Modified Median FxLMS for Impulsive Noise Reduction in ANC

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***Abstract*—Impulsive noise reduction is a special problem raised in active noise control (ANC) systems. The filtered-x least mean square (FxLMS) algorithm is a typical ANC method that has been successfully applied in practical applications. Some variants such as the logarithmic FxLMS and modified FxLMS algorithms have been proposed to deal with the impulsive noise for ANC. However, those methods require considerable computational loading in real-time applications. In this paper, a modified median FxLMS method is proposed for its low complexity to avoid the effect of the impulsive noise. Simulation results show that the proposed algorithm has better averaged noise reduction performance than the conventional FxLMS and other order statistics based FxLMS algorithms. In addition, the computational complexity stays the minimum requirement through using only the ordering operation, which is very suitable for the real-time implementation.**

***Index Terms*—Active noise control, Gaussian mixture impulsive noise, FxLMS algorithm, Median.**

I. Introduction

Noise cancellation methods can be generally divided into two different approaches: one is passive cancellation and the other is active cancellation. The passive cancellation approach basically relies on the material property to prevent from the noise. Due to practical limitations, the performance of the passive cancellation methods may be subject to the material engineering and physical design, usually also going with the cost of higher price. Thanks to the improvement of modern digital technology, the active cancellation approach has received a lot of attention, usually together with a satisfying performance and rich study in recent years.

In recent years, more and more audio applications have been introduced because of the advance in technology development. The requirement of active noise control (ANC) is widely found in cars, mobile phones, fans, etc. ANC goes to process the received sound waves in an earphone, for example, and the secondary path tries to generate the signal which is close to the noise appearing in the primary path. The noise is then canceled through a loudspeaker embedded in the earphone. ANC can improve the efficiency in noise control with lower volume and cost [1][2]. The most widely used method in the ANC approach is to apply the filtered-x least mean square (FxLMS) algorithm [2]. The algorithm has the advantages of robust performance [3], low computational complexity, and ease of implementation.

When the ANC system encounters the impulsive noise, the FxLMS algorithm will not reach a satisfying steady-state performance that can be achieved in the Gaussian noise condition. In [4], a new robust method called the filtered-x least mean p-power algorithm (FxLMP) was proposed, but its cost function $J(n) = E\{|e(n)|^p\} \approx |e(n)|^p$, where p is an integer number, indicates that the better results require knowing the prior parameter p, which is not an easy task. In [5], there is a simple variant of the FxLMS algorithms for ANC to deal with the impulsive noise. The reference signal $x(n)$ will be clipped if its amplitude is larger than a certain threshold. This algorithm can be more robust than the conventional FxLMS algorithm, but it needs to find two clipping parameters $[c1, c2]$ for the upper and lower amplitude levels, and moreover, the performance of this algorithm highly depends on the clipping parameters $c1$ and $c2$. In [6] and [7], Akhtar's algorithm improved the performance better than the Sun's algorithm [5], where if the reference signal is over a pre-determined threshold. In above mentioned algorithms, the common problem using those methods is to find appropriate threshold parameters, which may be not easy to be well used with on-line operation in general ANC systems.

In this paper, the proposed modified median FxLMS algorithm has lower computational complexity in $O(L)$, where L is the number of the filter taps in the secondary path. Compared with the previously proposed FxLMS algorithms for the problem caused by the impulsive noise, the new algorithm does not need to find clipping thresholds. Numerical results show that the proposed algorithm is more effective than other FxLMS algorithms.

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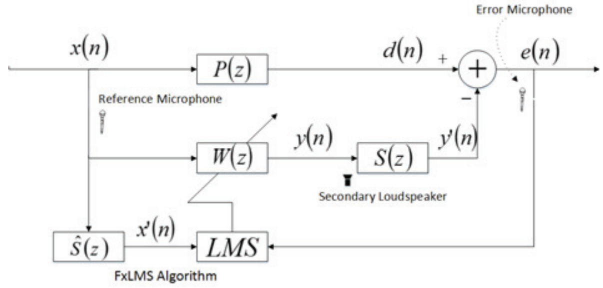

Fig. 1: Functional block of the conventional FxLMS algorithm.

II. FxLMS Algorithm

Fig. 1 depicts the functional block of the conventional single-channel feed-forward ANC structure using the FxLMS algorithm [2]. The noise source $x(n)$ is received from the receiver microphone, the system response $P(z)$ in the primary path is modeled for the physical channel between the receiver microphone and the error microphone, and the secondary-path response $S(z)$ models the characteristics of the secondary loudspeaker in the earphone. In addition, $\hat{S}(z)$ is the estimated response of $S(z)$ in the FxLMS algorithm, which can be obtained by on-line or off-line methods in practical applications. The objective of the adaptive filter $W(z)$ is to minimize the residual error signal $e(n)$, which essentially establishes the adaptation criterion in the FxLMS algorithm.

Assuming $W(z)$ is an finite impulse response (FIR) filter of the length of L taps, the corresponding output signal $y(n)$ is expressed as

$$y(n) = \mathbf{w}^T(n)\mathbf{x}(n), \tag{1}$$

where $\mathbf{w}(n) = [w_0(n), w_1(n), w_2(n), \cdots, w_{L-1}(n)]^T$ is the tap coefficient vector of $W(z)$ and $\mathbf{x}(n) = [x(n), x(n-1), \cdots, x(n-L+1)]^T$ is the $L \times 1$ input signal vector. The residual error signal $e(n)$ received by the error microphone is given by

$$e(n) = d(n) - y^{'}(n), \tag{2}$$

and $d(n) = p(n) * x(n)$ is the primary disturbance noise and $y^{'}(n) = s(n) * y(n)$ is the secondary canceling signal, where $*$ denotes convolution, $p(n)$ is the impulse response of the primary path model $P(z)$, and $s(n)$ is the impulse response of the secondary path model $S(z)$.

It is known that the least mean square (LMS) algorithm minimizes the mean square error (MSE) of the error signal to adaptively find the optimum filter coefficients. The negative gradient direction with a step size μ is established for the LMS algorithm with the tap update equation, usually written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mu}{2}\nabla J(n), \tag{3}$$

where ∇ denotes taking gradient, which is used to minimize the MSE of the cost function

$$J(n) = E[e^2(n)] \approx e^2(n), \tag{4}$$

where $E[\cdot]$ is the expectation operation. The FxLMS algorithm is modified by giving

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_1 e(n)\mathbf{x}^{'}(n), \tag{5}$$

where μ_1 is the step size for the FxLMS algorithm, $\mathbf{x}^{'}(n) = [x^{'}(n), x^{'}(n-1), \cdots, x^{'}(n-L+1)]^T$, and $x^{'}(n) = \hat{s}(n) * x(n)$, where $\hat{s}(n)$ is the impulse response of the estimated secondary path model $\hat{S}(z)$.

III. Modified Median FxLMS Algorithm

The performance of the ANC system will be seriously degraded if the conventional FxLMS algorithm is performed in the impulsive noise environment. The impulsive noise is a non-Gaussian noise with a long-tailed distribution such that there are possibly some noises of very large values, though with low probability, appearing at the input of the FxLMS structure. Some robust LMS algorithms have been studied to deal with the impulsive noise, however, the median LMS algorithm[8][9] is very appealing because of its low complexity. For the real-time ANC applications, computational complexity is critical and that is the main reason why we here propose a modified median FxLMS algorithm to improve the ANC performance in the impulsive noise.

A. Impulsive Noise

The FxLMS input $x(n)$, in general, is assumed to be a Gaussian noise $x_g(n)$. In this paper, for considering the interference of the impulsive noise, the interference can be modeled as a long-tailed distribution $x_\ell(n)$, say the exponential distribution. For simplicity, $x(n)$ can be modeled as

$$x(n) = (1-\eta)x_g(n) + \eta x_\ell(n), \tag{6}$$

where $x_g(n)$ and $x_\ell(n)$ are both independent and identically distributed (i.i.d.) zero-mean Gaussian sequences with variance σ_g^2 and σ_ℓ^2. η is an i.i.d. Bernoulli random sequence whose value is either one or zero with occurrence probabilities $P_r(\eta = 1) = p_r$ and $P_r(\eta = 0) = 1-p_r$, where p_r determines the percentage of appearing a large value of outliers in the mixture noise model. To characterize the impulsive noise, we have $\sigma_g^2 \ll \sigma_\ell^2$ and p_r can be near zero, say 0.001.

The overall noise variance in this sense becomes $\sigma_x^2 = (1-\eta)\sigma_g^2 + \eta\sigma_\ell^2$. If the occurrence probability of the impulsive noise is very small, the variance of $x(n)$ can be approximated as σ_g^2, which is the main reference value to set the threshold value of the proposed algorithm in the next subsection.

B. Modified Median FxLMS Algorithm

Instead of (5), we consider a new update innovation for the FxLMS filter as follows. Let

$$\mathbf{z}(n) = [e(n)x^{'}(n)\ e(n-1)x^{'}(n-1) \cdots e(n-L+1)x^{'}(n-L+1)]. \tag{7}$$

We define that the L ascending order statistics of the elements of $\mathbf{z}(n)$ as

$$z_{(1)}(n),\ z_{(2)}(n),\ \cdots, z_{(L)}(n).$$

The median of $\mathbf{z}(n)$ is then defined as

$$\psi_L(n) = \mathrm{med}\{\mathbf{z}(n)\}_L = z_{(\frac{L+1}{2})}(n), \tag{8}$$

where we suppose L is an odd number. The median FxLMS replaces the update innovation $e(n)\mathbf{x}^{'}(n)$ in (5) with $\Psi_L(n)$, where $\Psi_L(n) = [\psi_L(n)\ \psi_L(n-1) \cdots \psi_L(n-L+1)]^T$, which can be written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_2\Psi_L(n), \tag{9}$$

where μ_2 is the step size. It is worthy of note, the median FxLMS changes the convergence behavior on which the FxLMS algorithm is based. In sequence, the median FxLMS may have worse performance in the Gaussian noise. The incorporation of median also leads to a little computational overhead in $O(N)$ operations. In spite of above weakness, the median FxLMS has better resistance to the impulsive noise.

In practical applications, the interference of the impulsive noise usually has quite low probability. It is known that the median FxLMS may lead to worse performance than the standard FxLMS when the noise is Gaussian. Then, we develop a modified median FxLMS structure by introducing an impulse detection block in front of implementing the median.

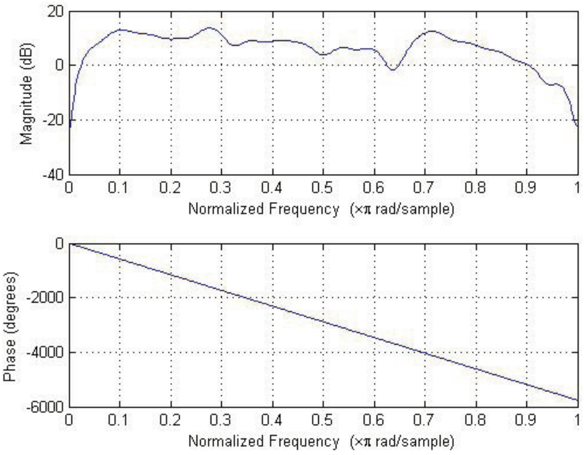

Fig. 2: Frequency response of the primary path model $P(z)$.

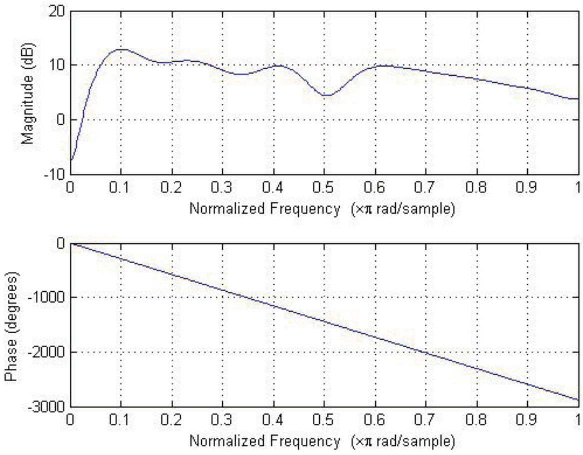

Fig. 3: Frequency response of the secondary path model $S(z)$.

Suppose $s(n)$ is normalized, the impulsive noise is determined by the following rule:

$$\max\{|\mathbf{x}^{'}(n)|\} \underset{\mathrm{H}_2}{\overset{\mathrm{H}_1}{\gtrless}} \gamma, \tag{10}$$

where γ is a pre-determined threshold, and H_1 indicates implementing (9) while H_2 implementing (5). One important concern in this algorithm is the setup of γ. As we have mentioned in above subsection, the impulsive interference actually has low occurrence probability in most of practical applications such that, by the three-sigma rule of thumb, nearly all Gaussian noise has the value below $3\sigma_g$ Therefore, we choose $\gamma = 3\sigma_g$ in this paper.

IV. Simulation Results

In this section, the performance of the modified median FxLMS algorithm is compared with that of the conventional FxLMS and other order statistics based FxLMS algorithms such as the mean FxLMS and the trimmed-mean FxLMS algorithms. The trimmed mean [10] is defined as calculating the mean after discarding given parts of the samples of values at high and low end. The tap lengths of the primary and secondary path models, $P(z)$ and $S(z)$, are 65 and 35, respectively. The frequency responses of the two models are plotted in Figs. 2 and 3. The tap length of the adaptive filter $W(z)$ is chosen as $L = 35$. Hence, the median in the proposed algorithm is chosen from the nearby 35 samples and the number of samples to calculate the mean in the trimmed-mean FxLMS algorithm are 11. In this paper, we suppose that the estimated secondary path model $\hat{S}(z)$ exactly equals to $S(z)$.

In this paper, the impulsive noise is simulated with the mixture Gaussian noise model written by

$$x(n) = (1-\eta)N(0,1) + \eta N(0,\sigma^2), \tag{11}$$

where $N(\alpha, \beta^2)$ denotes the Gaussian distribution with mean α and variance β^2, η is determined from the uniform distribution $U(0,1)$, and σ^2 will be set in the following experimental cases. In addition, if p_r is set as 0.001, that is, in average, one sample is considered as the impulsive interference in every 1000 input samples.

We compare the performance in terms of the mentioned 3σ decision rule for γ, that is, $\gamma = 3$ here, to analyze the performance metric of averaged noise reduction (ANR) [11]. Define ANR in decibel (dB) as

$$ANR(n)[\mathrm{dB}] = 20\log_{10}\left(\frac{A_e(n)}{A_d(n)}\right) \tag{12}$$

where

$$A_e(n) = \lambda A_e(n-1) + (1-\lambda)|e(n)| \tag{13}$$
$$A_d(n) = \lambda A_d(n-1) + (1-\lambda)|d(n)| \tag{14}$$

where $|\cdot|$ represents absolute value, λ is a forgetting factor and is set as 0.999.

A. Experimental Case 1

In Fig. 4, the variance of the signal $x(n)$ is set as $\sigma^2 = 1000$ and $p_r = 0.001$, the step sizes are the same $\mu_1 = 10^{-3}$ and $\mu_2 = 10^{-3}$ for all of the compared algorithms. The results of ANR are shown in Fig. 5. The proposed algorithm has better ANR performance than other three algorithms though the trimmed-mean FxLMS shows a little better results in parts of iterations. This indicates that median can effectively suppress the influence of the impulsive interference.

B. Experimental Case 2

In Fig. 6, the variance of the signal $x(n)$ is set as $\sigma^2 = 1000$ and $p_r = 0.003$, step sizes are $\mu_1 = 10^{-3}$ and $\mu_2 = 10^{-3}$ for all algorithms. The results of ANR are shown in Fig. 7. In this case, the occurrence probability of the impulsive noise is apparently more than that in case 1. The conventional FxLMS, the mean FxLMS, and even the trimmed-mean FxLMS algorithms obviously fail to give correct function. The proposed algorithm still maintains a better ANR performance.

V. Conclusion

The proposed modified median FxLMS has sufficient robustness to deal with the influence of the impulsive noise.

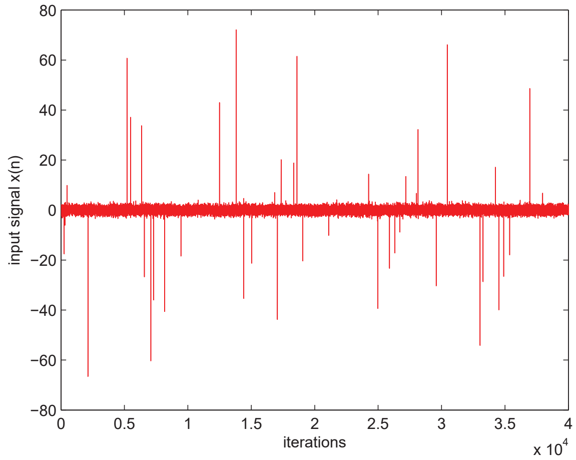

Fig. 4: Input $x(n)$ simulated by the Gaussian mixture impulsive noise with $p_r = 0.001$ and $\sigma^2 = 1000$ in Experiment case 1.

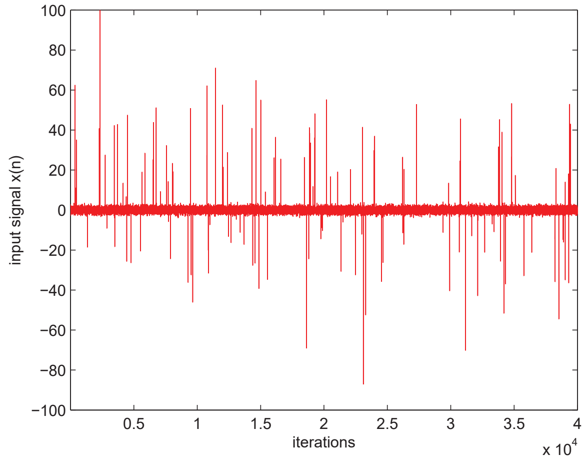

Fig. 6: Input $x(n)$ simulated by the Gaussian mixture impulsive noise with $p_r = 0.003$ and $\sigma^2 = 1000$ in Experiment case 2.

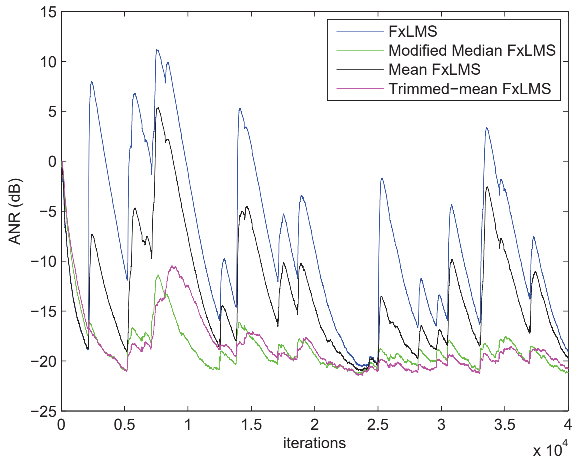

Fig. 5: ANR curves of the compared FxLMS algorithms in Experiment case 1.

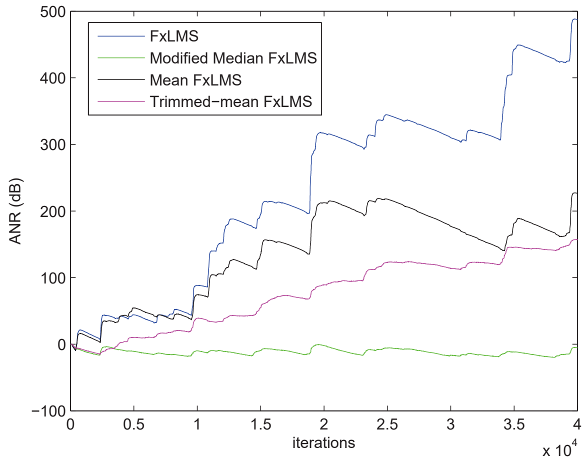

Fig. 7: ANR curves of the compared FxLMS algorithms in Experiment case 2.

According to the simulation results, even when the input signal has more intensively impulsive noises, the proposed algorithm still can effectively suppress the impulsive noise with a good ANR performance. We can see that the proposed method is robust to the impulsive noise.

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