On Rayleigh distance and absorption length of parametric loudspeakers

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Abstract— **This paper investigates the propagation of primary and secondary waves produced by parametric loudspeakers of different sizes. The theory of the parametric acoustic array describes the nonlinear interaction of waves to be confined to the near-field, but the nonlinearities may remain over the far-field, producing different results. Four simulations were done to compare the performance of loudspeakers with different Rayleigh distances. Based on the simulations, a new design was proposed to overcome the mismatching of absorption length and Rayleigh distance. Control over these distances is proposed as an improvement of the performance of the parametric loudspeaker, considering the required scope and proper application.**

I. *INTRODUCTION*

The parametric acoustic array was first described theoretically by Westervelt in 1963 [1]. This theoretical description was followed by experimentations in fluids, then in air, and finally by the application and development of the parametric loudspeaker. Due to an initial poor performance of the parametric array in air, its implementation has been slow, but developments in recent years have led to an increasing interest in new applications for parametric loudspeakers.

When two different frequency waves travel in the same direction, they will generate two new waves; one is the sum of the frequencies, and the other one is the difference frequency. The existence of the generated waves (known as secondary waves) is independent of the initial ones, known as primary waves [1]. Westervelt proposed a particular second order nonlinear acoustic wave equation, which is one of the main models describing the parametric array.

In 1965 Berktay developed a far-field equation [2], based on a quasi-linear solution for Westervelt's equation. In this solution the demodulated difference frequency wave is proportional to the square of the envelope of the original amplitude modulated signal. This analysis will eventually lead to the implementation of a modulation stage in the design of a parametric loudspeaker, as shown in figure 1. The use of modulation generates harmonics, so most of the improvements in this field have been focused on lowering harmonic distortion [3].

In this paper we present a different approach, by directing the attention to the adequate selection of Rayleigh distance and absorption length in the design process of a parametric loudspeaker. Only a few studies have considered before the suitable working distance of a parametric loudspeaker, but focusing on testing the sound quality of already constructed loudspeakers [4,5].

The rest of this paper is structured as follows. Section II describes the theory of the parametric acoustic array, section III describes the simulations, section IV shows the results of the comparisons, and section V the conclusions and future work.

II. *PARAMETRIC ACOUSTIC ARRAY*

Westervelt described the parametric acoustic array by introducing a second-order wave equation [1]:

$$\nabla^2 p_s - \frac{1}{c_0^2}\frac{\partial^2 p_s}{\partial t^2} = -\rho_0 \frac{\partial q}{\partial t} \quad (1)$$

Where ρ_0 is the density of medium, c_0 is the speed of sound, p_s is the pressure of the difference frequency wave and q is the virtual source density of the primary wave of pressure p_i, given by:

$$q = \frac{\beta}{\rho_0^2 c_0^4}\frac{\partial p_i^2}{\partial t} \quad (2)$$

Where β is the nonlinearity term.

A quasi-linear solution is given for the parametric array, including a linear part p_1 and a nonlinear correction p_2:

$$p = p_1 + p_2 \quad (3)$$

Berktay complemented this approach with a far-field solution of the parametric acoustic array [2]:

$$p_2(0, z, \tau) = \frac{\beta p_0^2 a^2}{16\rho_0 c_0^4 z \alpha_0}\frac{\partial^2}{\partial \tau^2}E^2(\tau) \quad (4)$$

Where a is the radius of the source, E is the envelope of the signal generated by the source, α_0 is absorption coefficient of the frequency ω_0 generated and z is the axis of propagation. This equation is known as Berktay's far-field solution. The audible sound wave is proportional to the second derivative of the square of the envelope, and to the square of the radius of the source and initial pressure p_0. Strong absorption is considered, so the nonlinearities should remain only in the near-field.

The Rayleigh distance z_0 describes the transition from the near-field to the far-field [6]:

$$z_0 = \frac{S}{\lambda} \quad (5)$$

Where S is the area of the source and λ is the wavelength of the mean frequency of the two primary waves, ω_0.

Strong absorption is given by [7]:

$$\alpha_0 z_0 \geq 1 \quad (6)$$

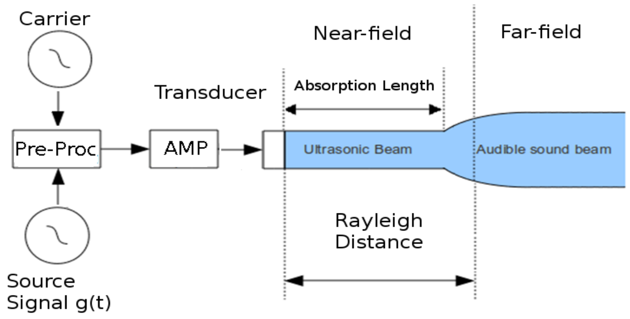

Fig. 1 Diagram of a parametric loudspeaker and its acoustic field.

In the case of the parametric acoustic array, α_T is a combined absorption coefficient that determines the extent of the nonlinear interaction [7]:

$$\alpha_T = \alpha_a + \alpha_b + \alpha_- \tag{7}$$

Where α_a, and α_b are the absorption coefficients of the primary frequencies, and α_- is the absorption coefficient of the difference frequency wave.

Due to the much smaller absorption coefficient of the difference frequency wave compared to the ultrasonic waves, the overall absorption coefficient can be approximated by:

$$\alpha_T = 2\alpha_a = 2\alpha \tag{8}$$

This term can replace the absorption coefficient in equation 6. The absorption length indicates an important drop in the pressure level of the high frequency waves, so it is considered the end of the nonlinear interaction. It is the reciprocal term of the absorption coefficient α_T:

$$L_a = 1/\alpha_T = 1/2\alpha \tag{9}$$

As a quasi-linear approach is used to describe the parametric acoustic array, so shock wave formation should not be present in the array. Strong absorption also establishes the Rayleigh distance to be longer than the absorption length, as shown in figure 1.

III. *EXPERIMENTAL SIMULATIONS*

Several simulations were done to compare different size loudspeakers. All of these simulations were done using, k-wave toolbox [8] over Nesi High-performance computing, to produce the acoustic field.

To test the capabilities of the simulator, the generation of frequencies by a Nicera AS195 loudspeaker was measured. A 2 kHz wave was projected from the loudspeaker, and the

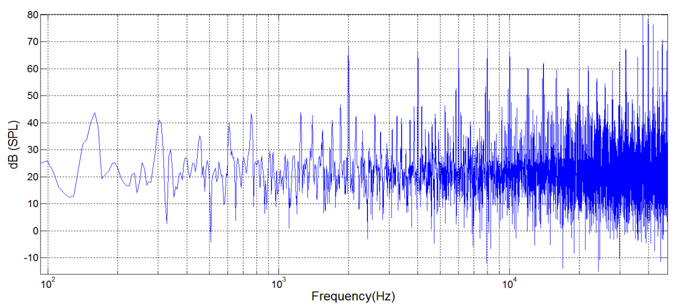

Fig. 2 FFT of the Nicera AS195 at 0 meters.

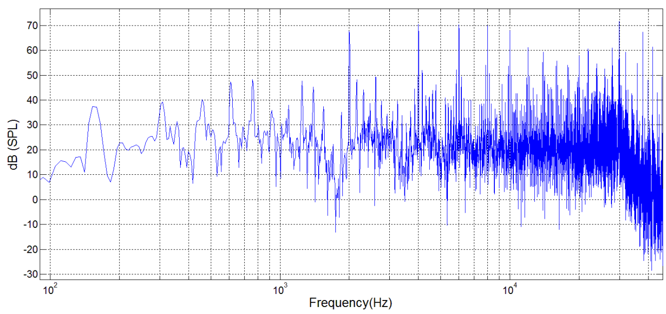

Fig. 3 FFT of the 195 transduces loudspeaker simulated.

acoustic field was measured with a Brüel & Kjær 2670 microphone, in an anechoic chamber.

Figure 2 shows the FFT of the NICERA AS195 measured at 0 meters from the source. Primary waves around 40 kHz are generated by the ultrasonic transducers, but also the demodulated sound wave and its harmonics are present.

Figure 3 shows the FFT generated by the simulator at 1 grid from the source, which is 1.98 mm long. The simulator shows the capability of working up to 50 kHz with no considerable differences with the real measurements. If the audible waves are compared, the results in both graphs are the same. Harmonics of the 2 kHz wave are present with a similar level up to 10 kHz in both cases.

After testing the simulator, four different size loudspeakers were simulated based on NICERA loudspeakers. The loudspeakers had 13, 50, 101 and 195 piezoelectric transducers, as shown in figure 4. In all cases, 2 sine waves of 40 kHz and 42 kHz were the input, with peak pressure of 320 Pascal. No modulation was used in these simulations, because the focus of this research is on the behaviour of the parametric

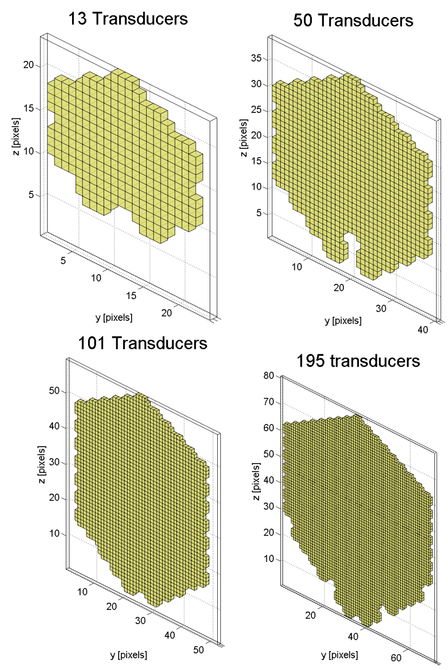

Fig. 4 Loudspeakers simulated of 13, 50, 101 and 195 transducers.

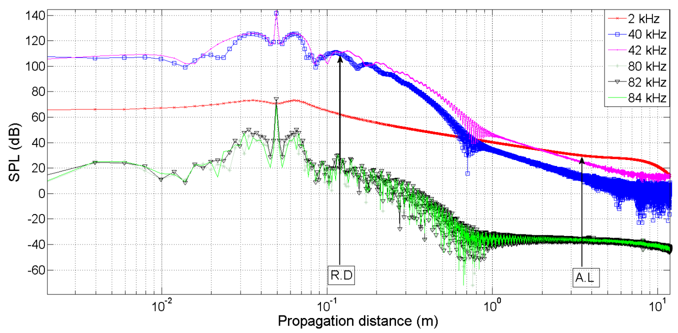

Fig. 5 On-axis sound pressure level of the 13 transducers loudspeaker

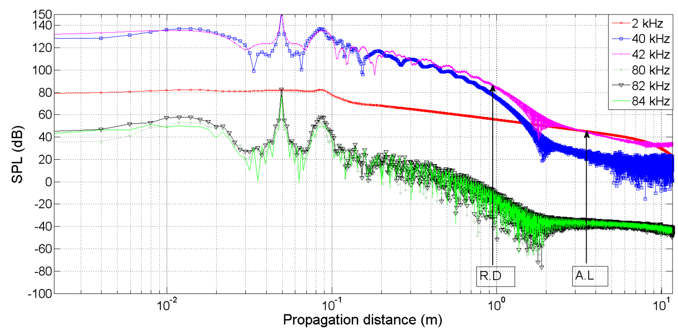

Fig. 7 Axial sound pressure level of the 101 transducers loudspeaker.

acoustic array, and not the generation of other frequencies due to modulation. Only on-axis measures were done in order to observe the changes of the waves along the propagation axis.

The distances of equations 5-9 were calculated. Absorption length is 3.5 m for every simulation, and Rayleigh distances are: 0.12, 0.46, 0.95 and 1.83 meters, for the 13, 50, 101 and 195 transducers loudspeakers, respectively.

IV. *RESULTS*

The axial pressure level graphs of the four loudspeakers simulated are presented in this section. The frequencies considered are the primary frequencies, 40 and 42 kHz, and the secondary waves 2 kHz, 80 kHz, 82 kHz and 84 kHz. Harmonics of the difference frequency wave, like 4 kHz, were not found, because they are produced by modulation.

Figure 5 shows the results of the 13 transducers loudspeaker. The Rayleigh distance is 0.12 meters, and the absorption length is 3.5 meters. In the far-field, and beyond the absorption length, the difference frequency wave has a level of 30 dB, which is higher than the ultrasonic waves.

Figure 6 shows the axial pressure level of the waves produced by the 50 transducers loudspeaker. Rayleigh distance is 0.46 meters this time. The difference frequency wave has a higher pressure level after the absorption length, about 33 dB, but the primary waves have a higher level as well.

Figure 7 shows the axial pressure level of the waves produced by the 101 transducers loudspeaker. The Rayleigh distance is 0.95 meters this time which is still shorter than the absorption length. The pressure level of the difference frequency wave is higher than the previous cases, reaching 45 dB at 3.5 meters from the source, and barely higher than the primary waves in the far-field.

Figure 8 shows the results of the 195 transducers loudspeaker simulated. The Rayleigh distance is 1.83 meters, which is about half the absorption length. The difference frequency wave has a lower level at the absorption length, if compared to the results in figure 5. This is because the 2 kHz wave has a lower level when it reaches the far-field, so all of the results after the Rayleigh distance will be lower than previous simulation.

In all cases the absorption length is longer than the Rayleigh distance. Therefore, the nonlinear generation continues over the far-field, producing an early attenuation of the difference frequency wave. The result is always an attenuated audible wave when the nonlinear interaction ends, by the absorption length.

As a design criteria we propose to match the absorption length and Rayleigh distance, by any of the following two options. The first one is to increase the operation frequency, to lower the absorption length. This option is useful when the parametric loudspeaker is needed in a shorter distance, like in installations at museums or videogames, but it would require a different type of piezoelectric transducer, with a higher resonant frequency. On the other hand, if the area of the loudspeaker is increased, the Rayleigh distance is increased as well. In this case, more piezoelectric transducers would be required, and the loudspeaker would reach longer distances. As the same type of transducers can be used in this case, we will increase the Rayleigh distance.

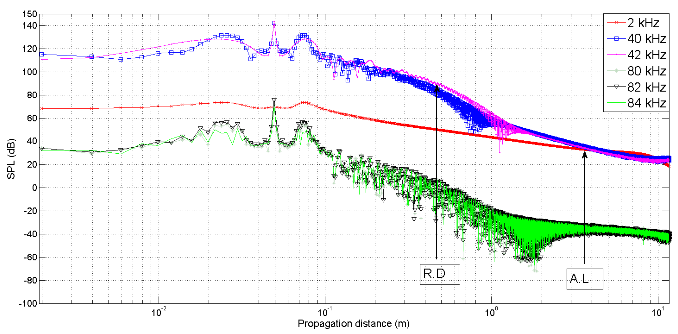

Fig. 6 On-axis sound pressure level of the 50 transducers loudspeaker.

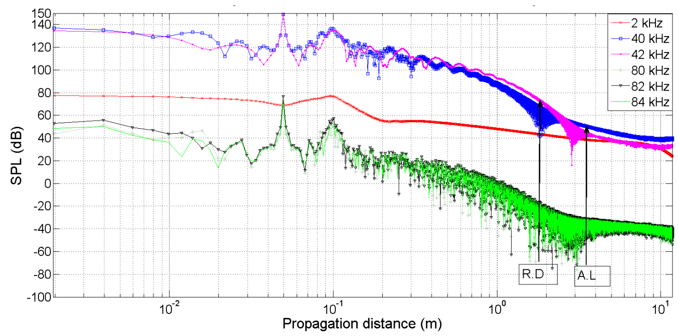

Fig. 8 Axial sound pressure level of the 195 transducers loudspeaker.

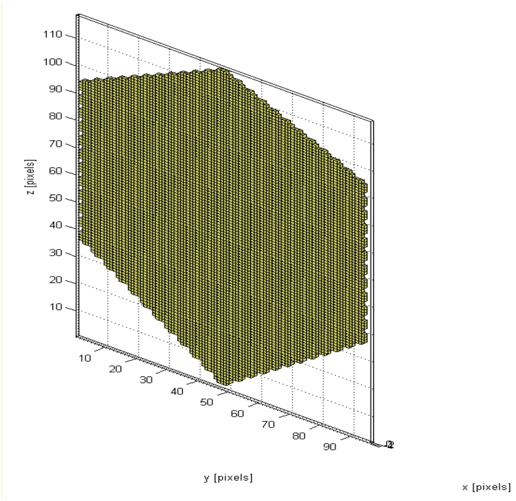

Fig. 9 New loudspeaker simulated, with 397 transducers.

A 397 transducers loudspeaker was simulated, keeping the hexagonal form of the previous loudspeakers, as shown in figure 9. Rayleigh distance is 3.6 meters in this case, slightly exceeding the absorption length previously presented. Finally, figure 10 shows the results of the axial pressure level of this loudspeaker, which present some improvements over the previous results. One of the improvements is the effective attenuation of the primary waves when reaching the far-field, getting a lower level than the difference-frequency wave. A second improvement is an overall higher level of the difference frequency wave, when compared to all of the previous cases. A difference of almost 10 dB is observed at 3.6 meters when compared to figure 5, which has the highest level at this distance.

V. *CONCLUSIONS*

The performances of the 4 first loudspeakers simulated are not optimal with the frequencies and the measures given. We recommend matching the Rayleigh distance with the absorption length to obtain a demodulated audible signal with a higher pressure level in the far-field.

Future designs of parametric loudspeakers should consider extending Rayleigh distance over the absorption length to obtain the best results of the parametric acoustic array in air.

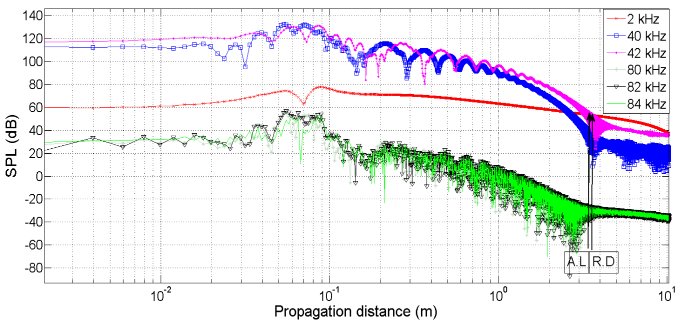

Fig. 10 On-axis sound pressure level of the 397 transducers loudspeaker.

If strong absorption criterion is not abided, then a review of the theory must be done in order to describe accurately the far-field behaviour of the parametric acoustic array in air.

In future works, a spectral analysis and subjective tests will be done to measure the improvements in sound quality provided by the use of this design method.

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