# A complete parallel narrowband active noise control system based on residual error separation using variable leaky LMS

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***Abstract*—In this paper, a residual error separation (RES) scheme using variable leakage LMS algorithm is proposed for a complete parallel narrowband active noise control (ANC, NANC) system. The RES subsystem is dedicated to separating individual sinusoid being targeted from the residual noise. Each separated sinusoid, instead of the whole residual noise, is used to update the corresponding control filter weights, which may significantly improve the overall performance of the system. A variable leakage (VL) LMS (VL-LMS) algorithm is developed for the RES subsystem to maintain its stability and to improve the system performance. Extensive simulations are provided to confirm the superiority of the proposed NANC system.**

## I. Introduction

In real environments, rotating machines or reciprocating motion devices produce a great deal of harmful noises that usually contain discrete low-frequencies [1], [2]. To eliminate such noises, the traditional narrowband active noise control (NANC) system has been extensively investigated and applied [1], [2].

For ANC of a narrowband noise with discrete frequencies, the NANC has been found very effective and efficient [2]. A typical conventional NANC system is shown in Fig.1. This system has uncorrelated sine and cosine reference input, and thus enjoys nice convergence properties. Based on this parallel structure, researchers have conducted a lot of analysis and modifications, and finally put forward a great many algorithms and strategies to improve its performance [3]-[6]. Statistical analysis of this NANC system was given in [3]. A robust NANC system using two-weight FIR subcontrollers was proposed in [4], which is capable of componsating for the influence of frequency mismatch between the reference waves and the primary noise. In [5], a parallel structure for x-filtering in NANC was introduced to reduce the complexity, making the system more efficient. A variable step-size (VSS) FXLMS (VSS-LMS) algorithm was derived in [6], that significantly improves the system convergence and tracking capability. Nevertheless, in all the above existing NANC systems, the control filters are updated by the same residual noise signal in all frequency channels. This implies that the subcontroller in a frequency channel is correlated to others in terms of both convergence and steady state performance. If the subcontrollers are allowed to work independently, the system performance may be greatly improved.

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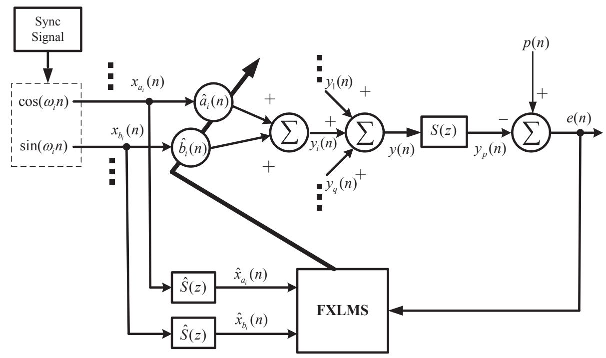



Fig. 1. Block diagram of conventional narrowband ANC system.

To overcome the statistical independence among frequency channels, Chang and Kuo proposed a complete parallel NANC system, where individual residual error signal used to update a subcontroller is generated by a bandpass filter derived from an IIR notch filter [7]. Although the bandpass filter does not bring any additional phase shift at the targeted frequency, it does introduce an additional group delay into the secondary path, which might limit the convergence of the system [8]. The idea proposed in [7] has inspired us in pursuing a new structure to increase the indpendence among the frequency channels.

This paper presents a new complete parallel NANC system that a residual error separation (RES) subsystem is added to extract individual residual error signal or sinusoid remaining in the residual noise using a variable leaky (VL) LMS (VL-LMS) algorithm. In the proposed system, every subcontroller is updated solely by its own residual error signal extracted by the RES subsystem. Furthermore, a VL-LMS algorithm is introduced to ensure the stability and to improve the convergence of the system. Various simulations are conducted to

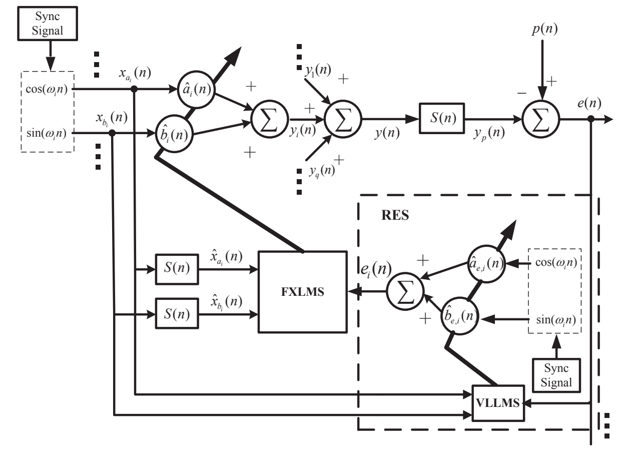



Fig. 2. The proposed NANC system with a VL-LMS based RES.

demonstrate the effectiveness and robustness of the proposed system.

## II. A NEW COMPLETE PARALLEL NARROWBAND ANC

Block diagram of a new complete parallel NANC system is depicted in Fig. 2. The VL-LMS based RES can separate the targeted sinusoids remaining in the residual noise, and each individual sinusoid so extracted is used to update its corresponding subcontroller. This way, convergence of the system may be significantly improved, as all frequency channels can work quite independently. Moreover, since the additive noise within the extracted individual sinusoid is much smaller than that within the residual noise, the steady-state performance of the overall system may be considerably improved.

The primary noise to be mitigated may be expressed as

$$\begin{aligned} p(n) &= \sum_{i=1}^{q} [a_i \cos(\omega_i n) + b_i \sin(\omega_i n)] + v_{\mathrm{p}}(n) \quad (1) \\ &= \sum_{i=1}^{q} p_i(n) + v_{\mathrm{p}}(n) \end{aligned}$$

where $q$ is the number of frequency components, $\omega_i$ is the frequency of the $i$th component, coefficients $\{a_i, b_i\}_{i=1}^{q}$ are discrete Fourier coefficients (DFC) of the noise components. $v_{\mathrm{p}}(n)$ is a zero-mean additive white noise with variance $\sigma_p^2$, $p_i(n)$ is the $i$th frequency component of $p(n)$. The true secondary path is indicated by $S(z)$, while $\hat{S}(z)$ stands for its estimate identified in advance. Both secondary paths, $S(z)$ and $\hat{S}(z)$, are FIR filters, having orders $M-1$ and $\hat{M}-1$, respectively. Their corresponding filter coefficients are $\{s_j\}_{j=0}^{M-1}$ and $\{\hat{s}_j\}_{j=0}^{M-1}$. The reference cosine and sine waves are denoted as $x_{a_i}(n) = \cos(\omega_i n)$, $x_{b_i}(n) = \sin(\omega_i n)$ and their x-filtered versions are expressed by $\hat{x}_{a_i}(n)$, $\hat{x}_{b_i}(n)$.

The output of the $i$th subcontroller is calculated by

$$y_i(n) = \hat{a}_i(n) x_{a_i}(n) + \hat{b}_i(n) x_{b_i}(n) \quad (2)$$

and its filtered version becomes

$$y_{p,i}(n) = \sum_{j=0}^{M-1} s_j y_i(n-j). \quad (3)$$

Then the residual noise can be calculated by

$$e(n) = \sum_{i=1}^{q} [p_i(n) - y_{p,i}(n)] + v_{\mathrm{p}}(n). \quad (4)$$

In order to avoid the interaction among frequency channels, the cost function is designed as follows:

$$J(n) = \frac{1}{2} \sum_{i=1}^{q} \{e_i^2(n)\} \quad (5)$$

where $e_i(n)$ is an individual residual noise signal generated by the RES subsystem and its ideal value is $p_i(n) - y_{p,i}(n)$. Note that the residual noise $e(n)$ is not involved explicitly at all in the above cost function. The gradient at time instant $n$ is given by

$$\begin{aligned} \nabla_{\hat{a}_i} J(n) &= -e_i(n) \hat{x}_{a_i}(n) \quad (6) \\ \nabla_{\hat{b}_i} J(n) &= -e_i(n) \hat{x}_{b_i}(n). \quad (7) \end{aligned}$$

The update of control filter weights becomes

$$\begin{aligned} \hat{a}_i(n+1) &= \hat{a}_i(n) + \mu e_i(n) \hat{x}_{a_i}(n) \quad (8) \\ \hat{b}_i(n+1) &= \hat{b}_i(n) + \mu e_i(n) \hat{x}_{b_i}(n) \quad (9) \end{aligned}$$

where step size $\mu$ usually takes a very small positive value.

The individual residual noise $e_i(n)$ of the $i$th channel can be calculated by the RES subsystem:

$$e_i(n) = \hat{a}_{\mathrm{e}_i}(n) x_{a_i}(n) + \hat{b}_{\mathrm{e}_i}(n) x_{b_i}(n) \quad (10)$$

A leaky factor $0 < \gamma < 1$ is introduced for the RES to improve its stability:

$$J_{\mathrm{RES}}(n) = \frac{1}{2} e^2(n) + \frac{1}{2} \gamma \sum_{i=1}^{q} \left[ \hat{a}_{\mathrm{e}_i}^2(n) + \hat{b}_{\mathrm{e}_i}^2(n) \right]. \quad (11)$$

Using (11) and (10) leads to

$$\begin{aligned} \nabla_{\hat{a}_{\mathrm{e}_i}} J_{\mathrm{RES}}(n) &= \frac{\partial J_{\mathrm{RES}}(n)}{\partial e(n)} \cdot \frac{\partial e(n)}{\partial \hat{a}_{\mathrm{e}_i}(n)} + \gamma \hat{a}_{\mathrm{e}_i}(n) \quad (12) \\ &\approx e(n) x_{a_i}(n) + \gamma \hat{a}_{\mathrm{e}_i}(n) \end{aligned}$$

$$\begin{aligned} \nabla_{\hat{b}_{\mathrm{e}_i}} J_{\mathrm{RES}}(n) &= \frac{\partial J_{\mathrm{RES}}(n)}{\partial e(n)} \cdot \frac{\partial e(n)}{\partial \hat{b}_{\mathrm{e}_i}(n)} + \gamma \hat{b}_{\mathrm{e}_i}(n) \quad (13) \\ &\approx e(n) x_{b_i}(n) + \gamma \hat{b}_{\mathrm{e}_i}(n). \end{aligned}$$

Note that bold approximations, i.e., $\frac{\partial e(n)}{\partial \hat{a}_{\mathrm{e}_i}(n)} \approx x_{a_i}(n)$ and $\frac{\partial e(n)}{\partial \hat{b}_{\mathrm{e}_i}(n)} \approx x_{b_i}(n)$, are made in (12) and (13), which make the implementation of RES possible but are expected to introduce some distortions to the algorithm that follows. The delicateness of some of the user parameters may come from these approximations. Further considerations are required.

The updating formula for RES subsystem are

$$\begin{aligned} \hat{a}_{\mathrm{e}_i}(n+1) &= \xi \hat{a}_{\mathrm{e}_i}(n) - \mu_{\mathrm{RES}} e(n) x_{a_i}(n) \quad (14) \\ \hat{b}_{\mathrm{e}_i}(n+1) &= \xi \hat{b}_{\mathrm{e}_i}(n) - \mu_{\mathrm{RES}} e(n) x_{b_i}(n) \end{aligned}$$

where $\xi = 1 - \mu_{\mathrm{RES}} \gamma$. The parameter $\xi$ plays an important role in the update equations, as it considerably affects the system performance. To make the RES work well, one has to make a compromise between convergence speed and steady-state performance. Here, a VL factor is used

$$\gamma(n) = \alpha\gamma(n-1) + (1-\alpha)\gamma_{\max} \tag{15}$$

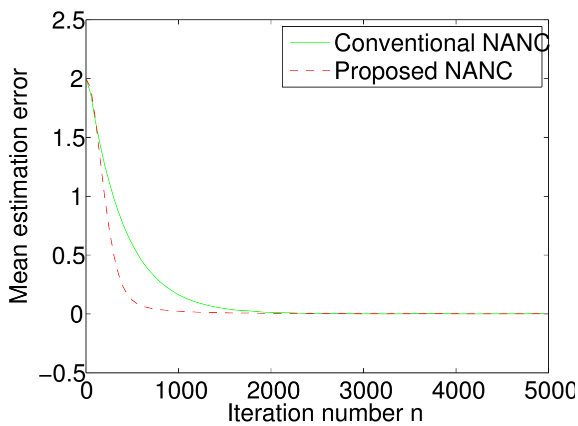
where $\alpha$ is a constant defined within $(0, 1]$, with typical values like 0.975, 0.980 etc., $\gamma_{\min}$ is the initial value of the leaky factor ($\gamma(0) = \gamma_{\min}$) that usually takes on a value like 0.08 or 0.1, while $P_{\max}$ is its steady-state value that should be quite close to 1, such as 0.8, 0.9, etc. This way, the VL-LMS based RES subsystem may converge very fast in the early stage of adaptation, quickly separating the residual noise $e(n)$ into individual resdiual noise $e_i(n)$ or targeted sinusoid within $e(n)$ without sacrificing the convergence of the overall system.

## III. Simulation

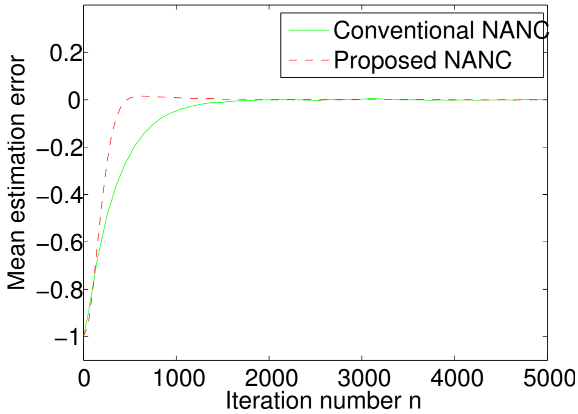
In order to demonstrate the effectiveness of the proposed NANC system, various simulations are carried out.

The primary sinusoidal noise contains three frequencies, i.e. $\omega_1 = 0.1\pi$, $\omega_2 = 0.2\pi$, and $\omega_3 = 0.3\pi$ ($q = 3$). Their corresponding DFCs are $a_1 = 2.0, b_1 = -1.0$, $a_2 = 1.0, b_2 = -0.5$, $a_3 = 0.5$, and $b_3 = 0.1$. The true secondary path $S(z)$ is a low-pass filter created by Matlab function (FIR1) with cutoff frequency $0.4\pi$ and its filter order $(M-1)$ is set to be 40. The secondary-path estimate $\hat{S}(z)$ with an order 30 $(= \hat{M} - 1)$ is obtained by using of an LMS algorithm based on the system identification configuration. An additive noise is zero-mean and has a variance $\sigma_p^2 = 0.1$. The chosen of step-size plays an important role on the convergence and steady-state error, we select the step-sizes that provide a better performance. The step size of conventional and proposed NANC systems are set to 0.005 and 0.002, respectively. The step size for the RES subsystem is 0.1.

First, comparisons between the conventional NANC system and the proposed one with RES are provided. The parameter $\alpha$ is set 0.9996. The maximum and minimum leaky factors are $\gamma_{\max} = 0.6$ and $\gamma_{\min} = 0.08$, respectively.

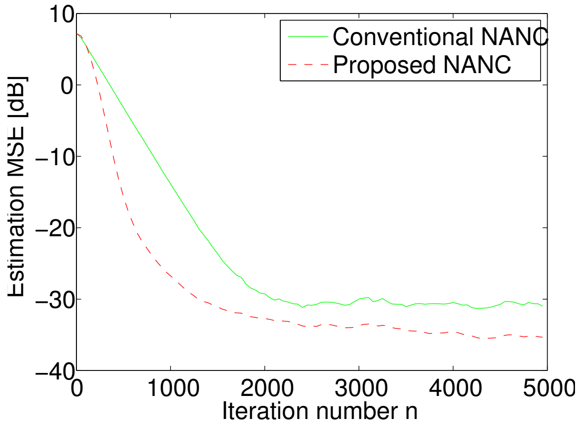



(a) $E[\varepsilon_{\hat{a}_1}(n)]$.

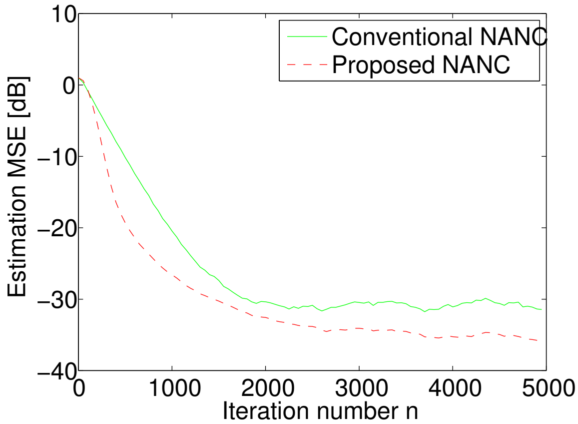



(b) $E[\varepsilon_{\hat{b}_1}(n)]$.

Fig. 3. Comparison of DFC estimate error between the conventional and the proposed narrowband ANC system.




(a) $\sum_{i=1}^{q} E[\varepsilon^2_{\hat{a}_i}(n)]$.




(b) $\sum_{i=1}^{q} E[\varepsilon^2_{\hat{b}_i}(n)]$.

Fig. 4. Comparison of DFC estimate MSE between the conventional and the proposed narrowband ANC system.

Fig. 3 shows comparisons of DFC estimation errors for the first frequency, $E[\varepsilon_{\hat{a}_1}(n)]$ $(= E[\hat{a}_{1,\text{opt}} - \hat{a}_1(n)])$ and $E[\varepsilon_{\hat{b}_1}(n)]$ $(= E[\hat{b}_{1,\text{opt}} - \hat{b}_1(n)])$, where $\hat{a}_{1,\text{opt}}$ and $\hat{b}_{1,\text{opt}}$ are optimal DFCs for the controller to converge to [5].

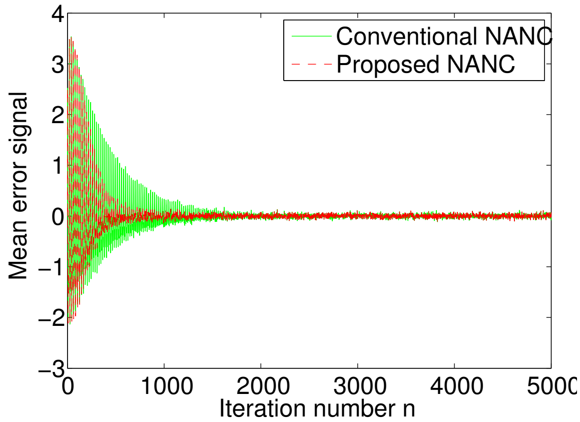



Fig. 5. Comparison of $E[e(n)]$ of the conventional and proposed system.

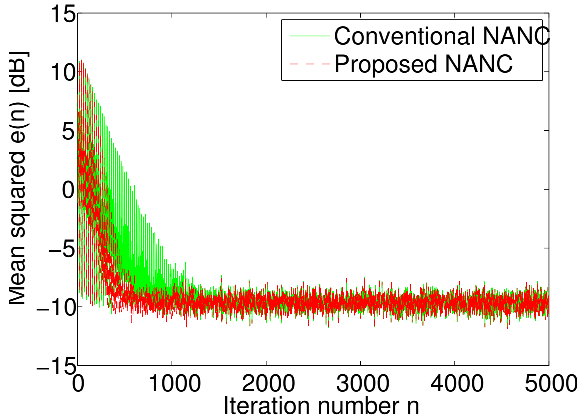
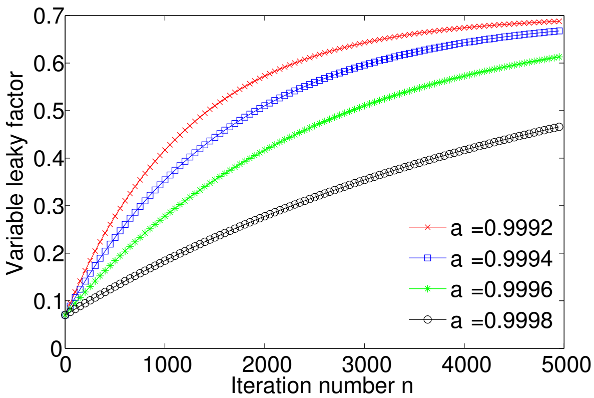


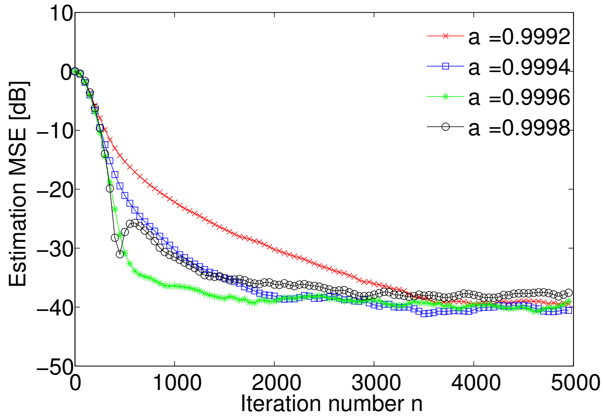

Fig. 6. Comparison of $E[e^2(n)]$ of the conventional and proposed system.

Fig. 4 presents the summation of estimation MSE of DFCs for all frequencies $E[\sum_{i=1}^{q}\varepsilon_{\hat{a}_i}^2(n)]$ and $E[\sum_{i=1}^{q}\varepsilon_{\hat{b}_i}^2(n)]$. To judge the system performance more clearly, the mean residual noise $E[e(n)]$ and its power $E[e^2(n)]$ of the conventional and the proposed systems are presented in Fig. 5 and Fig. 6, respectively.

Second, the selection of leaky factor is investigated by simulations. Different variable leaky factors are given in Fig. 7(a), and the corresponding DFC convergence curves are plotted in Fig. 7(b).




(a) Leaky factor changing curves




(b) Convergence of $E[\varepsilon_{\hat{b}_1}^2(n)]$

Fig. 7. Comparison of DFC convergence for different leaky factor control coefficients.

From the above comparisons, we have found the following insights.

The proposed system with a VL-LMS based RES works considerably better than the conventional system on the whole. See Figs. 3-6 for details. The new complete parallel system converges much faster than its counterpart does, and presents rather smaller steady-state DFC MSE. If the step sizes of two systems are set to the same $0.002$, the conventional system will convergence even slower, and still show a poorer steady-state performance compared with the proposed one.

The leaky factor plays an important role in determining the system performance. It must take positive values smaller than 1. A larger leaky factor will provide smaller steady-state DFC MSE, while the convergence will be very sluggish. If one sets the leaky factor to $\gamma = 1$, the update of the RES reduces exactly to the LMS, and the RES will present very poor stability.

Using a variable leaky factor will provide fast convergence in the transition phase, and yield small residual noise in the steady state. The user parameter $\alpha$ controls the dynamics of leaky factor as shown in Fig.7. The third parameter $\alpha = 0.9996$ provides the best system performance.

## IV. Conclusion

A new complete parallel NANC system has been proposed that is equipped with a VL-LMS based RES subsystem. The RES is capable of extracting every individual sinusoid or error element remining in the residual noise fast enough not to sacrifice the convergence of the overall system. The use of extracted individual sinusoid to update the subcontrollers in all frequency channels significantly enhances their performance. A lot of simulations have revealed that the proposed system enjoys better dynamics as well as steady-state performance as compared with its counterpart.

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