BPSK-OFDM Signal Detection in Time-Varying Channels Using Prior Information

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***Abstract*—The signals of BPSK-OFSM systems are real-valued, however, this property is ignored in state-of-the-art approaches, leading to detection performance degradation. In this paper, this prior knowledge is employed to develop a new equalizer based on minimum mean square error (MMSE) criterion. In particular, the proposed equalizer can achieve 3 dB gains comparing to the traditional MMSE equalizer in large signal-to-noise (SNR) region. Furthermore, the successive detection (SD) technique is used in the proposed equalizer is able to take the advantage of time diversity in time-varying channels. The computational complexity of the proposed methods are also analyzed and shown to be more efficient. Computer simulations are included to compare the proposed approaches with the traditional techniques in terms of detection performance.**

I. Introduction

Due to the use of Cyclic Prefix (CP), whose length is not less than the channel length, a low complexity one-tap equalizer is able to efficiently eliminate the inter-symbol interference (ISI) for OFDM systems in the quasi-static channels. However, in the time-varying channels, the time variations of the channel within single OFDM symbol duration could destroy subchannel orthogonality, incurring intercarrier interference (ICI) and eventually leading to system performance degradation. In order to combat the ICI, various methods, such as differential coding, ICI self-cancellation, Doppler diversity, and ICI cancellation-based equalization [1]-[5], have been proposed. Among them, the ICI cancellation-based equalization method is the most common and effective for its good performance under perfect channel estimation.

The linear equalizers, such as the least squares (LS) and minimum mean square error (MMSE) equalizers are widely used in the communication systems, because of their simple structure. However, for time-varying channels, the classical linear equalizers cannot efficiently exploit the diversity in the time domain, suffering serious performance degradation. To circumvent this issue, there are mainly two categories in the existing methods.

An alternative OFDM signals detection method is widely linear (WL) filtering techniques [1], who exploit the non-circularity of the received data to obtain extra degrees of freedom. Nevertheless, to implement OFDM signals detection, two complicated equalization matrices need to be determined, requiring very high computational complexity. Another popular OFDM signals detection method is the nonlinear equalizers who have decision feedback or ICI cancellation in the procedure of detection, such as MMSE with successive detection (MMSE-SD) [2]. However, when the number of subcarriers in one OFDM symbol increases, their computational loads also increase rapidly. To combat the problem, some papers [3]-[4] reduce computational complexity by ignoring small ICI coefficients in channel matrix. Unfortunately, this will lead to performance degradation. Recently, a space alternating generalized expectation-maximisation (SAGE) technique is applied to detect signals [5]. It has been revealed that it not only has low computational complexity but also has superior detection performance. In time-varying channels, in order to guarantee the reliability of the detection performance, BPSK-OFDM are usually used in communication systems. Nevertheless, the aforementioned approaches neglect the fact that the BPSK-OFDM signals are real-valued. Therefore, a novel signals detection algorithm will be proposed. This can be achieved by forming a novel equalization matrix. It comprises the prior information that the transmitted data is a real-valued vector. After detection, the theoretical analysis and simulation results illustrate the proposed scheme has complexity advantages and significant performance over the existing signal detection methods. In order to further improve the detection performance, SD technology could be introduced in our proposed algorithm. It's success is due to the fact that it could make good use of time diversity in time-varying channels.

The following notations are used throughout the paper. Boldface lowercase and uppercase letters are used for vectors and matrices, respectively. Superscripts T and H denote transpose and conjugate transpose, respectively. The notation $\mathbb{E}(\cdot)$ and $\mathrm{tr}(\cdot)$ are reserved for the expectation operator and the trace of a matrix, respectively. The matrix $\mathbf{F}$ denotes the discrete Fourier transform (DFT) matrix and the matrix $\mathbf{F}^H$ denotes the inverse discrete Fourier transform (IDFT) matrix. Furthermore, $\mathbf{e}_i$ stands for an N-size column vector with the i-th element being one while elsewhere zero.

II. System Model

We consider an OFDM system with N subcarriers. Over time-varying channels, the received signals for each OFDM symbol can be expressed as

$$\mathbf{r} = \mathbf{Hd} + \mathbf{v} \tag{1}$$

where $\mathbf{d} = [d_0, \cdots, d_{N-1}]^T$, $\mathbf{r}$ and $\mathbf{v}$ are the N-point DFTs of the transmitted signal vector, received signal vector and noise vector, respectively. $\mathbf{H}$ is an $N \times N$ channel impulse response matrix in the frequency domain.

It is assumed that $\mathbf{H}$, which can be obtained by pilot-assisted channel estimation methods, is available at the receiver. The principal task of this paper is to detect the signal $\mathbf{d}$ with received $\mathbf{r}$.

III. Our proposed equalizer for BPSK-OFDM systems

In this section, we mainly discuss the linear equalizers which have no decision feedback in signal detection for BPSK-OFDM systems. First, Part A will introduce the traditional MMSE equalizer. However, the prior information that the transmitted data are real-valued is neglected in the procedure of detection, leading to performance degradation. Hence, our modified MMSE (M-MMSE) equalizer, which applies the prior information is proposed and presented in Part B.

A. Traditional MMSE equalizer

It is well known that the MMSE equalizer, which minimizes the total power of the ICI component and Gaussian noise at the output, is usually better than the LS equalizer. Based on MMSE criterion, an equalization matrix is firstly constructed, then the transmitted data $\mathbf{d}$ can be detected by multiplying the received signal $\mathbf{r}$ by the equalization matrix $\mathbf{W}$ [2], that is

$$\hat{\mathbf{d}}_{\text{MMSE}} = \mathbf{W}\mathbf{r} \tag{2}$$

where

$$\mathbf{W} = \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H + \frac{\sigma_{\mathbf{v}}^2}{\sigma_{\mathbf{d}}^2}\mathbf{I}_N\right)^{-1}$$

in which $\sigma_{\mathbf{v}}^2$ and $\sigma_{\mathbf{d}}^2$ denote the variances of the elements of $\mathbf{v}$ and $\mathbf{d}$, respectively. For simplicity but without loss of generality, the power of transmitted signals, $\sigma_{\mathbf{d}}^2$, is assumed to be 1 throughout this paper.

B. M-MMSE equalizer for BPSK-OFDM systems

Being different from that in MQAM-OFDM systems, the transmitted signal $\mathbf{d}$ in BPSK-OFDM systems is not complex-valued but real-valued. According to the information theory, if this prior information can be considered in detection, the system performance will be improved. For this purpose, the relation between the observed data vector and the detected parameter vector should comprise this prior information.

According to (1), the real component and imaginal components of $\mathbf{r}$ can be expressed as

$$\mathbf{r}_{\text{Re}}=\mathbf{H}_{\text{Re}}\mathbf{d}+\mathbf{v}_{\text{Re}} \tag{3}$$

and

$$\mathbf{r}_{\text{Im}}=\mathbf{H}_{\text{Im}}\mathbf{d}+\mathbf{v}_{\text{Im}}, \tag{4}$$

respectively.

In the sequel, the received data in (1) can be repressed as

$$\mathbf{y} = \mathbf{Q}\mathbf{d} + \mathbf{n} \tag{5}$$

with

$$\mathbf{y} = [\ \mathbf{r}_{\text{Re}}{}^T \quad \mathbf{r}_{\text{Im}}{}^T\ ]^T\ ,\ \mathbf{Q} = \left[\ \mathbf{H}_{\text{Re}}{}^T \quad \mathbf{H}_{\text{Im}}{}^T\ \right]^T$$

and

$$\mathbf{n} = \left[\ \mathbf{v}_{\text{Re}}{}^T \quad \mathbf{v}_{\text{Im}}{}^T\ \right]^T.$$

Obviously, (5) has indicated that the transmitted signal $\mathbf{d}$ is a real-valued vector. It's worth noting that (5) is also suitable to other systems with real-valued constellation, such as, MPAM-OFDM and MASK-OFDM. Further, the relation between the detected parameter vector and the new constructed observed vector can be supposed as linear, which is given as

$$\hat{\mathbf{d}}_{\text{M-MMSE}} = \mathbf{G}\mathbf{y} \tag{6}$$

where $G \in \mathbb{R}^{N\times 2N}$ is a new equalizer matrix.

Assuming J represent the cost function of detection and according to MMSE criterion, we have

$$\min J = \min \mathbb{E}\left[\left\|\mathbf{d} - \hat{\mathbf{d}}_{\text{M-MMSE}}\right\|^2\right] = \min_{\mathbf{G}\in\mathbb{R}^{N\times 2N}} \mathbb{E}\left[\|\mathbf{d} - \mathbf{G}\mathbf{y}\|^2\right] \tag{7}$$

By setting the derivative of J with respect to $\mathbf{G}$ to zero, we have

$$\begin{aligned}
&\tfrac{\partial}{\partial \mathbf{G}}\left\{\mathbb{E}\left[(\mathbf{d} - \mathbf{G}\mathbf{y})^T(\mathbf{d} - \mathbf{G}\mathbf{y})\right]\right\}\\
&= \mathbb{E}\left\{\tfrac{\partial}{\partial \mathbf{G}}\left[\mathrm{tr}\left((\mathbf{d} - \mathbf{G}\mathbf{y})(\mathbf{d} - \mathbf{G}\mathbf{y})^T\right)\right]\right\}\\
&= 2\mathbb{E}\left[-\mathbf{d}\mathbf{y}^T + \mathbf{G}\mathbf{y}\mathbf{y}^T\right]\\
&= 0
\end{aligned} \tag{8}$$

Before going on further, we will make the following assumption:

- The transmitted vector $\mathbf{d}$ is uncorrelated with the Gauss noise vector $\mathbf{v}$, i.e., $\mathbb{E}\{\mathbf{d}^T\mathbf{v}\} = 0$.

Then, substituting (5) into (8) yields

$$\begin{aligned}
&\mathbb{E}\left[\mathbf{G}(\mathbf{Q}\mathbf{d} + \mathbf{n})(\mathbf{Q}\mathbf{d} + \mathbf{n})^T\right] - \mathbb{E}\left[\mathbf{d}(\mathbf{Q}\mathbf{d} + \mathbf{n})^T\right]\\
&= \mathbf{G}\left(\mathbf{Q}\mathbb{E}\left[\mathbf{d}\mathbf{d}^T\right]\mathbf{Q}^T + \mathbb{E}\left[\mathbf{n}\mathbf{n}^T\right]\right) - \mathbb{E}\left[\mathbf{d}\mathbf{d}^T\right]\mathbf{Q}^T\\
&= 0
\end{aligned} \tag{9}$$

which leads to the solution for $\mathbf{G}$ given by

$$\mathbf{G} = \mathbf{Q}^T\left(\mathbf{Q}\mathbf{Q}^T + \frac{\sigma_{\mathbf{n}}^2}{\sigma_{\mathbf{d}}^2}\mathbf{I}_{2N}\right)^{-1} \tag{10}$$

where $\sigma_{\mathbf{n}}^2$ denotes the variances of noise.

As $\mathbf{v}$ is AWGN vector, its real component has the same power as its imaginal component, i.e., $\sigma_{\mathbf{n}}^2 = \frac{1}{2}\sigma_{\mathbf{v}}^2$. substituting this into (10), we have

$$\mathbf{G} = \mathbf{Q}^T\left(\mathbf{Q}\mathbf{Q}^T + \frac{\sigma_{\mathbf{v}}^2}{2\sigma_{\mathbf{d}}^2}\mathbf{I}_{2N}\right)^{-1} \tag{11}$$

Since $\mathbf{Q}\mathbf{Q}^T + \frac{\sigma_{\mathbf{v}}^2}{2\sigma_{\mathbf{d}}^2}\mathbf{I}_{2N}$ in (11) is a much big matrix whose size is $2N \times 2N$, the operation of $\left(\mathbf{Q}\mathbf{Q}^T + \frac{\sigma_{\mathbf{v}}^2}{2\sigma_{\mathbf{d}}^2}\mathbf{I}_{2N}\right)^{-1}$ will incur much computational load. In order to circumvent this problem, (11) is rewritten as

$$\mathbf{G} = \left(\mathbf{Q}^T\mathbf{Q} + \frac{\sigma_{\mathbf{v}}^2}{2\sigma_{\mathbf{d}}^2}\mathbf{I}_N\right)^{-1}\mathbf{Q}^T \tag{12}$$

The proof of (12) is seen in Appendix III in [2]. Apparently, $\mathbf{Q}^T\mathbf{Q} + \frac{\sigma_{\mathbf{v}}^2}{2\sigma_{\mathbf{d}}^2}\mathbf{I}_N$ is an $N \times N$ matrix enabling us to determine $\mathbf{G}$ with lower computational requirement than (11).

IV. M-MMSE equalizer with SD technique for BPSK-OFDM systems

It is pointed out in [2] that though the time-varying channels destroy the orthogonality of subcarriers, they provide us with time diversity, especially in rapidly time-varying channels. However, the residual interference and noise enhancement grow as well. In order to make good use of time diversity. A nonlinear equalizer, i.e., MMSE-SD, is proposed and proved that it can achieve better performance of detection. This can be achieved by detecting the transmitted data one-by-one, and eliminating the ICI one-by-one. Similarly, the SD technique also can be adopted in our M-MMSE equalizer, which, for convenience, can be named as M-MMSE-SD. M-MMSE-SD's procedure listed in Algorithm 1.

Algorithm 1 M-MMSE-SD algorithm

Step **1**: Set $i = 0$.

Step **2**: Obtain the M-MMSE equalization matrix by (12).

Step **3**: Find the subcarrier who has the highest post-detection SNR:

$$\mathrm{l}_i = \arg\max_k \mathrm{SINR}_k = \frac{|\langle \mathbf{g}_k, \mathbf{q}_k \rangle|^2}{\sum\limits_{m,m\neq k} |\langle \mathbf{g}_k, \mathbf{q}_m \rangle|^2 + \frac{\sigma_v^2}{2\sigma_{\mathbf{d}}^2}||\mathbf{g}_k||^2} \tag{13}$$

where $\mathbf{g}_k$ and $\mathbf{q}_k$ are the k-th column vector of $\mathbf{G}^T$ and $\mathbf{Q}$, respectively.

Step **4**: Make a soft decision of the l_i-th subcarrier by

$$\tilde{d}_{\mathrm{l}_i} = \mathbf{g}_{\mathrm{l}_i}{}^T \mathbf{y} \tag{14}$$

Step **5**: Obtain the detected data $\hat{d}_{\mathrm{l}_i}$ by making a hard decision on $\tilde{d}_{\mathrm{l}_i}$.

Step **6**: Modified the vector $\mathbf{y}$ by subtracting the terms coming from the contributions of the detected data $\hat{d}_{\mathrm{l}_i}$.

$$\mathbf{y} = \mathbf{y} - \mathbf{q}_{\mathrm{l}_i}\hat{d}_{\mathrm{l}_i} \tag{15}$$

Step **7**: Update the matrix $\mathbf{Q}$ by

$$\mathbf{Q} = [\mathbf{q}_0 \ \cdots \ \mathbf{q}_{\mathrm{l}_i-1} \ \mathbf{0} \ \mathbf{q}_{\mathrm{l}_i+1} \cdots \ \ \mathbf{q}_{N-1}] \tag{16}$$

Step **8**: If $i = N - 1$, set $i = i + 1$ and jump to Step **2**.

V. Complexity Requirement

The computational complexity of the proposed algorithms is evaluated by the number of real multiplications. Assume that the operation of $N \times N$ complex matrix inversion requires $O(N^3)$ complex multiplications and a complex multiplication is equivalent to three real multiplications. For our proposed M-MMSE algorithm, the main complexity is in solving (12). The computation of $\mathbf{Q}^T\mathbf{Q} + \frac{\sigma_v^2}{2\sigma_{\mathbf{d}}^2}\mathbf{I}_N$ in (12) requires $O(2N^3 + N)$ real multiplications. Then, the matrix inversion and matrix multiplication in (12) involve $O(N^3)$, respectively. Therefore, the whole algorithm requires $O(4N^3 + N)$ real multiplications. On the other hand, the number of real multiplication required to calculate each step of our proposed M-MMSE-SD algorithm is listed in Table I. Then, we can easily obtain that this algorithm requires a total of $O(4N^4 + 2N^3 + 8N^2)$ real multiplications.

In other to compare the complexity of different methods, the complexity of our proposed algorithms, LS, MMSE, SAGE and MMSE-SD methods are all presented in Table II. It is concluded that these methods can be ordered in terms of the complexity in an ascending manner as M-MMSE, LS, MMSE, SAGE, M-MMSE-SD , and finally MMSE-SD methods. Apparently, among all the linear equalizers, our proposed M-MMSE algorithm has the lowest computation complexity. For the nonlinear equalizers, though our proposed M-MMSE-SD algorithm is slightly higher than the SAGE method, the simulation results in next section will show that it has better performance of detection, especially in high normalized Doppler frequency.

TABLE I
Computation complexity of each step in M-MMSE-SD.

	Number of real multiplications
Step 1	0
Step 2	$O(4N^3 + N)$
Step 3	$O(2N^2 + 3N)$
Step 4	$O(2N)$
Step 5	0
Step 6	$O(2N)$
Step 7	0
Step 8	0

TABLE II
Computation complexity of each method

	Number of real multiplications
LS	$O(9N^3)$
MMSE	$O(9N^3 + 3N)$
M-MMSE	$O(4N^3 + N)$
SAGE	$O(3N^4 + 3N^3 + 3N^2)$
MMSE-SD	$O(9N^4 + 3N^3 + 15N^2)$
M-MMSE-SD	$O(4N^4 + 2N^3 + 8N^2)$

Note that, in order to further reduce the computational complexity of various detection methods, the channel impulse respond matrix $\mathbf{H}$ can be approximated as a banded matrix. As the same as that in [5], their computational complexity could be easily computed.

VI. Simulation Results

In this section, simulation results are presented to assess the performances of the proposed algorithms. The parameters of the system selected are in concordance with the standard WiMAX IEEE 802.16e. The system operates with a 5 MHz bandwidth and is divided into 512 subcarriers. The length of CP is 64. The scheme given in [6] will be used for generating the time-varying channels. And typical urban (TU) area model which is considered in the COST-207 project [7] is adopted in the channel parameters.The normalized Doppler frequencies

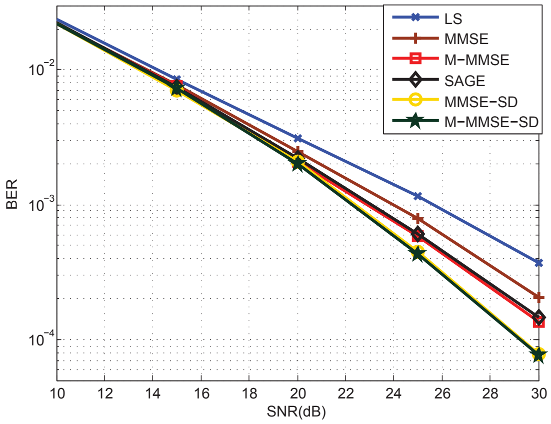

Fig. 1. Performance comparisons of detection algorithms (BPSK-OFDM, f_n = 0.0307).

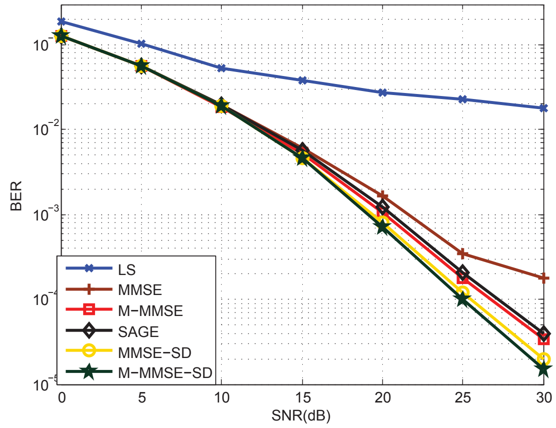

Fig. 2. Performance comparisons of detection algorithms (BPSK-OFDM, f_n = 0.1075).

are set at $f_n = 0.0307$ and $f_n = 0.1075$, corresponding to a mobile object moving with speeds of 120 and 420 km/h, respectively, at a carrier frequency of 2.4 GHz. This velocity can be computed as [3]

$$v = f_n \frac{c}{f_c T}$$

where c, T and f_c represent the speed of light, overall symbol duration and carrier frequency, respectively.

To compare the detection performance of our proposed algorithms with that of the existing algorithms, the BER of BPSK-OFDM is shown as functions of SNR for $f_n = 0.0307$ and $f_n = 0.1075$ in Figs. 1 and 2, respectively. Results are shown for LS, MMSE, SAGE, M-MMSE, MMSE-SD and M-MMSE-SD equalizers. As expected, it is seen that these algorithms can be ordered in terms of the performance in an ascending manner as the LS, MMSE, SAGE, M-MMSE, MMSE-SD and M-MMSE-SD.

It's observed in Fig. 1 that M-MMSE equalizer exhibits a detection gain of about 1.8 dB over the traditional MMSE equalizer at BER=2×10^{-4} for $f_n = 0.0307$. while the M-MMSE-SD equalizer performance is only slightly better than that of the linear equalizers. This is due to small time diversity in slowly time-varying channels. In addition, it is seen in Fig. 2 that the M-MMSE equalizer performance exhibits a detection gain of about 5 dB over the traditional MMSE equalizer at BER=2×10^{-4} for $f_n = 0.1075$. Such big gains are achieved by applying the prior information that the transmitted data are real-valued for BPSK-OFDM systems. The curves in Fig. 2 also illustrate that the nonlinear equalizers perform much better than the traditional linear equalizers. This is because the nonlinear equalizers, such as SAGE, MMSE-SD and M-MMSE-SD, not only minimize the powers of the residual interference and noise, but also make good use of the large time diversity in high normalized Doppler frequency.

VII. Conclusion

In this paper, we propose the novel signals detection algorithms for BPSK-OFDM systems. The M-MMSE algorithm, i.e., a linear equalizer, performs better than the traditional MMSE equalizer with low computational complexity. This is because that the prior information is considered when we construct the equalization matrix. In order to further improve the detection performance, the M-MMSE-SD algorithm is also proposed, which has been proved that it has the best performance in the existing equalizers.

In fact, the M-MMSE and M-MMSE-SD algorithms not only can be applied in BPSK-OFDM systems, but also can be applied in the other systems, such as MPAM-OFDM and MASK-OFDM, who have real-valued constellation.

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