MIMO-OFDM Pilot Symbol Design For Sparse Channel Estimation

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***Abstract*—Channel equalization is a crucial part of the OFDM communications protocols, which in turn requires channel estimation. In this paper, we consider the problem of orthogonal pilot design in MIMO-OFDM systems for sparse channel estimation. The pilot design in MIMO scenarios compared to the conventional SISO case has the additional constraint that the capability of recovery should be uniformly provided for all single channels. For instance, perfect estimation of a channel at the cost of another one is not permitted. This requirement is even more significant in the emerging Massive-MIMO systems. Our pilot design is based on the compressed sensing technique of minimizing the coherence measure of the Fourier submatrix associated with the pilot subcarriers. However, we desire to minimize the coherence value of such matrices simultaneously for all channels. Here, we optimize over both the pilot locations and values. As finding the global minimizer is a combinatorial problem, we resort to a greedy method. Although there is no guarantee for the optimality of the achieved patterns, our simulation results confirm their suitability in practice.**

***Index Terms*—OFDM, Pilot Design, Sparse channel estimation, Compressive Sensing, Massive MIMO**

I. Introduction

OFDM channel estimation has been one of the main concerns of the researchers, and many OFDM channel estimation methods have been developed so far. Recent studies reveal that some wireless channels such as mmWave channels, high definition television (HDTV), and underwater acoustic channels have sparse structures [1]–[4]. Considering the sparse structures of the channels results in the design of algorithms with better performance and lower pilot overhead [5]–[7]. Therefore, many researchers have used Compressive Sensing (CS) theory [8], [9] for sparse channel estimation [5]–[7].

To efficiently utilize CS for sparse channel estimation in OFDM systems, the pilot symbols should be designed properly. In [7] it is shown that random pilot locations can guarantee the perfect channel reconstruction. However, random pilots are not applicable in practical systems [10]. Therefore, some pilot design schemes are proposed to design deterministic pilots [10]–[14]. The deterministic pilots are designed based on minimizing the coherence of the measurement matrix [10]–[14]. Under some specific conditions, it is shown in [10] and [12] that the pilot locations designed according to the cyclic difference sets (CDS) are optimum. However, in practical systems the necessary conditions for existence of the CDSs are not satisfied. Therefore, a greedy method is proposed in [10], to design suboptimal pilot patterns. The authors of [12] propose three different schemes for suboptimal pilot pattern design, where two methods are based on stochastic search and the other one applies a tree-based searching structure. In [13] and [14], it is shown that joint design of pilot pattern and power results in better performance. Both the methods in [13] and [14] decouple the joint design of pilot pattern and power into the disjoint pattern and power design sub-problems and solve them sequentially. The pilot design method of [13] is only applicable for the cases where either CDS or almost different set (ADS) exist. In [13], the exhaustive search for finding the optimal pilot pattern and powers is implemented using backtracking which reduces the computational cost in many cases. Another OFDM pilot design is introduced in [14] for the application of cognitive radios. Replacing the exhaustive search for the pilot locations with a sequential stochastic search is shown to achieve descent results with tolerable computational cost [12].

To have interference-free channel estimation in multiuser and MIMO OFDM systems, orthogonal pilot design is a common technique; in this approach, pilot subcarriers of all users occupy distinct frequencies and do not coincide with any of the data subcarriers. In this paper we do not consider non-orthogonal settings, as they suffer from pilot contamination, an undesired effect that can severely limit the overall performance [15], [16]. Beside the orthogonality, multiple pilot sequences should be designed in a fair manner. By fair design of pilots, we mean all the pilot sequences should estimate CSI with the same quality under the same conditions.

In CS based channel estimation, fair design of pilots results in the design of measurement matrices with equal coherence value. MIMO-OFDM pilot pattern design for sparse channel estimation is considered in [11], [12]. To have fair pilot sequences, the authors in [11] use genetic algorithm (GA) to generate a core pilot sequence and design the other pilot sequences by shifting the entries of the core pilot sequence. In [12] the single user pilot pattern design algorithms are extended to the MIMO case, and two methods are proposed based on the stochastic search. In one method they sequentially design pilot sequences for multiple antennas. In the other method, to prevent unfair pilot design, they jointly design multiple pilot sequences.

In this paper, we consider the pilot design for MIMO-

OFDM sparse channel estimation. The joint design of pilot pattern and power is considered based on minimizing the coherence of the measurement matrix. We decouple the problem in to the disjoint subproblems of pattern design and pilot allocation, and sequentially solve them. Different from the methods in [11] and [12], the pilot patterns are determined deterministically in a greedy manner. To find the pilot powers, like [14] we cast the pilot power problem as a SOCP problem. In order to design fair pilot sequences we design the algorithm based on minimizing the maximum coherence value of the transmitters. Simulation results show that the pilot pattern designed by the proposed method outperforms the others in terms of designing sensing matrices with smaller coherence and estimating channels with lower MSE.

The rest of the paper is organized as follows. The MIMO-OFDM pilot design problem is formulated in Section II. Section III considers the pilot power allocation. Section IV presents the MIMO-OFDM pilot design algorithm. The simulation results are given in Section V. Finally, conclusions are provided in Section VI.

The following notations are used throughout the paper. Matrices and vectors are represented by boldface upper case and boldface lower case letters, respectively. The entry at the i-th row and the j-th column of $\mathbf{A}$ is denoted by $[\mathbf{A}]_{i,j}$. $a(i)$ stands for the i-th entry of a vector $\mathbf{a}$. $\mathbf{1}$ and $\mathbf{0}$ mean a vector with all-one entries and a vector with all-zero entries, respectively. $\mathbf{a} > \mathbf{0}$ means that all the entries of the vector $\mathbf{a}$ are greater than 0. $[\,\cdot\,]^T$ and $\mathrm{diag}\{\cdot\}$ denote the transpose and the diagonal matrix, respectively. $\langle \mathbf{a}, \mathbf{b} \rangle$ denotes the inner product of $\mathbf{a}$ and $\mathbf{b}$. The sets $\mathcal{N}$ and $\mathcal{L}$ are defined as $\mathcal{N} = \{1, 2, \cdots, N\}$ and $\mathcal{L} = \{1, 2, \cdots, L-1\}$. $\mathbb{Z}$ stands for the integer numbers set, and $\emptyset$ shows the empty set.

II. Problem Formulation

Consider a MIMO-OFDM system, where the transmitter and the receiver employ n_T and n_R antennas, respectively. The signals are transmitted on N subcarriers. To estimate the channels at the receiver, each transmit antenna occupies N_P subcarriers for pilot symbol transmission. The pilot subcarriers assigned to the i-th transmit antenna are denoted as $\mathcal{P}_i = \{p_{i,1}, p_{i,2}, \cdots, p_{i,N_P}\}$, where we assume that $1 \le p_{i,1} < p_{i,2} < \cdots < p_{i,N_P} \le N$. In order to have interference free channel estimation, we use frequency orthogonal pilot transmission strategy, which means that $\mathcal{P}_i \cap \mathcal{P}_j = \emptyset$ for $1 \le i, j \le n_T$ and $i \ne j$. Under such assumption, MIMO-OFDM channel estimation is decomposed into simultaneous estimation of $n_T \times n_R$ SISO-OFDM channels. Representing the pilot symbols transmitted by i-th transmit antenna as $\mathbf{x}_i = [x_i(1), x_i(2), \cdots, x_i(N_P)]^T$, the associated received vector at j-th receive antenna, $\mathbf{y}_{j,i} = [y_j(p_{i,1}), \cdots, y_j(p_{i,N_P})]^T$, can be written as,

$$\mathbf{y}_{j,i} = \mathbf{X}_i \mathbf{F}_i \mathbf{h}_{j,i} + \mathbf{n}_{j,i}. \tag{1}$$

Here, $\mathbf{X}_i = \mathrm{diag}\{x_i(1), x_i(2), \cdots, x_i(N_P)\}$, $\mathbf{h}_{j,i} = [h_{j,i}(1), \cdots, h_{j,i}(L)]^T$ represents the channel impulse response between (j,i)-th receiver-transmitter pair, and $\mathbf{n}_{j,i} = [n_j(p_{i,1}), n(p_{i,2}), \cdots, n(p_{i,N_P})]^T$ shows the additive white Gaussian noise (AWGN), which is modeled as $\mathbf{n}_{j,i} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{N_P})$. the j-th receive antenna noise at the pilot subcarriers assigned to the transmitter i. Also, $\mathbf{F}_i$ is a DFT submatrix with elements defined as $[\mathbf{F}_i]_{n,l} = e^{-j\frac{2\pi}{N}(n-1)(l-1)}$ for $n \in \{p_{i,1}, p_{i,2}, \cdots, p_{i,N_P}\}$ and $l \in \{1, \cdots, L\}$. Defining $\mathbf{\Phi}_i = \mathbf{X}_i \mathbf{F}_i$, (1) can be written as

$$\mathbf{y}_{j,i} = \mathbf{\Phi}_i \mathbf{h}_{j,i} + \mathbf{n}_{j,i}. \tag{2}$$

Here, we refer to $\mathbf{\Phi}_i$ as the measurement matrix for i-th transmitter.

We assume that the channel $\mathbf{h}_{j,i}$ is a k-sparse vector of length L, meaning that $\mathbf{h}_{j,i}$ has at most k non-zero elements where $k \ll L$. Therefore, the CS theory can be applied for estimation of $\mathbf{h}_{j,i}$ with significantly less number of pilots than conventional methods. In [8], it is shown that if the measurement matrix satisfies the Restricted Isometry Property (RIP), then $\mathbf{h}_{j,i}$ can be reconstructed from $\mathbf{y}_{j,i}$ with an overwhelming probability. However, there is no polynomial time algorithm to check whether a matrix satisfies the RIP [17]. A common alternative to guarantee the perfect reconstruction of sparse signals is the coherence of the measurement matrix. Coherence of a matrix is defined as the maximum cross-correlations between its normalized columns. The coherence condition is stronger than RIP, and can be evaluated easily. Therefore, many sparse channel estimation methods have used coherence as a basis for designing pilot signals.

In (2), the coherence of $\mathbf{\Phi}_i$ is formulated as

$$\begin{aligned} \mu_{\mathbf{\Phi}_i} &= \max_{\substack{1 \le m,n \le L \\ m \ne n}} \frac{|\langle \phi_m, \phi_n \rangle|}{\|\phi_m\|_2 \cdot \|\phi_n\|_2} \\ &= \max_{\substack{1 \le m,n \le L \\ m \ne n}} \frac{\left| \sum_{j=1}^{N_P} |x_i(j)|^2 e^{-j\frac{2\pi}{N} p_{i,j}(m-n)} \right|}{\sum_{j=1}^{N_P} |x_i(j)|^2}. \end{aligned} \tag{3}$$

Here, $\mu_{\mathbf{\Phi}_i}$ represents the coherence of $\mathbf{\Phi}_i$, and ϕ_m denotes the m-th column of $\mathbf{\Phi}_i$. According to the periodic structure of $\mathbf{\Phi}_i$, the cross-correlation between ϕ_m and ϕ_n only depends on the difference between the indices m and n. Therefore, letting $r = m - n$, $\mu_{\mathbf{\Phi}_i}$ can be written as,

$$\mu_{\mathbf{\Phi}_i} = \max_{r \in \mathcal{L}} \frac{\left| \sum_{j=1}^{N_P} |x_i(j)|^2 e^{-j\frac{2\pi}{N} p_{i,j} r} \right|}{\sum_{j=1}^{N_P} |x_i(j)|^2} \tag{4}$$

The sparse recovery methods such as ℓ_1 minimization and greedy methods, are guaranteed to perfectly recover $\mathbf{h}_{j,i}$ when $\mu_{\mathbf{\Phi}_i} < \frac{1}{2k}$ [18]. Therefore, we design the pilot symbols based on minimizing the coherence of the measurement matrix. Most of the OFDM pilot design methods proposed for sparse channel estimation only consider the design of pilot pattern and assume equal powers for all pilots. However, according to (4), it is obvious that besides the pilot locations, the pilot powers can control the value of $\mu_{\mathbf{\Phi}_i}$. Consequently, as in [13] and [14], we consider the joint design of pilot pattern and pilot power based on minimizing the coherence of the measurement matrix.

Therefore, the pilot design problem for i-th transmit antenna is formulated as

$$\begin{aligned}\Omega_i^{\text{opt}} &= \arg\min_{\Omega} \mu_{\mathbf{\Phi}_i} && (5)\\ &= \arg\min_{\Omega}\max_{r\in\mathcal{L}} \frac{\left|\sum_{j=1}^{N_P} v_i(j) e^{-j\frac{2\pi}{N}p_{i,j}r}\right|}{\sum_{j=1}^{N_P} v_i(j)}.\end{aligned}$$

Here, $v_i(j)$ denotes the power assigned to the j-th pilot symbol of transmitter i, i.e., $v_i(j) = |x_i(j)|^2$. Ω is the feasible set and is defined as $\Omega = \{\mathbf{v}_i, \mathcal{P}_i\}$, where $\mathbf{v}_i = [v_i(1), \cdots, v_i(N_P)]^T$. Also in (5), $\Omega_i^{\text{opt}} = \{\mathbf{v}_i^{\text{opt}}, \mathcal{P}_i^{\text{opt}}\}$, where

$$\begin{aligned}\mathbf{v}_i^{\text{opt}} &= [v_i^{\text{opt}}(1), \cdots, v_i^{\text{opt}}(N_P)]\\ \mathcal{P}_i^{\text{opt}} &= \{p_{i,1}^{\text{opt}}, \cdots, p_{i,N_P}^{\text{opt}}\}. && (6)\end{aligned}$$

The pilot design problem of (5) is a joint optimization of continues and discrete variables, and finding optimal solution for it is almost impossible. Therefore, to find the suboptimal pilot powers and pattern, we disjoint (5) into the pilot and pattern optimization subproblems and solve them in a sequential manner.

III. Pilot Power Design

Assume that $\mathcal{P} = \{p_{i,1}, \cdots, p_{i,m}\}$ represents the m pilot locations for i-th transmit antenna. For the given pilot pattern, the optimal pilot powers are found through the following optimization problem.

$$\begin{aligned}\mathbf{v}_i^{\text{opt}} = \arg \quad &\min_{\mathbf{v}_i} \quad \max_{r\in\mathcal{L}} \frac{\left|\sum_{j=1}^{m} v_i(j) e^{-j\frac{2\pi}{N}p_{i,j}r}\right|}{\sum_{j=1}^{m} v_i(j)}\\ &\text{s.t.} \quad \mathbf{v}_i > \mathbf{0} \qquad (7)\end{aligned}$$

According to (4), if $\mathbf{v}_i^{\text{opt}}$ is a solution for (7), then $\mathbf{v}_i = [\alpha v_i(1), \cdots, \alpha v_i(m)]^T$ will be another solution for (7) for any $\alpha > 0$. Therefore, we propose to put the following constraint on the pilot powers.

$$\sum_{j=1}^{m} v_i(j) = 1. \qquad (8)$$

Obviously, while making the search region as small as possible, the proposed power constraint preserves all the solutions of (7). Considering the pilot powers constraint of (8), and defining $g(\mathcal{P}, \mathbf{v}_i)$ as

$$g(\mathcal{P}, \mathbf{v}_i) = \max_{r\in\mathcal{L}} \left|\sum_{j=1}^{m} v_i(j) e^{-j\frac{2\pi}{N}p_{i,j}r}\right|, \qquad (9)$$

the power allocation problem for a given pilot pattern of $\mathcal{P}$ is written as

$$\mathbf{v}_i^{\text{opt}} = \arg\min_{\mathbf{v}_i\in\mathcal{V}} g(\mathcal{P}, \mathbf{v}_i) \qquad (10)$$

where

$$\mathcal{V} = \{\mathbf{v}_i | \sum_{j=1}^{m} v_i(j) = 1, v_i(j) > 0 \ \forall j\}.$$

As mentioned in [14], (10) can be casted as a SOCP problem which can be solved using convex optimization packages such as CVX [19] and MOSEK [20].

IV. MIMO-OFDM Pilot Design Algorithm

To design orthogonal pilot sequences for all the transmit antennas, (5) should be considered simultaneously for $i = 1, \cdots, n_T$. Since there is no priority between the transmit antennas, the pilots should be designed fairly. By fair design of pilots, we mean that all the measurement matrices should have the same coherence value. According to (4), it is obvious that if $\mathcal{P}_i^* = \{p_{i,1}^*, \cdots, p_{i,N_P}^*\}$ is a solution for (5), then $\mathcal{P}_i = \{p_{i,1}^* + q, \cdots, p_{i,N_P}^* + q\}$, achieved by shifting the entries of $\mathcal{P}_i^*$ by $q \in \mathbb{Z}$, is also a solution for (5). Therefore, as proposed in [11], one possible method for designing fair pilot sequences is to generate a core pilot sequence and then design the other pilot sequences by shifting the entries of the core sequence. However, this method depends on the available number of subcarriers N, the number of transmit antennas n_T, and the number of pilots N_P. For instance, consider a case where $N = 256$, $n_T = 16$ and $N_P = 16$. To design orthogonal pilot sequences using the method proposed in [11], each two consecutive pilot subcarriers of the core sequence should be spaced at least 16 subcarriers apart, which is similar to the conventional pilot design methods.

Therefore, in this section we propose a pilot design algorithm to design multiple pilot sequences as fair as possible. The proposed algorithm is consisted of two parts. In the first part of the algorithm, n_T orthogonal pilot sequences are designed jointly in $N_P - 1$ steps. Then, in the second part of the algorithm, we try to improve the achieved pilot patterns in a limited number of iterations. The detailed algorithm is presented in Table I. N, n_T, N_P, L and $I_{\max}$ are the inputs of the algorithm, where $I_{\max}$ determines the maximum iterations of the second part of the algorithm.

The first part of the algorithm, indicated from step 1 to step 6, contains one outer loop and one inner loop. The outer loop and the inner loop involve $N_P - 1$ and n_T iterations, respectively. Let us define $\widehat{\mathcal{P}}_i(n) = \{\widehat{p}_{i,1}, \cdots, \widehat{p}_{i,n}\}$ as the i-th optimal pilot pattern achieved in the n-th iteration of the outer loop. In the initialization step we arbitrarily set $\widehat{\mathcal{P}}_i(1) = \{i\}$ for $i = 1, \cdots, n_T$. The other pilot locations are determined as follows.

- Given $\widehat{\mathcal{P}}_i(n-1)$, in the n-th iteration of the outer loop, we sequentially determine the n-th pilot subcarrier of each transmit antenna through n_T iterations of the inner loop.
- In the i-th iteration of the inner loop, form all possible $\mathcal{P} = \{\widehat{\mathcal{P}}_i(n-1) \cup p\}$ and find $f(\mathcal{P})$ as,

  $$f(\mathcal{P}) = \min_{\mathbf{v}\in\mathcal{V}} g(\mathcal{P}, \mathbf{v}). \qquad (11)$$

  In the definition of $\mathcal{P}$, p is an element in set $\mathcal{A}_{n,i}$, where $\mathcal{A}_{n,i}$ contains the available subcarriers which is defined as

  $$\mathcal{A}_{n,i} = \mathcal{N} \setminus \Big(\bigcup_{j=1}^{i-1} \widehat{\mathcal{P}}_j(n) \bigcup_{j=i}^{n_T} \widehat{\mathcal{P}}_j(n-1)\Big). \qquad (12)$$

  Evaluating $f(\mathcal{P})$ for all possible sets of $\mathcal{P}$, we find $\widehat{\mathcal{P}}_i(n)$ as

  $$\widehat{\mathcal{P}}_i(n) = \arg\min_{\mathcal{P}} f(\mathcal{P}). \qquad (13)$$

TABLE I
THE PROPOSED MIMO-OFDM PILOT DESIGN ALGORITHM.

Input: N, n_T, N_P, L, $I_{\max}$.
1: Initialization: $\widehat{\mathcal{P}}_i(1) \Leftarrow \{i\}$ for $i = \{1, \cdots, n_T\}$.
2: **for** $n = 2, \cdots, N_P$
3: **for** $i = 1, \cdots, n_T$
4: Obtain $\widehat{\mathcal{P}}_i(n)$ according to (13).
5: **end for**(i)
6: **end for**(n)
7: $\mathcal{P}_i^{\text{opt}} \Leftarrow \widehat{\mathcal{P}}_i(N_P)$ and $C(i) \Leftarrow f(\mathcal{P}_i^{\text{opt}})$ for $i = 1, \cdots, n_T$.
8: **for** $l = 1, \cdots, I_{\max}$
9: Find s according to (14).
10: $\mathcal{P}_{\text{aux}} \Leftarrow \mathcal{P}_s^{\text{opt}}$.
11: **for** $n = 1, \cdots, N_P$
12: Obtain $\mathcal{P}_n$ according to (15).
13: **if** $f(\mathcal{P}_n) < C(s)$
14: $\mathcal{P}_s^{\text{opt}} \Leftarrow \mathcal{P}_n$ and $C(s) \Leftarrow f(\mathcal{P}_n)$.
16: **end if**
17: **end for**(n)
18: **if** $\mathcal{P}_s^{\text{opt}} \neq \mathcal{P}_{\text{aux}}$
19: Go to step 8.
20: **end if**
21: **for** $n = 1, \cdots, n_T$, $n \neq s$.
22: **for** $m = 1, \cdots, N_P$
23: Evaluate $C_{\text{perm}}(n, m)$.
24: **end for**(m)
25: **end for**(n)
26: Find $\mathcal{P}_s^{\text{opt}}$ and $\mathcal{P}_{n^*}^{\text{opt}}$ using (16).
27: **if** $\mathcal{P}_s^{\text{opt}} = \mathcal{P}_{\text{aux}}$
28: break.
29: **end if**
30: **end for**(l)
31: Using (10), design the pilot powers for each pilot pattern.

At the end of the first part of the algorithm, step 7, we set $\mathcal{P}_i^{\text{opt}} = \widehat{\mathcal{P}}_i(N_P)$, and evaluate $\mu_{\mathbf{\Phi}_i}$ for each transmit antenna, i.e., $C(i) = f(\mathcal{P}_i^{\text{opt}})$.

The second part of the algorithm, indicated from step 8 to step 31, contains one outer loop and two inner loops. The outer loop, the first inner loop and the second inner loop contain $I_{\max}$, N_P, and n_T number of iterations, respectively. In each iteration of the outer loop we reduce the maximum coherence of the pilot sequences through the following procedure.

- Given the pilot sequences resulted from iteration $l-1$ of the outer loop, i.e., $\mathcal{P}_i^{\text{opt}}$ for $i = 1, \cdots, n_T$, in the l-th iteration of the outer loop we find the set with the maximum coherence,
$$s = \arg\max_i C(i). \tag{14}$$
- Finding s, in the n-th iteration of the first inner-loop, we evaluate $f(\mathcal{P})$ for all possible $\mathcal{P} = \{\mathcal{P}_s^{\text{opt}} \setminus p_{s,n}^{\text{opt}}\} \cup \{p\}$. Here, p is an element in the set $\mathcal{N} \setminus \bigcup_{i=1}^{n_T} \mathcal{P}_i^{\text{opt}}$. Then we find $\mathcal{P}_n$ as,
$$\mathcal{P}_n = \arg\min_{\mathcal{P}} f(\mathcal{P}). \tag{15}$$
Now, if $f(\mathcal{P}_n) < C(s)$, then we set $\mathcal{P}_s^{\text{opt}} = \mathcal{P}_n$ and $C(s) = f(\mathcal{P}_n)$.
- If the maximum coherence is reduced in the first inner loop, the second inner loop will be skipped; otherwise we go through the second inner loop. This procedure, indicated from the step 18 to step 20, is done using the auxiliary set $\mathcal{P}_{\text{aux}}$ defined in the step 10.

TABLE II
FREQUENCY-ORTHOGONAL PILOT PATTERNS DESIGNED FOR SPARSE CHANNEL ESTIMATION IN MIMO-OFDM SYSTEMS.

1st antenna	1, 33, 41, 45, 53, 57, 81, 85, 113 133, 149, 157, 173, 233, 245, 253
2nd antenna	2, 34, 42, 46, 54, 58, 82, 86, 114 134, 150, 158, 174, 234, 246, 254
3rd antenna	3, 15, 35, 55, 71, 91, 95, 103, 107 111, 119, 155, 179, 199, 227, 231
4th antenna	4, 16, 36, 56, 72, 92, 96, 104, 108 112, 120, 156, 180, 200, 228, 232
5th antenna	5, 61, 77, 93, 97, 101, 137, 141, 145 165, 169, 197, 205, 217, 225, 249
6th antenna	6, 62, 78, 94, 98, 102, 138, 142, 146 166, 170, 198, 206, 218, 226, 250
7st antenna	7, 19, 47, 51, 79, 99, 123, 159, 167 171, 175, 183, 187, 207, 223, 243
8th antenna	8, 20, 188, 48, 52, 80, 100, 160, 168 176, 208, 124, 224, 184, 244, 172

- In the n-th iteration of the second inner loop, where $n \neq s$, we search whether exchanging the entries of $\mathcal{P}_s^{\text{opt}}$ with $\mathcal{P}_n^{\text{opt}}$ reduces the maximum coherence. Therefore, we repeat the following steps for $m = 1, \cdots, N_P$.
  1) Form $\mathcal{P}_1$ and $\mathcal{P}_2$ as,
  $$\begin{aligned} \mathcal{P}_1 &= \{\mathcal{P}_s^{\text{opt}} \setminus p_{s,m}^{\text{opt}}\} \cup \{p_{n,m}^{\text{opt}}\} \\ \mathcal{P}_2 &= \{\mathcal{P}_n^{\text{opt}} \setminus p_{n,m}^{\text{opt}}\} \cup \{p_{s,m}^{\text{opt}}\}. \end{aligned}$$
  and evaluate $f(\mathcal{P}_1)$ and $f(\mathcal{P}_2)$.
  2) If $\max\{f(\mathcal{P}_1), f(\mathcal{P}_2)\} < C(s)$ then we set $C_{\text{perm}}(n, m) = \max\{f(\mathcal{P}_1), f(\mathcal{P}_2)\}$; otherwise we set $C_{\text{perm}}(n, m) = 1$.
- At the end of the second inner loop, we search for the best permutation. The best permutation results in the largest reduction in the maximum coherence. Therefore, the best pilot sequences are determined as follows.

$$\begin{aligned} (n^*, m^*) &= \arg\min_{\substack{n,m \\ n=1,\cdots,n_T \text{ and } n\neq s \\ m=1,\cdots,N_P}} C_{\text{perm}}(n, m) \\ \mathcal{P}_s^{\text{opt}} &= \{\mathcal{P}_s^{\text{opt}} \setminus p_{s,m^*}^{\text{opt}}\} \cup \{p_{n^*,m^*}^{\text{opt}}\} \\ \mathcal{P}_{n^*}^{\text{opt}} &= \{\mathcal{P}_{n^*}^{\text{opt}} \setminus p_{n^*,m^*}^{\text{opt}}\} \cup \{p_{s,m^*}^{\text{opt}}\}. \end{aligned} \tag{16}$$

Note that if the maximum coherence does not reduce at the end of l-th iteration of the outer loop, then no improvement will happen in the next iterations of the outer loop. Therefore, we check this situation in the steps $27-29$. Finally, at step 31, we find the optimum pilot powers for the resulted pilot patterns, i.e., $\mathcal{P}_i^{\text{opt}}$.

V. SIMULATION RESULTS

In this section, a number of simulations are conducted to show the performance of the proposed pilot design algorithm. Obviously, employing orthogonal pilot sequences decouples a MIMO-OFDM channel estimation into a several SISO-OFDM ones. Therefore, to evaluate the performance of the proposed pilot design method, we consider MISO-OFDM system in all simulations. Note that in all simulations we set $I_{\max} = 20$.

In the first simulation, we compare the proposed pilot design algorithm with the method proposed in [12]. We consider

TABLE III
FREQUENCY-ORTHOGONAL PILOT PATTERNS DESIGNED BY THE METHOD IN [12] FOR SPARSE CHANNEL ESTIMATION IN MIMO-OFDM SYSTEMS.

1st antenna	8, 40, 48, 52, 72, 82, 99, 142, 145 154, 158, 161, 183, 209, 212, 230
2nd antenna	9, 41, 49, 53, 73, 83, 100, 143, 146 155, 159, 162, 184, 210, 213, 231
3rd antenna	10, 42, 50, 54, 74, 84, 101, 144, 147 156, 160, 163, 185, 211, 214, 232
4th antenna	17, 25, 47, 56, 59, 63, 75, 111, 115 130, 141, 149, 153, 174, 200, 250
5th antenna	12, 34, 55, 64, 67, 109, 112, 148, 173 215, 222, 233, 238, 241, 249, 252
6th antenna	2, 15, 45, 58, 62, 66, 96, 103, 107 132, 165, 181, 186, 189, 204, 206
7st antenna	18, 22, 33, 68, 76, 80, 88, 91, 95 116, 133, 167, 198, 205, 229, 246
8th antenna	7, 79, 92, 117, 120, 152, 168, 180 187, 197, 219, 223, 239, 243, 251, 255

an OFDM system with $N = 256$ and $N_P = 16$. Two different scenarios are considered for this system. In the first scenario we set $n_T = 16$, and in the other one the number of transmitters is set to be $n_T = 8$. The channel length is set to be $L = 60$ for $i = 1, \cdots, n_T$ in both scenarios. The Extension Scheme 2 proposed in [12] are used for generating multiple pilot sequences, where the maximum numbers of iterations for the outer and inner loops are set to be 1000 and 15, respectively. In all simulations, we have used Mosek package [20] for power allocation problem, while we have considered equal pilot powers for the method of [12], ie., $v_i(n) = \frac{1}{N_P}$ for $n = 1, \cdots, N_P$ and $i = 1, \cdots, n_T$.

Figures 1 and 2 present the coherence of the measurement matrices for different transmit antennas. According to these figures, the coherence of the measurement matrices designed by our method have smaller values than those designed by the Extension Scheme 2 proposed in [12]. Also, the lower differences between the maximum and minimum coherence values verify the ability of the proposed method in preserving the fairness among different pilot sequences. As shown in Figures 1 and 2, the variance of the coherence values in the second scenario is less than the variance of the coherences in the first scenario. This is due to fact that in the case of $n_T = 16$, all the subcarriers are occupied by the transmitters, therefore the only way for improving the pattern design is permutation. But in the second case, $n_T = 8$, only 128 subcarriers should be allocated to transmitters, therefore we have many choices to improve the pilot pattern design.

We next compare the channel estimation performance of the pilot patterns used in the second scenario. The pilot patterns designed by the proposed algorithm and the Extension Scheme 2 [12] are given in Tables II and III, respectively. The performance is evaluated in terms of the normalized MSE of all the channel estimates. We define the normalized MSE as

$$\mathrm{NMSE} = \frac{1}{R \times n_T} \sum_{n=1}^{R} \sum_{i=1}^{n_T} \frac{\| h_{1,i}^{(n)} - \hat{h}_{1,i}^{(n)} \|_2^2}{\| h_{1,i}^{(n)} \|_2^2},$$

where R is the number of channel realizations, and $\hat{h}_{1,i}^{(n)}$ and

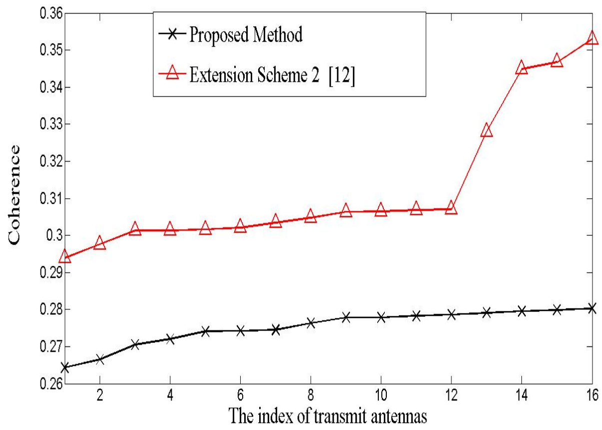

Fig. 1. The Coherence of the measurement matrix for $n_T = 16$ transmit antennas.

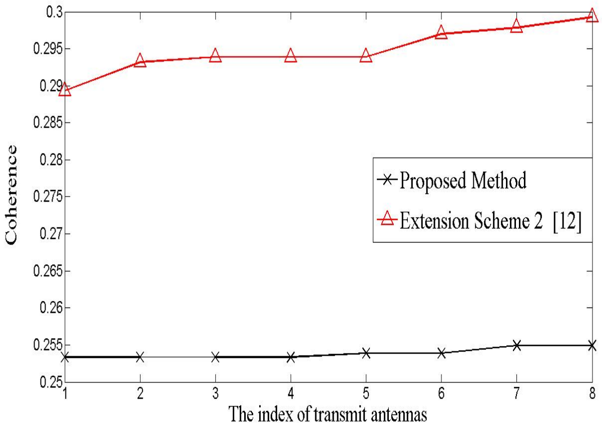

Fig. 2. The Coherence of the measurement matrix for $n_T = 8$ transmit antennas.

$\hat{h}_{1,i}^{(n)}$ represent the n-th channel realization and its estimate, respectively. The multipath channel is modeled as a k-sparse vector with $k = 5$ non-zero taps. The non-zero elements are randomly positioned among $L = 60$ taps, and their values are generated according to the i.i.d. complex Gaussian distribution with zero mean and unit variance, i.e., $\mathcal{CN}(0, 1)$. Based on the described channel model, 5000 channel realizations are used in the simulations. The channel vectors are estimated by Orthogonal Matching Pursuit (OMP) algorithm [18]. Fig. 3 presents the MSE performance for channel estimation. It is observed that the pilot sequences designed by the proposed method outperform the sequences obtained from the method of [12].

In the second simulation, we compare the proposed pilot design algorithm with the GA based method in [11]. To make a fair comparison, we use exactly the same parameters as in [11]. Therefore, we consider an OFDM system with $N = 512$, $N_P = 24$, $n_T = 4$ and $L = 50$. By making some straight forward changes in the proposed MIMO pilot

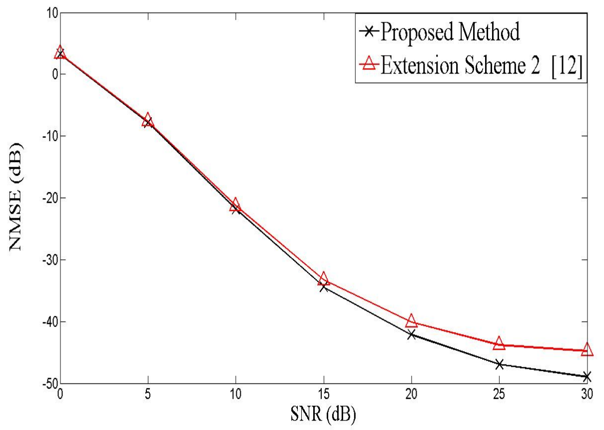

Fig. 3. NMSE performance comparisons of channel estimation for different pilot design schemes.

TABLE IV
COHERENCE OF THE MEASUREMENT MATRIX USING THE SHIFTING MECHANISM.

Scheme	Coherence value
Proposed Method	0.1763
GA based Method [11]	0.2186

design algorithm, we generate a core pilot sequence. Then, to determine the pilot patterns of other transmitters we use shifting mechanism proposed in [11]. The resulted coherence values of different pilot design schemes are summarized in Table IV. According to results given in Table IV, using the proposed method we achieve exactly fair pilot sequences with coherence of 0.1763, which is approximately 0.04 smaller than the coherence achieved from the GA algorithm of [11].

VI. CONCLUSIONS

In this paper, the pilot design for sparse channel estimation in MIMO-OFDM systems is investigated. Based on minimizing the coherence of the measurement matrix, we have proposed a greedy algorithm to design orthogonal pilot sequences. To generate pilot sequences with lower coherence values, we consider the joint optimization of pilot patterns and powers. In order to preserve the fairness between multiple pilot sequences, we have designed the algorithm based on joint optimization of multiple pilot sequences and minimizing the maximum coherence. Simulation results verify that the proposed method outperforms the others in terms of designing sensing matrices with smaller coherence and estimating channels with lower NMSE.

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