# WIRELESS DIGITAL DEMODULATION SYSTEM VIA HIERARCHICAL MULTIRESOLUTION EMPIRICAL MODE DECOMPOSITION APPROACH

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#### ABSTRACT

This paper proposes a hierarchical multiresolution based empirical mode decomposition approach for performing a wireless digital demodulation. The waveform corresponding to each digital symbol is represented as the sum of the intrinsic mode functions. Each obtained intrinsic mode function of each symbol is further decomposed via a discrete cosine transform approach. First, zeros are inserted in each intrinsic mode function in the discrete cosine transform domain. Then, the next level empirical mode decomposition is performed in the time domain. The discrete cosine transform coefficients of the next level intrinsic mode functions where the zeros are added are removed in the discrete cosine transform domain. This step is repeated and the obtained intrinsic mode functions in each level form a dictionary. The received signals corrupted by the additive noises with unknown distributions and distorted by nonlinear channels are represented by both the weight vectors and the residue vectors based on the dictionary at each level of decomposition. The decoding scheme is to find the corresponding symbol such that the sum of the residue vectors in various levels of the decomposition is minimized. Experimental results show that the decoding accuracy is higher than that of the conventional matched filter approach.

### **1. INTRODUCTION**

Many devices such as home appliances, office equipments, mobile handsets and laptop computers are connected together via wireless communication networks. The common wireless communication networks include Wifi, Zigbee, Bluetooth and cellular mobile networks [1]-[3]. To transmit signals through wireless channels, signals are usually modulated to high frequency bands [4]. This is because high frequency signals can transmit more far away than low frequency signals. The received signals are then demodulated back to the baseband signals. Hence, modulation and demodulation are usually employed in wireless communication systems and they play a very important role in our daily life [1]-[4].

The common digital demodulation system is via the matched filter approach [5], [6]. A received signal is projected to a bank of matched filters. The symbol corresponding to the maximum absolute projection value is taken as the corresponding decoded symbol. Here, the

waveforms representing the symbols have to be orthonormal to each others. Nevertheless, the waveforms are usually distorted by nonlinear channels (The channel characteristics are nonlinear with respect to the transmitted signals.). Hence, the received waveforms in the practical situations are not orthonormal to each others. Also, the wireless channel noise is required to be an additive white Gaussian distributed. However, this is also not the case in many practical situations.

A hierarchical multiresolution based empirical mode decomposition of signals is implemented via the discrete Fourier transform [9]. However, as the discrete Fourier transform is a complex valued transform, the required computational power for the further processing is high. In this paper, a hierarchical multiresolution based empirical mode decomposition of signals is implemented via the discrete cosine transform. As the discrete cosine transform is a real valued transform, the required computational power for the further processing is significantly reduced. Moreover, this paper further applies the hierarchical multiresolution based empirical mode decomposition method for performing the wireless digital demodulation. It is worth noting that the conventional matched filter approach is a linear approach based on both the orthonormality among the waveforms and the additive white Gaussian distributed channel noises. On the other hand, although the signals are represented as the sum of the intrinsic mode functions, the intrinsic mode functions are nonlinear with respect to the signals. Hence, our proposed approach is a nonlinear approach which does not require the orthonormality among the waveforms and the additive white Gaussian distributed channel noises. Therefore, our approach would be more suitable for nonlinear channels [7], [8]. Moreover, each symbol in the conventional matched filter approach is represented by a single input single output linear time invariant filter while each symbol in our proposed approach is represented by a weighted vector. As the decoding process of the conventional matched filter approach is to compare the magnitudes of the projected values in the one dimensional space while the decoding process of our proposed approach is find the solution of an optimization problem in a high dimensional space, the degree of freedom of our proposed approach is much higher than that of the matched filter approach.

The outline of this paper is as follows. The proposed digital demodulation scheme is formulated as an optimization problem via a hierarchical mutliresolution based empirical mode decomposition approach. The details are discussed in Section 2. Computer numerical simulation results are presented in Section 3. Finally, a conclusion is drawn in Section 4.

### 2. PROBLEM FORMULATION

Suppose that there are C symbols in a wireless digital communication system and each symbol is represented by an N point discrete time sequence. Let  $\mathbf{x}_c$  for  $c = 0, \dots, C-1$ be the vectors representing these sequences. Assume that  $\mathbf{x}_{a}$ for  $c = 0, \dots, C-1$  are distorted by a nonlinear channel. Let  $\bar{\mathbf{x}}_c$  for  $c = 0, \dots, C-1$  be the vectors representing these distorted sequences. Suppose that  $\overline{\mathbf{x}}_{c}$  for  $c = 0, \dots, C-1$  can be represented by the sum of its intrinsic mode functions. Let  $\mathbf{F}_c$  be the matrix with its columns being the vectors representing the intrinsic mode functions of  $\overline{\mathbf{x}}_{a}$  for  $c = 0, \dots, C-1$ . Obviously, **F** can be obtained via performing the empirical mode decomposition on  $\mathbf{x}_c$  for  $c = 0, \dots, C-1$ . Define  $\mathbf{F} = \begin{bmatrix} \mathbf{F}_0 & \dots & \mathbf{F}_{C-1} \end{bmatrix}$ . Here, the total number of rows of  $\mathbf{F}$  is equal to the signal length and the total number of columns is equal to the total number of intrinsic mode functions. It is worth noting that the total number of intrinsic mode functions is usually smaller than the length of the signal. Hence, **F** is a tall matrix. Moreover, the intrinsic mode functions are usually linear independent. Hence, we can assume that  $\mathbf{F}^T \mathbf{F}$  is invertible. Let  $\mathbf{I}_c$  for  $c = 0, \dots, C-1$  be the weight vector representing  $\overline{\mathbf{x}}_{c}$  using **F**. That is,  $\overline{\mathbf{x}}_c = \mathbf{F}\mathbf{I}_c$  for  $c = 0, \dots, C-1$ . Then, we have  $\mathbf{I}_{c} = (\mathbf{F}^{T}\mathbf{F})^{-1}\mathbf{F}^{T}\mathbf{\bar{x}}_{c}$  for  $c = 0, \dots, C-1$ . It is well known in the wavelet theory that the frequency bands at high scale wavelets are narrower than those at the low frequency bands. Hence, fine details of the signals can be extracted out if the signals are represented using high scales of wavelets. Similarly, if the intrinsic mode functions can be further decomposed, then more fine information can be employed for the demodulation. Nevertheless, it is worth noting that the intrinsic mode functions cannot be further decomposed if the empirical mode decomposition is directly applied to the intrinsic mode functions. In order to address this difficulty, the decomposition of the signal is performed via a discrete cosine transform approach. The detail procedures are as follow. First, the discrete cosine transforms of the intrinsic mode functions are computed. Second, zeros are inserted in the intrinsic mode functions in the discrete cosine transform domain. Third, the inverse discrete cosine transforms of these zero inserted intrinsic mode functions are computed. Fourth, the next level empirical mode decomposition of each zero inserted intrinsic mode function

is performed in the time domain. Fifth, the discrete cosine transforms of these next level intrinsic mode functions are computed. Sixth, the discrete cosine transform coefficients of the next level intrinsic mode functions are removed where the removed coefficients are located exactly the same as that the zeros are added in the discrete cosine transform domain. Seventh, the inverse cosine transforms of these zero removed intrinsic mode functions are computed. As a result, the next level intrinsic mode functions are obtained. Similarly, denote  $\tilde{\mathbf{F}}$  as the matrix with its columns being the vectors representing the next level intrinsic mode functions. Let  $\widetilde{l}_{c}$  for  $c = 0, \dots, C-1$  be the weight vector representing  $\overline{\mathbf{x}}_c$  using  $\widetilde{\mathbf{F}}$ . That is,  $\overline{\mathbf{x}}_c = \widetilde{\mathbf{F}}\widetilde{\mathbf{l}}_c$  for  $c = 0, \dots, C-1$ . Similarly, we also assume that  $\tilde{\mathbf{F}}^T \tilde{\mathbf{F}}$  is invertible. Hence, we have  $\tilde{\mathbf{I}}_{c} = (\tilde{\mathbf{F}}^{T}\tilde{\mathbf{F}})^{-1}\tilde{\mathbf{F}}^{T}\bar{\mathbf{x}}_{c}$  for  $c = 0, \dots, C-1$ . In fact, more intrinsic mode functions in higher levels of the decomposition can be obtained by repeating the above procedures. Now consider the case that the transmitted signals are corrupted by the noises with an unknown distribution and distorted by a nonlinear channel. Let y be the received signal. In this case, it is not guaranteed that v can be represented as a linear combination of the columns of F or  $\widetilde{\mathbf{F}}$  and we have the reconstruction errors. Denote  $\mathbf{r}$  and  $\mathbf{l}$  as the reconstruction error and the weight vector by representing v using F, respectively. Similarly, denote  $\tilde{r}$ and  $\tilde{\mathbf{i}}$  as the reconstruction error and the weight vector by representing v using  $\widetilde{\mathbf{F}}$ , respectively. That is,  $\mathbf{v} = \mathbf{F}\mathbf{I} + \mathbf{r}$  and  $\mathbf{v} = \widetilde{\mathbf{F}}\widetilde{\mathbf{l}} + \widetilde{\mathbf{r}}$ . It is worth noting that a new set of intrinsic mode functions are required to be computed for a new signal

approach. However, as the difference between  $\mathbf{y}$  and  $\overline{\mathbf{x}}_c$  for a particular value of  $c \in \{0, \dots, C-1\}$  is only due to the presence of the additive noise and the difference due to the time varying channel, the required computational power can be reduced if the new signal is represented by the linear combination of the intrinsic mode functions of the original signal plus an error signal without performing the empirical mode decomposition again. The robustness of the empirical mode decomposition depends on the energy of the additive noise and the stationarity of the channel. From the demodulation viewpoint, we need to determine I

based on the conventional empirical mode decomposition

From the demodulation viewpoint, we need to determine **I** and  $\tilde{\mathbf{i}}$  for a new received signal  $\mathbf{y}$ . For a slow varying channel and a low noise energy environment, both  $\|\mathbf{y} - \mathbf{FI}\|^2$  and  $\|\mathbf{y} - \mathbf{\widetilde{FI}}\|^2$  are small. Therefore, the demodulation scheme can be formulated as the following optimization problem: Problem (P)

$$\min_{c} \|\mathbf{y} - \mathbf{F} \mathbf{l}_{c}\|^{2} + \|\mathbf{y} - \widetilde{\mathbf{F}} \widetilde{\mathbf{l}}_{c}\|^{2},$$
  
subject to  $\mathbf{l}_{c} \in {\mathbf{l}_{0}, \dots, \mathbf{l}_{C-1}}$  and  $\widetilde{\mathbf{l}}_{c} \in {\widetilde{\mathbf{l}}_{0}, \dots, \widetilde{\mathbf{l}}_{C-1}}.$ 

Denote  $c^*$  as the optimal solution of Problem (P). Then, the decoding rule is to assign y to  $c^*$ .

Denote **X** and  $\overline{\mathbf{X}}$  as the matrices with their columns being  $\mathbf{x}_c$  and  $\mathbf{\overline{x}}_c$  for  $c = 0, \dots, C-1$ . Let  $\mathbf{I}_c$  be the  $C \times C$  identity matrix. Suppose that the waveforms for representing the symbols are orthonormal to each others. Then, we have  $\mathbf{X}^T \mathbf{X} = \mathbf{I}_C$ . If there is no noise corrupted in the channel and there is no distortion in the channel, then only one element in  $\mathbf{X}^T \mathbf{y}$  is one and all other elements are zero. Hence, by locating the position of the nonzero element in  $\mathbf{X}^T \mathbf{v}$ , the transmitted symbol can be decoded. This is the working principal of the conventional matched filter approach. However, as the channel is nonlinear, in general  $\overline{\mathbf{X}}^T \overline{\mathbf{X}} \neq \mathbf{I}_C$ . Even though there is no noise corrupted in the channel, it is no longer true that there is only one nonzero element in  $\bar{\mathbf{X}}^{T}\mathbf{y}$ . Therefore, the decoding error based on the conventional matched filter approach could be very large. On the other hand, it is still true in the nonlinear channel environment that both  $\overline{\mathbf{x}}_{c} = \mathbf{F}\mathbf{I}_{c}$  and  $\overline{\mathbf{x}}_{c} = \mathbf{\tilde{F}}\mathbf{\tilde{I}}_{c}$  for  $c = 0, \dots, C-1$ . If there is no noise corrupted to the channel and the nonlinear channel is stationary, then there is no decoding error based on our proposed approach. As our proposed approach can eliminate the decoding error due to the stationary nonlinear channel characteristics, our proposed approach in general can achieve a lower decoding error compared to that of the matched filter approach.

### 3. NUMERICAL COMPUTER SIMULATION RESULTS

In this paper, C = 2 is chosen for the demonstration. This is because the binary communication system is the most common wireless digital communication systems employed in practical situations. To represent a waveform by a discrete time sequence, the sampling rate is at least higher than the Nyquist sampling rate of the signal. Hence, *N* could not be too small. In this paper, N = 256 is chosen which is large enough for most simple waveforms. To order to have a fair comparison to the matched filter approach,  $\mathbf{x}_c$ for  $c = 0, \dots, C-1$  should be chosen as the signals which are orthonormal to each others. Here,  $x_0[n] = \sin\left(\frac{2\pi n}{N}\right)$  and

 $x_1[n] = \cos\left(\frac{2\pi n}{N}\right)$  for  $n = 0, \dots, N-1$  are chosen. This is

because they are the most common orthonormal waveforms used in binary communication systems. To demonstrate the effectiveness of our proposed method, the following three cases are considered. The first case considers the situation that there is no deterministic distortion introduced by the channel. The second case considers the situation that the deterministic distortion introduced by the channel is linear and time varying. In this case, the deterministic channel distortion can be modeled by a matrix multiplication. Let the matrix be **H**. Here, we assume that there is no attenuation introduced by this deterministic distortion because of the simplicity reason. This implies that **H** is unitary. The last case considers the situation that the deterministic distortion introduced by the channel is nonlinear but time invariant. Here, the deterministic channel distortion function is modeled by a polynomial function. This is because Taylor series can be employed for modeling a wide class of nonlinear functions. Let the order, the DC gain and the roots of the polynomial be P,  $G^P$  and  $\alpha_p$  for  $p = 0, \dots, P-1$ , respectively, as well as the polynomial function be  $h(\cdot)$ .

That is, 
$$\overline{x}_{c}[n] = h(x_{c}[n]) = \prod_{p=0}^{p-1} G(x_{c}[n] - \alpha_{p})$$
 for

 $c = 0, \dots, C-1$ . Here,  $\alpha_p$  for  $p = 0, \dots, P-1$  are assumed to be uniformly disturbed between -1 and 1 because the dynamical ranges of these sinusoidal waveforms are between -1 and 1. Besides, G = 1.3 is chosen because the DC gain of this nonlinear distortion function is approximately preserved. Moreover, P = 54 is chosen because more terms in the polynomial can achieve a more accurate approximation of a nonlinear function. On the other hand, the channel is also corrupted by an additive random noise. Denote the random noise vector be  $\boldsymbol{v}$  . Hence, we have  $\overline{\mathbf{x}}_{c} = \mathbf{x}_{c}$  for  $c = 0, \dots, C-1$  and  $\mathbf{y} = \mathbf{x}_{c} + \mathbf{v}$  for some  $c \in \{0, \dots, C-1\}$  for the first case,  $\overline{\mathbf{x}}_c = \mathbf{H}\mathbf{x}_c$  for  $c = 0, \dots, C-1$ and  $\mathbf{y} = \mathbf{H}\mathbf{x}_{c} + \mathbf{v}$  for some  $c \in \{0, \dots, C-1\}$  for the second case, and  $\mathbf{y} = \overline{\mathbf{x}}_c + \mathbf{v}$  for some  $c \in \{0, \dots, C-1\}$  for the last case. In this paper, two types of random noises are considered. They are the Gaussian disturbed random noise and the Rayleigh distributed random noise. They are chosen because they are commonly employed for wireless communication channels. For the Rayleigh distributed noise, it is in the form of  $p_R(\eta) = \frac{\eta}{\sigma^2} e^{\frac{\eta^2}{2\sigma^2}}$ , where  $\sigma$  is the

parameter of the Rayleigh distribution controlling the noise energy. For the Gaussian distributed noise, it is in the form

of  $p_G(\eta) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{(\eta-\mu)^2}{2\sigma^2}}$ , where  $\mu$  and  $\sigma$  are the mean and

the standard deviation of the distribution, respectively. Here,  $\sigma$  controls the noise energy and  $\mu = 0$  is chosen because of the simplicity reason.

In the hierarchical multiresolution based empirical mode decomposition algorithm, only two levels of the decompositions are performed because of the simplicity reason. Hence, we only have F and  $\tilde{F}$ . Also, the total numbers of zeros to be inserted and removed are exactly equal to the length of the signal. Also, they are inserted and removed at the highest frequency band.

Define the signal to noise ratio as  $SNR = E\left(\frac{\overline{\mathbf{x}}_c^T \overline{\mathbf{x}}_c}{\mathbf{v}^T \mathbf{v}}\right)$ . Here,

 $E(\cdot)$  is the expectation operator. Figure 1 and Figure 2 plot the bit error rates against the signal to noise ratios for the channel without any deterministic distortion but with the Gaussian disturbed noise and the Rayleigh disturbed noise, respectively. From Figure 1 and Figure 2, we can see that our proposed method achieves the same performances as those of the matched filter approach for both the Gaussian disturbed noise and the Rayleigh disturbed noise. Since the matched filter approach achieves the optimal solutions for both the Gaussian disturbed noise and the Rayleigh disturbed noise for the channel without any deterministic distortion, this implies that our proposed method also achieves the optimal solutions for the channel without any deterministic distortion for both the Gaussian disturbed noise and the Rayleigh disturbed noise. Figure 3 and Figure 4 plot the bit error rates against the signal to noise ratios for the channel with the linear time varying deterministic distortion as well as with the Gaussian disturbed noise and the Rayleigh disturbed noise, respectively. It can be seen from Figure 3 and Figure 4 that the bit error rates for the matched filter approach of both the Gaussian disturbed noise and the Rayleigh disturbed noise drop very slowly as the signal to noise ratios increase. This implies that the matched filter approach is not good for the channel with the linear time varying deterministic distortion. This is because  $\mathbf{X}^T \mathbf{H} \mathbf{x}_{\circ}$  is no longer equal to the vector with the unique nonzero element located at the  $c^{th}$  element. On the other hand, our proposed method still performs very well for the channel with the linear time varying deterministic distortion for both the Gaussian disturbed noise and the Rayleigh disturbed noise. This is because the linear time varying characteristic of the channel is approximated by its nonlinear characteristic. Finally, Figure 5 and Figure 6 plot the bit error rates against the signal to noise ratios for the channel with the nonlinear time invariant deterministic distortion as well as with the Gaussian disturbed noise and the Rayleigh disturbed noise, respectively. Similarly to the above, it can be seen from Figure 5 and Figure 6 that the bit error rates of both the Gaussian disturbed noise and the Rayleigh disturbed noise for the matched filter approach are saturated. Here, "saturated" means that the probability of error per symbol is equal to 0.5. This implies that the matched filter approach basically fails for performing the demodulation for the channel with the nonlinear time invariant deterministic distortion. On the other hand, it can be seen from Figure 5 and Figure 6 that our proposed method still performs very well for the channel with the nonlinear time invariant deterministic distortion for both the Gaussian disturbed noise and the Rayleigh disturbed noise.



<sup>10<sup>3</sup></sup> Signal to noise ratio <sup>10</sup> Figure 1. Channel without any deterministic distortion but

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Figure 2. Channel without any deterministic distortion but with additive Rayleigh distributed noise.



Figure 3. Channel with linear time varying deterministic distortion and additive Gaussian distributed noise.



Figure 4. Channel with linear time varying deterministic distortion and additive Rayleigh distributed noise.



Figure 5. Channel with nonlinear time invariant deterministic distortion and additive Gaussian distributed noise.



deterministic distortion and additive Rayleigh distributed noise.

### 4. CONCLUSION

This paper proposes a hierarchical multiresolution based empirical mode decomposition approach for performing the demodulation for wireless digital communication systems. The waveform corresponding to each digital symbol is represented as the sum of the intrinsic mode functions. Each obtained intrinsic mode function of each symbol is further decomposed in higher levels. The obtained intrinsic mode functions in each level form a dictionary. The received signals corrupted by the additive noises with unknown distributions and distorted by nonlinear channels are represented by both the weight vectors and the residue vectors based on the dictionary at each level of decomposition. The decoding scheme is to find the corresponding symbol such that the sum of the residue vectors in various levels of the decomposition is minimized. Since the empirical mode decomposition approach can capture the nonlinear characteristics of the channel, the experimental results show that the decoding accuracy based on our proposed method is higher than that based on the conventional matched filter approach.

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