proposed [5][6]. These algorithms are often termed as ZA-NLMF, RZA-NLMF, RL1-NLMF, LP-NLMF and L0-NLMF, respectively. Without loss of generality, two typical algorithms, i.e., ZA-NLMF and RZA-NLMF, are considered in throughout paper as for benchmarks.

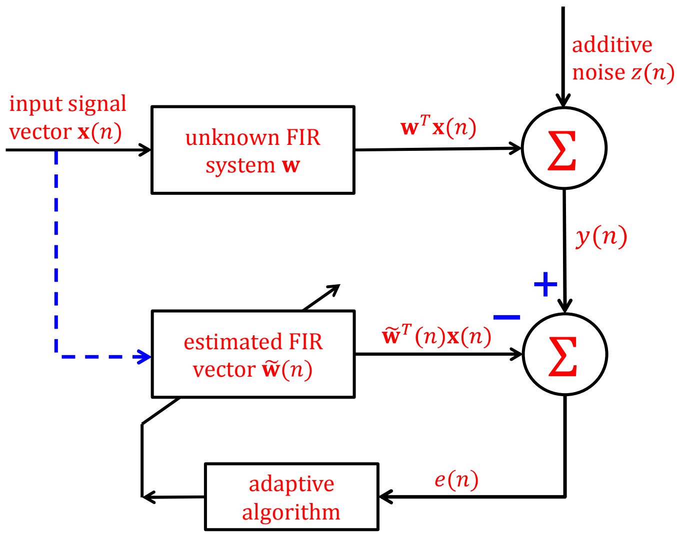

Fig. 1. Framework of adaptive system identification technique.

In real system identification scenarios, both steady-state performance and convergence speed of the ASIDE techniques are two important evaluation criteria. However, the convergence speed of the conventional NLMF-type algorithms is constrained by many factors, i.e., *initial step-size*, *input signal power*, *noise power* and *initial estimation error* [7]. In the low SNR case, the NLMF-type algorithms can perform fast and also can achieve better estimation performance than NLMS-type algorithms. In the high SNR case, however, the convergence speed of the NLMS-type algorithms is very slow. We can find that the conventional NLMF-type algorithms cannot be applied in all SNR cases. Hence, it is necessary to develop new NLMF-type algorithms so that they can perform fast convergence speed in all SNR but without sacrificing steady-state MSE performance.

B. Main Contributions

In this paper, we propose a fast sparse NLMF-type algorithms, termed as fast zero-attracting NLMF (ZA-NLMF) and fast reweighted zero-attracting NLMF (RZA-NLMF), which provide better steady-state performance and faster convergence speed than the conventional NLMF-type algorithms. Specifically, we first review the conventional algorithms and point out the reason of slow convergence speed. By virtue of the approximation theory, the proposed sparse algorithm is effective for exploiting system sparsity and stable for all statistics of input signal, noise, and initial setting of the algorithm. Finally, the proposed algorithms are evaluated by the Monte Carlo simulations against SNR and system sparsity.

The rest of the paper is organized as follows. A system model is described and problem formulation is introduced in Section II. In Section III, fast NLMF-type algorithms are proposed for ASIDE. Computer experiments are conducted in Section IV in order to verify the effectiveness of proposed algorithm for ASIDE. Finally, we conclude the paper in Section V.

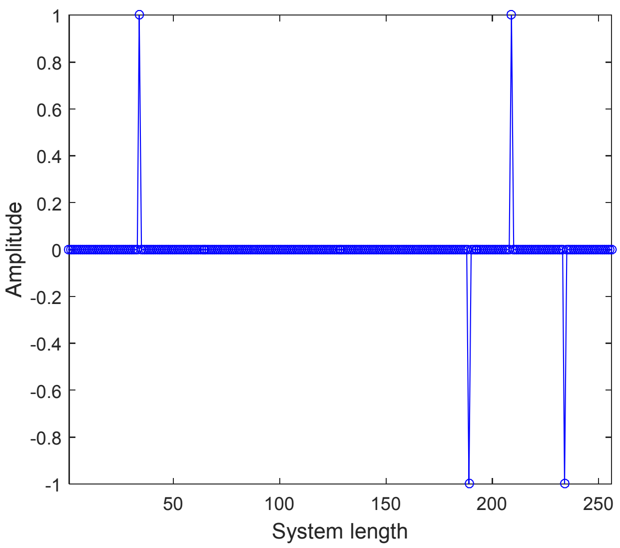

Fig. 2. A typical example of sparse FIR system, where the length is 256 while the number of nonzero coefficient is 4.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Considering an unknown sparse system, its FIR vector is described by N-length $\mathbf{w}=[w_1,w_2,\ldots,w_N]^T$, where the number of nonzero coefficient is K ($K \ll N$), and $(\cdot)^T$ is transpose operator. Assume that an input signal $\mathbf{x}(n)$ is input to the system, then the system output signal $y(n)$ is given by

$$y(n)=\mathbf{w}^T\mathbf{x}(n)+z(n), \tag{1}$$

where $\mathbf{x}(n)=[x(n),x(n-1),\cdots,x(n-N+1)]^T$ denotes the N-length vector of input signal $x(n)$, and $z(n)$ is the observation additive Gaussian noise assumed to be independent of $\mathbf{x}(n)$. The objective of the ASIDE is to adaptively estimate the unknown FIR vector $\mathbf{w}$ using the input signal $\mathbf{x}(n)$ and the system output signal $y(n)$. One of effective identification techniques is adopting the standard NLMF algorithm [4]. The system identification error is defined as

$$e(n)=y(n)-\tilde{\mathbf{w}}^T(n)\mathbf{x}(n), \tag{2}$$

where $\tilde{\mathbf{w}}(n)=[\tilde{w}_0(n),\tilde{w}_1(n),\cdots,\tilde{w}_{N-1}(n)]^T$ denotes n-th filter updating weight vector. To enable mathematical tractability of the performance analysis, throughout the paper, we resort to the following independent assumptions.

- ***Assumption A1**: The training sequence $\mathbf{x}(n)$ and additive noise $z(n)$ are mutually independent.*
- ***Assumption A2**: The training sequence $\mathbf{x}(n)$ satisfies independent and identically distributed (i.i.d.) Gaussian distribution with finite variance σ_x^2.*
- ***Assumption A3**: Additive Gaussian noise $z(n)$ satisfies an i.i.d. with zero and variance σ_n^2.*

Letting $\mathcal{S}(\tilde{\mathbf{w}}(n))$ denotes a sparsity-aware function which can take advantage of channel sparsity. By combining the fourth-order error criterion (i.e., $e^4(n)$) and sparsity-aware function i.e., $\mathcal{S}(\tilde{\mathbf{w}}(n))$, the cost function of sparse NLMF algorithms [10], [11] can be devised as

$$G\left(\tilde{\mathbf{w}}(n)\right)=\frac{1}{4}e^4(n)+\lambda\mathcal{S}\left(\tilde{\mathbf{w}}(n)\right), \tag{3}$$

where λ denotes a nonnegative regularization parameter which can adjust the estimation error and sparse penalty strength. According to (3), the corresponding update equation of sparse NLMF algorithms can be derived as

$$\begin{aligned}\tilde{\mathbf{w}}(n+1)&=\tilde{\mathbf{w}}(n)+\mu\frac{e^3(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2\left(\|\mathbf{x}(n)\|_2^2+e^2(n)\right)}\\&\quad-\mu\lambda\nabla_{\mathbf{w}}\mathcal{S}\left(\tilde{\mathbf{w}}(n)\right)\\&=\tilde{\mathbf{w}}(n)+\mu(n)e(n)\mathbf{x}(n)-\rho\nabla_{\mathbf{w}}\mathcal{S}\left(\tilde{\mathbf{w}}(n)\right),\end{aligned} \tag{4}$$

where $\rho=\mu\lambda$ stands for the sparse constraint parameter and $\mu(n)$ denotes a variable step-size as

$$\mu(n)=\frac{\mu e^2(n)}{\|\mathbf{x}(n)\|_2^2\left(\|\mathbf{x}(n)\|_2^2+e^2(n)\right)}. \tag{5}$$

According to (5), we can find that the step $\mu(n)$ depends on four parameters: initial step-size μ , SNR, initial estimation error $e^2(n)$, and input signal $\|\mathbf{x}(n)\|_2^2$. In the low SNR case, the big $e^2(n)$ promotes the big $\mu(n)$ which can ensure the fast tacking speed during gradient descend. In the high SNR case, the small initial error $e^2(n)$ leads to small $\mu(n)$ which causes the very slow convergence speed. In addition, calculating the (8) will consume high computational complexity. Considering above two limitations, the conventional NLMF-type algorithms are inappropriately applied in all SNR scenarios for ASIDE.

III. Proposed Fast NLMF-Type Algorihtms

To reduce the unwanted complexity and to improve the fast convergence speed, $\mu(n)$ in (5) is approximated as

$$\mu(n) \approx \frac{\mu\sigma_e^2}{N\sigma_x^2\left(N\sigma_x^2+\sigma_e^2\right)}. \quad (6)$$

The approximation in (6) is reasonable for large N in steady-state. Hence, $\|\mathbf{x}(n)\|_2^2$ can be replaced by $N\sigma_x^2$ and $e^2(n)$ can be replaced by σ_e^2. Here, σ_x^2 and σ_e^2 denote input signal variance and transient MSE variance, respectively. By comparing with sparse NLMS-type algorithms, the sparse NLMF-type algorithms can highly improve the estimation error by the variable step-size $\mu(n)$ with higher-order error criterion, especially in the case of low SNR regimes [4]. However, transient MSD and initial input signal variance can control the value of the step-size $\mu(n)$, where the small step-size decelerates the convergence speed and large one accelerates the convergence speed. Hence, the sparse NLMF algorithms can perform very well but sacrifice the convergence speed in high SNR regimes. This motivates us to develop a fast sparse NLMF algorithms using non-constraint step-size $\mu(n)$ which do not depends on the transient MSD and initial input signal variance.

The fact that the minimum steady-state MSE of the sparse NLMF-type algorithm using (6) cannot reach the non-constrained minimum steady-state MSE for high SNR case. In order to get rid of the constraints, the step-size $\mu(n)$ of the fast sparse NLMF-type algorithms can be approximated as

$$\mu(n) = \frac{\mu e^2(n)}{\|\mathbf{x}(n)\|_2^2 \max\left(\delta\|\mathbf{x}(n)\|_2^2, e^2(n)\right)} \approx \frac{\mu\sigma_e^2}{N\sigma_x^2 \max\left(\delta N\sigma_x^2, \sigma_e^2\right)}, \quad (7)$$

provided that μ satisfies the stability condition of the sparse NLMF algorithms [12]; i.e.

$$0 < \mu < 2. \quad (8)$$

In the steady-state phase, $\delta\|\mathbf{x}(n)\|_2^2$ dominants $e^2(n)$. Hence, $\max(\delta\|\mathbf{x}(n)\|_2^2, e^2(n)) = \delta\|\mathbf{x}(n)\|_2^2 \approx \delta N\sigma_x^2$ for large N. In (7), one can find that the step-size can be controlled by choosing suitable δ which can ensure proposed algorithms to keep fast tacking ability. In the transient phase, especially for large initial MSD, $e^2(n)$ dominates $\delta\|\mathbf{x}(n)\|_2^2$. Consequently, the proposed algorithms using step-size $\mu(n)$ in (7) behave as NLMS-type algorithm with a unity step-size, which yields a fast convergence.

By adopting zero-attracting factor $\mathcal{S}(\tilde{\mathbf{w}}(n)) = \|\tilde{\mathbf{w}}(n)\|_1$ [13] in (4), fast ZA-NLMF algorithm can be derived as

$$\tilde{\mathbf{w}}(n+1) = \tilde{\mathbf{w}}(n) + \mu(n)e(n)\mathbf{x}(n) - \rho_{ZA}\text{sign}\left(\tilde{\mathbf{w}}(n)\right). \quad (9)$$

Similarly, by adopting reweighted zero-attracting factor $\mathcal{S}(\tilde{\mathbf{w}}(n)) = \sum_{i=0}^{N-1}\log\left(1+\varepsilon_{RZA}|\tilde{w}_i(n)|\right)$ [13] in (4), fast RZA-NLMF algorithm can be derived as

$$\tilde{\mathbf{w}}(n+1) = \tilde{\mathbf{w}}(n) + \mu(n)e(n)\mathbf{x}(n) - \frac{\rho_{RZA}\,\text{sgn}\left(\mathbf{w}(n)\right)}{1+\varepsilon_{RZA}|\mathbf{w}(n)|}, \quad (10)$$

where $\rho_{RZA} = \rho\varepsilon_{RZA}$ and ε_{RZA} denotes the reweighted factor, which can ensure the fast RZA-NLMF algorithm to exploit more sparsity than the fast ZA-NLMF algorithm. In the right of [13], $\varepsilon_{RZA} = 20$ is adopted in this paper as well.

Tab. I. Simulation parameters.

Parameters	Values
Input signal	Pseudo-random Binary sequence
Additive noise distribution	$\mathcal{CN}(0,1)$
Length of FIR system	$N = 256$
No. of nonzero coefficients	$Sparsity \in \{4,16,32\}$
Nonzero coefficient	$\{-1,+1\}$
Received SNR	SNR $\in$ {0dB,5dB,10dB,20dB}
Step-size	$\mu = 1.5$
Sparse constraint parameters	$\rho_{ZA} = \rho_{RZA} = 3\times10^{-5}$
Reweight factor of RZA-NLMF	$\varepsilon_{RZA} = 20$

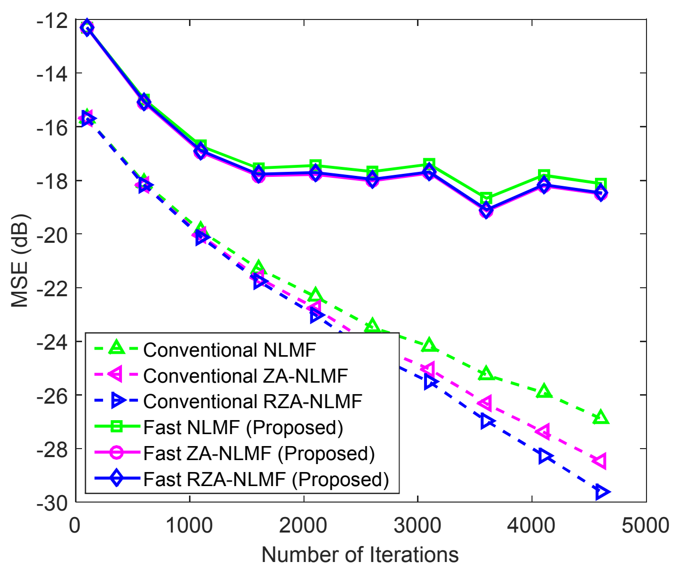

Fig. 3. MSE comparsions in the case of SNR=0dB.

IV. NUMERICAL SIMULATIONS

In this section, the proposed fast sparse NLMF-type algorithms are evaluated in different scenarios, SNR and system sparsity K. For achieving average performance, 100 independent Monte-Carlo runs are adopted. The simulation setup is configured according to typical broadband wireless communication system [14]. The maximum length of FIR system $\mathbf{w}$ is $N = 256$ and its number of nonzero coefficients is set as $K \in \{4,16,32\}$. Each nonzero coefficient is set as either $+1$ or -1 and their positions are randomly decided within the $\mathbf{w}$. To validate the effectiveness of the proposed algorithms, we adopt average MSE metric which is defined as

$$\mathrm{MSE}\{\tilde{\mathbf{w}}(n)\}(\mathrm{dB}) \triangleq 10\log_{10} E\{\tilde{\mathbf{w}}(n) - \mathbf{w}\}, \quad (11)$$

where $\mathbf{w}$ and $\tilde{\mathbf{w}}(n)$ are the actual signal vector and reconstruction vector, respectively. The received SNR is defined as P_0/σ_n^2, where P_0 is the received power of the pseudo-random noise (PN)-sequence for training signal. In addition, to achieve better steady-state estimation performance, reweighted factor of RZA-NLMF-type algorithms is set as $\varepsilon_{RZA} = 20$ [15]. Detailed parameters for computer simulations are given in Tab. I.

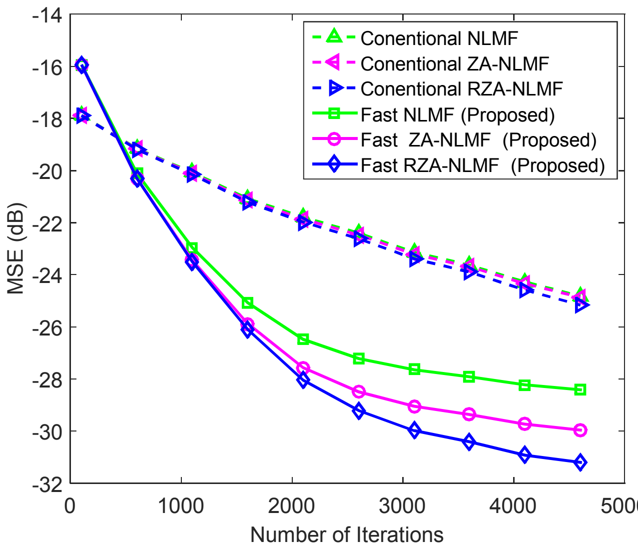

Fig. 4. MSE comparsions in the case of SNR=5dB.

In the first experiment, average MSE curves of the proposed methods are depicted in the case of system sparsity K=4 under four SNR regimes (i.e., 0dB, 5dB, 10dB, and 20dB), as shown in Figs. 3-6. In the low SNR case (e.g., 0dB), the proposed fast NLMF-type algorithms do not have the performance advantage and fast convergence speed. In the median (e.g., 5dB or 10dB) or high SNR (e.g, 20dB) cases, the proposed fast NLMF-type algorithms can achieve fast convergence speed and lower MSE performance than conventional NLMF-type algorithms. In addition, the proposed fast sparse NLMF-type algorithms (i.e., RZA-NLMF and ZA-NLMF) can get lower MSE performance than the fast NLMF algorithm without sparse constraint because the former algorithms can exploit the inherent sparse structure information in the system. Let us take Fig. 6 as for the example. The proposed RZA-NLMF algorithm can get at least 10dB performance gain when comparing to the fast NLMF algorithm. Hence, the first experiment implies that the proposed fast NLMF-type algorithms can accelerate convergence speed comparing to conventional NLMF-type algorithms. In the case of sparse system, fast sparse NLMF-type algorithms can perform better than fast NLMF algorithm.

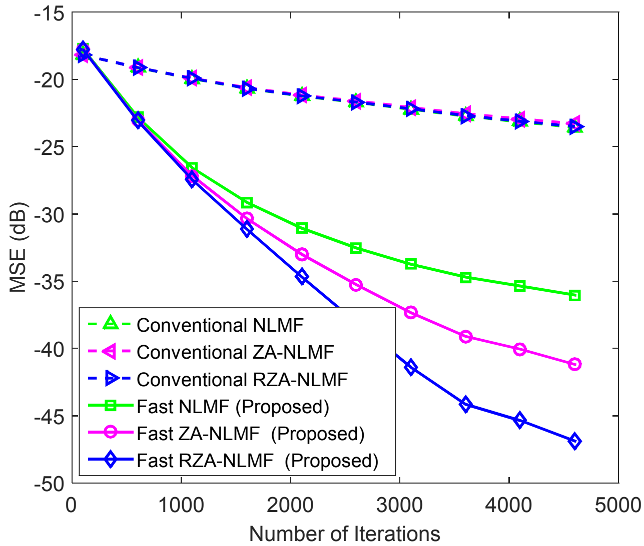

Fig. 5. MSE comparisons in the case of SNR=10dB.

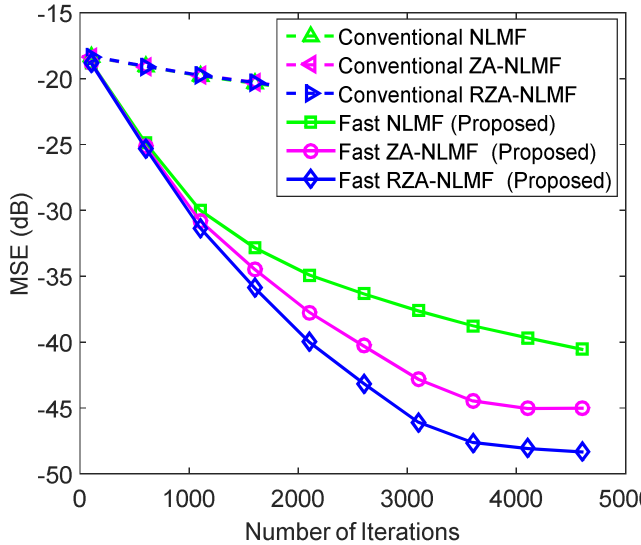

Fig. 6. MSE comparisons in the case of SNR=20dB.

In the second experiment, the proposed methods are evaluated by MSE against system sparsity, $K \in \{16, 32\}$ in the case of SNR=10dB, as shown in Figs. 7-8. We can find that performance gap of the proposed sparse NLMF algorithms and NLMF algorithm is decreased as the number of nonzero coefficients K. Also, Figs. 7-8 show that the proposed fast NLMF-type algorithms are close to conventional NLMF-type algorithms. We can deduce that the proposed fast NLMF-type algorithms may perform very well in the scenario of sparse system. In the scenario of quasi-sparse system, the fast convergence speed of proposed NLMF-type algorithms can keep while the performance advantage will be vanished. Hence, the second experiment imply that the proposed NLMF-type algorithms can keep fast convergence speed while identification performance may depend on real system sparsity.

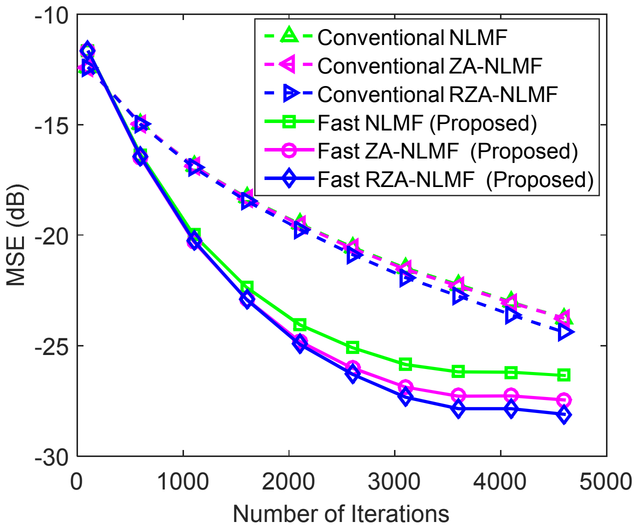

Fig. 7. MSE comparsions in the case of SNR=10dB and $K = 16$.

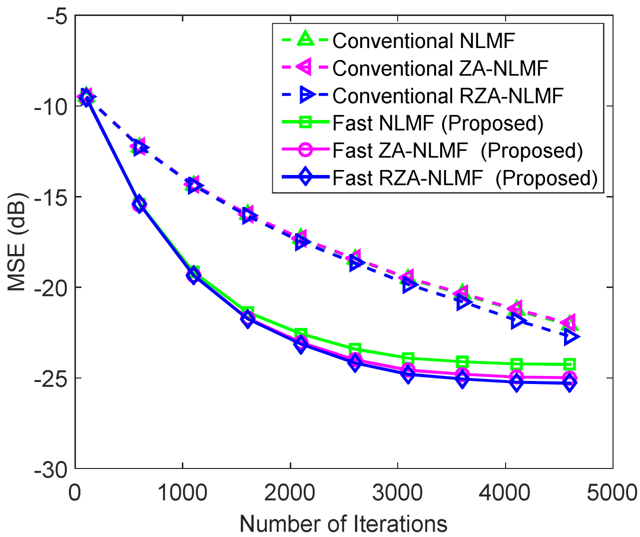

Fig. 8. MSE comparsions in the case of SNR=10dB and $K = 32$.

V. Conclusions

Fast sparse NLMF-type algorithms have been proposed to improve the convergence speed as well as to exploit system sparsity. To avoid the constraints of initial input signal variance and initial estimation error variance to conventional NLMF algorithm, we resorted to statistical approximation method and then derived the fast NLMF-type algorithms. To deal with ASIDE problems, fast ZA-NLMF and fast RZA-NLMF algorithms has been developed. By means of Monte Carlo simulation, the proposed fast NLMF-type algorithms have been confirmed in convergence speed and identification performance.

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